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Symmetry breaking by fermions

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When fermions are coupled sufficiently strongly to spinless fields, the zero-point energies of the fermions can move the minimum of the effective potential away from zero thereby breaking symmetries, including supersymmetry, and generating mass.

The Higgs mechanism is usually viewed as an artificial feature of the standard model because the renormalized squared mass μ^2 of the Higgs boson is contrived to be negative (or zero in the Coleman–Weinberg limit). In a theory containing only bosons and light fermions, this criticism is surely valid; but in the standard model with a heavy top quark, the Higgs mechanism may be much less artificial. For in theories with sufficiently strong Yukawa couplings, the zero-point energies of the fermions can move the minimum of the effective potential away from zero so as to break symmetries and generate masses. In what follows, I shall illustrate this phenomenon by means of two examples, the Higgs mechanism and the massless Wess–Zumino model. I shall argue that the top quark may make the renormalized squared mass μ^2 of the Higgs boson negative and that fermion zero-point energies can break supersymmetry and chiral symmetry.

Before discussing either example, let me remind the reader of a particularly simple effective potential due to Weinberg [1]. Let us consider a theory which may contain spinless fields ϕ_i, fermions ψ_i, and gauge bosons B_i^μ and in which the tree-level potential is $V(\phi)$. If the spinless fields assume classical or mean values $\phi_{i,\mathrm{cl}}$, then the particles of the theory may acquire masses $m_i(\phi_{\mathrm{cl}})$. To lowest order these masses are determined: for the scalar fields, by the eigenvalues of the matrix of second derivatives of the tree-level potential $V(\phi_{\mathrm{cl}})$; for the fermions, by the matrix of Yukawa couplings (if we exclude fixed mass terms); and for the gauge fields, by the eigenvalues of the mass matrix arising from the covariant derivatives of the scalar fields. With these definitions, the one-loop effective potential is the sum of the tree-level potential and the zero-point energies of the various particles,

$$V_{\mathrm{eff}}(\phi_{\mathrm{cl}}) = V(\phi_{\mathrm{cl}}) + \sum_i (-1)^{2j} n_i \cdot \tfrac{1}{2} \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \sqrt{\boldsymbol{k}^2 + m_i^2(\phi_{\mathrm{cl}})}\,, \qquad (1)$$

in which j is the spin and n_i the number of degrees of freedom of particle i, e.g., four for a Dirac fermion, three for a massive vector boson, and two for a massless one.

Let us now use this effective potential to examine symmetry breaking in the case of the Higgs mechanism. Suppose first that there are no fermions in the theory. Then $(-1)^{2j}$ is positive, and so the one-loop contributions to the effective potential $V_{\mathrm{eff}}(\phi_{\mathrm{cl}})$ are all positive. Thus if the tree-level squared masses $m_i^2(\phi_{\mathrm{cl}})$ of the scalar fields are also positive, then the effective potential $V_{\mathrm{eff}}(\phi_{\mathrm{cl}})$ is an increasing function of the classical or mean values $\phi_{i,\mathrm{cl}}$ of the scalar fields. Hence the only way to break the symmetry is to do so by hand: to stipulate that some of the squared masses $m_i^2(\phi_{\mathrm{cl}})$ of the scalar fields (i.e., the coefficients of the quadratic powers of the scalar fields $\phi_{i,\mathrm{cl}}$ in the potential $V(\phi_{\mathrm{cl}})$) are negative. In fact since the changes in the squared masses δm_i^2 due to the one-loop corrections are positive and quadratically diver-

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gent, the squared masses in the potential must be negative and similarly divergent.

Let us now consider the same theory with fermions. In this case the zero-point energies of the fields contribute to the effective potential $V_{\mathrm{eff}}(\phi_{\mathrm{cl}})$ a quadratic polynomial in the scalar fields of the form $\Lambda^2 \sum_i (-1)^{2j} n_i m_i^2(\phi_{\mathrm{cl}})$, where Λ is an ultraviolet cutoff. (There are also quartic terms of the Coleman–Weinberg form $\phi_{\mathrm{cl}}^4[a+b\log(\Lambda^2/\phi_{\mathrm{cl}}^2)]$, but these are numerically irrelevant unless the preceding terms vanish.) Now for $j=\frac{1}{2}$, the sign $(-1)^{2j}$ is negative. Thus if the fermions have sufficiently strong Yukawa couplings, then some of the eigenvalues of the matrix of partial derivatives

