$$\eta^{00} = \eta^{0i}\eta^{0j}\eta_{ij}$$

$$= (-1)(-1)\,\eta_{00} = \eta_{00}$$

$$L^{-1\,m}{}_{j}\, L^{j}{}_{l} = \delta^{m}_{l}$$

$$L^{-1\,m}{}_{j} = \eta_{mk}\, L^{i}{}_{k}\, \eta_{ij}$$

$$= L^{i\,\cancel{k}m} = L_{j}{}^{m}$$

$$L^{-1\,m}{}_{j} = L_{j}{}^{m}$$

$$X'_{i} = \eta_{ij}\, L^{j}{}_{k}\, X^{k}$$

$$= \eta_{ij} \Lambda^j{}_k \eta^{km} X_m$$

$$= \Lambda_i{}^m X_m$$

$$= \Lambda^{-1\,m}{}_i X_m$$

$$X^i y_i = \Lambda^{-1\,m}{}_i y_m \Lambda^i{}_j X^j$$

$$= \Lambda^{-1\,m}{}_i \Lambda^i{}_j y_m X^j$$

$$= \delta^m_j y_m X^j = X^j y_j$$

$$\Lambda^{T\,i}{}_j = \Lambda^j{}_i$$

$$\Lambda^{T\,i}{}_j \neq \Lambda^j{}_i$$

$$p = L^{-1} x' = x$$

$$\frac{\partial}{\partial x^i} = \partial_i$$

$$\frac{\partial}{\partial x'^i} = \frac{\partial x^k}{\partial x'^i} \frac{\partial}{\partial x^k}$$

$$= L^{-1\,k}{}_i \, \partial_k$$

$$M^{\cdots k} \, V_{\cdots k \cdots}$$

$$= M^{\cdots i} \, L^k{}_i \, L^{-1\,j}{}_k \, V_{\cdots j \cdots}$$

$$= M^{\cdots i} \, L^{-1\,j}{}_k \, L^k{}_j \, V_{\cdots i \cdots}$$

$$= M^{i\ldots} \delta^{j}{}_{i} V_{\ldots j \ldots}$$

$$= M^{j\ldots} V_{\ldots j \ldots}$$

$$\sum_{k=0}^{3} {}_{k}{}^{k} \quad \text{is invariant}$$

time $d\tau$ in rest frame

$$d\tau = \sqrt{1 - \frac{v^2}{c^2}}\, dt$$

lab time

$$A'(x) = A(x) + q \partial \cdot \lambda(x$$

local symmetry

$$\psi'_i(x) = e^{iq\Lambda(x)}\,\psi(x)$$

charge

$$\left[(\partial_i + iA_i)\,\psi\right]'$$

$$= \partial_i\left[e^{iq\Lambda}\psi\right]$$

$$+\, iA_i(+\, q\,\partial_i\Lambda\,\psi$$

$$= e^{iq\Lambda}\,\partial_i\psi + \cancel{(i\partial_i(q\Lambda)}\,\psi$$

$\hbar \partial_i A_i e^{iq\lambda} + iq\,\partial_i\lambda\,\psi$

$$= e^{iq\lambda}\left[\partial_i + A_i\right]$$

$$(D_i\psi)' = e^{iq\lambda} D_i\psi$$

$$\psi' = e^{iq\lambda}\psi$$

$$(D_i\psi)^* D_i\psi \quad \text{invariant}$$

$$\psi^*\psi \qquad\qquad ''$$

$$\partial_i A_k = \frac{\partial A_k}{}$$

$$\partial x^j$$

$$\partial^j A_k = \frac{\partial A_k}{\partial x_j}$$

A^i contravariant

∂_i covariant

A_i

∂^i

$$du \wedge dv = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} dx \wedge dy$$