$$\frac{\partial^2}{\partial\phi_{j,\mathrm{cl}}\,\partial\phi_{k,\mathrm{cl}}} \sum (-1)^{2j} n_i m_i^2(\phi_{\mathrm{cl}}) \tag{2}$$

will be negative at $\phi_{\mathrm{cl}}=0$, and the symmetry typically will break regardless of the tree-level potential $V(\phi_{\mathrm{cl}})$. In this case the only way to preserve the symmetry is to stipulate that some of the squared masses $m_i^2(\phi_{\mathrm{cl}})$ of the spinless bosons are not only positive but of the order of Λ^2 or larger. In other words, if there are fermions in the theory with sufficiently strong Yukawa couplings, then the only way to preserve the symmetry is to do so by hand.

Let me illustrate this argument by the example of the standard model of the electroweak interactions. For simplicity I will keep track only of the particles that are sufficiently massive to contribute importantly to the effective potential – the top quark t, the W and Z gauge bosons, and the Higgs boson. If $\phi_{\mathrm{cl},0}$ is the mean value of the Higgs field at the minimum of the effective potential, then for these heavy particles the masses that determine the zero-point energies are simply $m_t^2(\phi_{\mathrm{cl}})=(m_t^2/\phi_{\mathrm{cl},0}^\dagger\phi_{\mathrm{cl},0})\phi_{\mathrm{cl}}^\dagger\phi_{\mathrm{cl}}$ for the t, $m_W^2(\phi_{\mathrm{cl}})=(m_W^2/\phi_{\mathrm{cl},0}^\dagger\phi_{\mathrm{cl},0})\phi_{\mathrm{cl}}^\dagger\phi_{\mathrm{cl}}$ for the W, and $m_Z^2(\phi_{\mathrm{cl}})=(m_Z^2/\phi_{\mathrm{cl},0}^\dagger\phi_{\mathrm{cl},0})\phi_{\mathrm{cl}}^\dagger\phi_{\mathrm{cl}}$ for the Z. If the tree-level Higgs potential is $V(\phi_{\mathrm{cl}})=\mu^2\phi_{\mathrm{cl}}^\dagger\phi_{\mathrm{cl}}+\lambda(\phi_{\mathrm{cl}}^\dagger\phi_{\mathrm{cl}})^2$, then the squared mass of the Higgs boson is $m_H^2(\phi_{\mathrm{cl}})=6\lambda\phi_{\mathrm{cl}}^\dagger\phi_{\mathrm{cl}}+\mu^2$. Suppose now that $\mu^2>0$. In this case if the top quark is so heavy that

$$m_t^2 > \tfrac{1}{2}m_W^2+\tfrac{1}{4}m_Z^2+\tfrac{1}{12}m_H^2\,, \tag{3}$$

then the top quark breaks the symmetry, despite the fact that $\mu^2>0$. The threshold for the mass of the top is about 78 GeV/c^2 if the mass of the Higgs is 100 GeV/c^2 (93 GeV/c^2 if $m_H=200$ GeV/c^2). Since the mass of the top exceeds 89 GeV/c^2 [2], it may well be the Yukawa coupling of the top quark that breaks SU(2)⊗U(1) down to U(1). The Higgs mechanism may be less artificial than it seems to be.

The weak point in the preceding argument is that it relies on there being a large ultraviolet contribution to the effective potential from the Higgs sector of the standard model. Since the Higgs sector appears to define a trivial theory [3] (i.e., one in which the renormalized couplings must be sent to zero as the cutoff is removed), such an argument can be only qualitatively suggestive at best. There is, however, some evidence from lattice simulations [4] for the breaking of symmetry by Yukawa interactions.

Whether Yukawa interactions really do break the symmetry of the electroweak interactions, depends upon how the triviality of the theory is repaired. The standard model is actually doubly troubled: both the U(1) sector [5] and the Higgs sector [3] seem to be trivial. One can cure the triviality of the U(1) sector by embedding the U(1) symmetry in a grandly unified theory [6] with a semi-simple gauge group. Now a positive beta function can cause triviality [7]. But in a supersymmetric Yang–Mills theory, both the Yukawa coupling constant and the quartic coupling constant of the spinless fields are simply related to the gauge coupling constant, and this gauge-boson coupling constant is asymptotically free if there are not too many kinds of fermions [8]. Thus by making the standard model supersymmetric, one can presumably fix the triviality of the Higgs sector. Logical consistency may require the extension of standard model to a SUSY-GUT.

Because the example of the Higgs mechanism is tained with quadratic ultraviolet divergences, let us consider a non-trivial theory with Yukawa interactions, the massless Wess–Zumino model [9]. As we shall see, the zero-point energies of the fermions can break both the chiral symmetry and the supersymmetry of this model and can generate masses for all the particles of the theory.

The Lagrange density of the massless Wess–Zumino model is

$$\begin{aligned}\mathscr{L}=&\tfrac{1}{2}\partial_\mu A\,\partial^\mu A+\tfrac{1}{2}\partial_\mu B\,\partial^\mu B+\tfrac{1}{2}\bar\psi\mathrm{i}\gamma^\mu\,\partial_\mu\psi\\&-g\bar\psi(A+\mathrm{i}\gamma_5 B)\psi-V(A,B)\,,\end{aligned} \tag{4}$$

where $V(A,B)$ is the potential

$$V(A,B)=\tfrac{1}{2}g^2(A^2+B^2)^2 . \quad (5)$$

Apart from the supersymmetry, the theory has a global chiral symmetry under the transformations $\psi'=e^{i\alpha\gamma_5/2}\psi$, $A'=\cos\alpha A+\sin\alpha B$, and $B'=-\sin\alpha A+\cos\alpha B$. This potential has a unique minimum: $(A, B)=(0,0)$ which preserves supersymmetry and the chiral symmetry. At this minimum, the scalar field A, the pseudoscalar field B, and the Majorana fermion ψ are all massless.

For this model the Weinberg effective potential [1] is

$$V_{\text{eff}}(A,B)=V(A,B)+\sum(-1)^{2j}(2j+1) \times\tfrac{1}{2}\int\frac{d^3k}{(2\pi)^3}\sqrt{k^2+m_j^2(A,B)} , \quad (6)$$

in which the sum is over the two bosons and the fermion. The tree-level squared masses are for the two bosons

$$m_{0,\pm}^2(A,B)=\tfrac{1}{2}[V(A,B)_{AA}+V(A,B)_{BB} \pm\{[V(A,B)_{AA}-V(A,B)_{BB}]^2+V(A,B)_{AB}^2\}^{1/2}] =2g^2[2(A^2+B^2)\pm(A^4-A^2B^2+B^4)^{1/2}] , \quad (7)$$

in which subscripts denote differentiation, and for the fermion

$$m_{1/2}^2(A,B)=4g^2(A^2+B^2) . \quad (8)$$

Due to the equality of the number of bosonic and fermionic degrees of freedom, the quartic divergences in the effective potential cancel for all values of A and B. Since the masses $m_{0,\pm}^2(A,B)$ and $m_{1/2}^2(A,B)$ satisfy the Ferrara–Girardello–Palumbo mass formula [10]

$$\sum(-1)^{2j}(2j+1)m_j^2(A,B)=0 , \quad (9)$$

the quadratic divergences in the effective potential also cancel for all values of A and B, leaving only a logarithmic divergence. If we cut off the momentum integrals at $|\boldsymbol{k}|=\Lambda$, then the effective potential assumes the form

$$V_{\text{eff}}(A,B)=V(A,B) +\sum(-1)^{2j}\frac{(2j+1)m_j^4(A,B)}{128\pi^2} \times\left(1-2\log\frac{[\Lambda+\sqrt{\Lambda^2+m_j^2(A,B)}\,]^2}{m_j^2(A,B)}\right) \quad (10)$$

plus terms of the order of $m_j^8(A,B)/\Lambda^4$.

After letting the cutoff $\Lambda=1$ TeV and the coupling constant $g=2$, I found numerically that the effective potential $V_{\text{eff}}(A,B)$ has four minima: $(A,B)=(\pm a_1, 0)$ and $(A,B)=(0,\pm a_1)$, where $a_1\approx 1.042$ GeV. At these minima $V_{\text{eff}}(A,B)\approx -0.119$ GeV^4. The mass of the fermion to lowest order is $m_{1/2}\approx 4.2$ GeV, and the masses of the bosons as given by the eigenvalues of the matrix of second partial derivatives of the potential $V(A,B)$ are $m_{0,+}\approx 5.1$ GeV and $m_{0,-}\approx 2.9$ GeV. Thus in the massless Wess–Zumino model, the fermion zero-point energies give masses to the particles and break both the supersymmetry and the chiral symmetry. For the massive Wess–Zumino model, the story is much the same if the mass is not too great, except that the chiral symmetry is not present to begin with.

One may also dimensionally regularize the divergent integrals in the effective potential (6) by changing the three-dimensional integration over d^3k to an integration over d^dk and introducing a mass scale μ and a dimensional coupling constant $g(d)=\mu^{(3-d)/2}g$. With $d=2.85$, $\mu=1.64$ GeV$/c^2$, and $g=2$, I found the same masses for the spinless particles and for the fermion as with a momentum cutoff Λ of 1 TeV.

How does the present discussion differ from the conventional ones? In renormalized perturbation theory, one divides the effective potential into a prescribed finite part and some infinite counterterms. As one computes the radiative corrections, one adjusts the infinite counterterms so that they cancel the divergences of the radiative corrections and preserve the finite part in accordance with the renormalization conditions. Thus one retains the net rather than the gross radiative corrections. By keeping the cutoff finite, I have separated the gross radiative corrections, which are the whole contributions of the high-energy physics, from the canceling counterterms. This retention of the gross radiative corrections is the main difference between the present discussion and the usual treatments.

The net and gross contributions can be qualitatively different. Thus, for example, the gross contribution of a particle of spin j to the effective potential is of the form

$$(-1)^{2j}\{a^2\Lambda^4+b^2\Lambda^2\phi^2+c^2\phi^4[1-d^2\log(\Lambda^2/\phi^2)]\}, \tag{11}$$

in which a^2, b^2, c^2, and d^2 are all positive. If one sends the cutoff to infinity, interpreting the tree-level potential as a finite part plus counterterms, then after canceling the divergent parts of the radiative correction and satisfying the renormalization conditions, one is left with the net contribution

$$(-1)^{2j}c^2d^2\phi^4\log(\phi^2/M^2), \tag{12}$$

where M is a mass scale. For $\phi^2<M^2<\Lambda^2$, the gross contribution of the high-energy physics to the effective potential tends to force the field ϕ toward zero if $2j$ is even and away from zero if $2j$ is odd; the net contribution has the opposite effects.

This difference between gross and net radiative corrections bears on the relationship between the present discussion and the classic paper on the effective potential by Coleman and Weinberg [11]. They computed the effective potential $V(\phi)$ for a scalar field ϕ coupled to scalar fields, gauge fields, and fermions. They showed that in one-loop order these fields contribute to the effective potential $V(\phi)$ terms of the form $(-1)^{2j}e^4\phi^4\log\phi^2/M^2$ where j is the spin of the field, e is the coupling constant (except for self-coupled scalar fields for which $e=\sqrt{\lambda}$), and M is a mass scale. They concluded that for theories in which the renormalized effective potential has zero curvature at the origin, $V''(0)=0$, the gauge fields can cause the Higgs field ϕ to assume a non-zero mean value in the vacuum, but that fermions tend to suppress such symmetry breaking. Yet I found that fermions can stimulate symmetry breaking by making the renormalized coefficient ϕ^2 negative, $V''(0)\leqslant 0$. The resolution of this paradox is that Coleman and Weinberg chose the counterterms of the lagrangian so as to keep the renormalized coefficient of ϕ^2 equal to zero, $V''(0)=0$; while I allowed $V''(0)$ to be determined by the bare parameters of the lagrangian and by the dynamics. There is no contradiction because we were in effect talking about theories with different bare parameters. They were talking about the net radiative corrections, while I have been emphasizing the gross ones.

Conclusions: If particles acquire masses dynamically, then the zero-point energies of these particles depend on their masses and therefore influence the choice of the vacuum state. In models in which fermions are sufficiently strongly coupled to spinless bosons, the zero-point energies of the fermions can force some of the bosons to assume non-zero mean values in the vacuum, thereby generating fermion masses and making the vacuum asymmetric. Equivalently, in theories with substantial Yukawa couplings, fermion loops can make the effective, renormalized coefficient of the square of a scalar field negative $\mu^2<0$, even if the corresponding bare or tree-level coefficient is positive. If the top quark is heavy enough, its loops can drive symmetry breaking in the standard model. Thus the Higgs mechanism may be less artificial than it seems. Fermion zero-point energies can also break supersymmetry and chiral symmetry in the massless Wess–Zumino model.

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