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STATIC FORCES IN NONCOMPACT SU(2)

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Wilson loops have been measured at strong coupling, $\beta = 0.5$, on a 12^4 lattice in noncompact simulations of pure SU(2) without gauge fixing. In the loops that have been well measured, there is no sign of quark confinement.

In 1980 Creutz [1] displayed quark confinement at moderate coupling in lattice simulations [2] of both abelian and nonabelian gauge theories. Whether nonabelian confinement is as much an artifact of Wilson's action as is abelian confinement remains unclear.

Wilson's basic variables are elements of a compact group and enter the action only through the traces of their products. His action has extra minima [3]. Mack and Pietarinen [4] and Grady [5] have shown that these false vacua affect the string tension. In their simulations of SU(2), they placed gauge-invariant infinite potential barriers between the true vacuum and the false vacua. Mack and Pietarinen saw a sharp drop in the string tension; Grady found that it vanished.

To avoid using an action that has confinement built in, some physicists have introduced lattice actions that are noncompact discretizations of the continuum action with fields as the basic variables [3, 6–11]. For U(1) these noncompact formulations are accurate for general coupling strengths [8]; for SU(2) they agree well with perturbation theory at very weak coupling [9].

This report relates the results of measuring Wilson loops at strong coupling, $\beta \equiv 4/g^2 = 0.5$, on a 12^4 lattice in a noncompact simulation of SU(2) gauge theory without gauge fixing or fermions. Creutz ratios of large Wilson loops provide a lattice estimate of the $q\bar{q}$-force for heavy quarks. There is no sign of quark confinement in the loops that have been measured precisely. The force at six lattice spacings is stronger than at five, but the statistics are not sufficient to evaluate this signal.

Patrascioiu, Seiler, Stamatescu, Wolff, and Zwanziger [6] performed the first noncompact simulations of SU(2) by using discretizations of the classical action. They fixed the gauge and saw a force rather like Coulomb's.

In the present simulations, the action is free of spurious zero modes, and it is not necessary to fix the gauge. The fields are constant on the links of length a, the lattice spacing, but are interpolated linearly throughout the plaquettes. In the plaquette with vertices n, $n+e_\mu$, $n+e_\nu$, and $n+e_\mu+e_\nu$, the field is

$$A^a_\mu(x) = (\frac{x_\nu}{a} - n_\nu)A^a_\mu(n+e_\nu) + (n_\nu + 1 - \frac{x_\nu}{a})A^a_\mu(n), \qquad (1)$$

and the field strength is

$$F^a_{\mu\nu}(x) = \partial_\nu A^a_\mu(x) - \partial_\mu A^a_\nu(x) + g f^a_{bc} A^b_\mu(x) A^c_\nu(x). \qquad (2)$$

The action S is the sum over all plaquettes of the integral over each plaquette of the squared field strength,

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$$S = \sum_{P_{\mu\nu}} \frac{a^2}{2} \int dx_\mu dx_\nu F^c_{\mu\nu}(x)^2. \quad (3)$$

The mean-value in the vacuum of a euclidean-time-ordered operator $Q(A)$ is approximated by a normalized multiple integral over the $A^a_\mu(n)$'s

$$\langle \mathcal{T} Q(A) \rangle_0 \approx \frac{\int e^{-S(A)} Q(A) \prod_{\mu,a,n} dA^a_\mu(n)}{\int e^{-S(A)} \prod_{\mu,a,n} dA^a_\mu(n)} \quad (4)$$

which one may compute numerically [1]. I used Macsyma to write the Fortran code [10].

The quantity normally used to study confinement in quarkless gauge theories is the Wilson loop, $W(r,t)$, which is the mean-value in the vacuum of the path-and-time-ordered exponential

$$W(r,t) = \frac{1}{d} \langle \mathcal{P}\mathcal{T} e^{-ig \oint A^a_\mu T_a dx_\mu} \rangle_0 \quad (5)$$

divided by the dimension d of the matrices T_a that represent the generators of the gauge group. Although Wilson loops vanish [12] in the exact theory, Creutz ratios $\chi(r,t)$ of Wilson loops defined [1] as double differences of logarithms of Wilson loops

$$\chi(r,t) = -\log W(r,t) - \log W(r-a,t-a) + \log W(r-a,t) + \log W(r,t-a) \quad (6)$$

are finite. For large t, $\chi(r,t)$ approximates (a^2 times) the force between a quark and an antiquark separated by the distance r.

For a compact Lie group with N generators T_a normalized as $\mathrm{Tr}(t_a t_b) = k\delta_{a,b}$, the lowest-order perturbative formula for the Creutz ratio is

$$\chi(r,t) = \frac{N}{2\pi^2\beta}[-f(r,t) - f(r-a,t-a) + f(r,t-a) + f(r-a,t)] \quad (7)$$

where the function $f(r,t)$ is

$$f(r,t) = \frac{r}{t}\arctan\frac{r}{t} + \frac{t}{r}\arctan\frac{t}{r} - \log\left(\frac{a^2}{r^2} + \frac{a^2}{t^2}\right) \quad (8)$$

and β is the inverse coupling $\beta = d/(kg^2)$.

To measure Wilson loops and their Creutz ratios $\chi(r,t)$, I used a 12^4 periodic lattice, a heat bath, and ten independent runs with cold starts. The first run began with 25,000 thermalizing sweeps at $\beta = 2$ followed by 5000 at $\beta = 0.5$; the other nine runs began at $\beta = 0.5$ with 20,000 thermalizing sweeps. In all I made 29,750 measurements, separating successive measurements by twenty sweeps and using a version of Parisi's trick [13] that respects the dependencies in the corners of the loops. The values of the Creutz ratios so obtained are listed in the table along with the theoretical values given by the formulas (7–8). I estimated the errors by the jackknife method [14], assuming that all measurements were independent. Binning in small groups made little difference.

Noncompact Creutz ratios at $\beta = 0.5$

$\frac{r}{a} \times \frac{t}{a}$	Monte Carlo	Order $1/\beta$
2×2	0.23111(5)	0.39648
3×3	0.03567(15)	0.13092
4×4	0.00494(41)	0.06529
5×5	0.00165(117)	0.03913
6×6	0.00298(322)	0.02608

If the static force between heavy quarks is independent of distance, then the Creutz ratios $\chi(r,t)$ for large t should be independent of r and t. In the loops that are well measured there is no sign of confinement. The measured $\chi(r,t)$'s are smaller than their tree-level perturbative values (7–8). The ratio $\chi(6a,6a)$ is bigger than $\chi(5a,5a)$. But the error in $\chi(6a,6a)$ is huge, and the large value of $\chi(6a,6a)$ may be a transient.

Why don't noncompact simulations display quark confinement? Here are some answers:

Noncompact methods lack an exact lattice gauge invariance. They have approximate forms of all continuum symmetries, including gauge invariance, which they respect more at weak coupling than at strong. So noncompact methods may not be sufficiently accurate at strong coupling and may accommodate too small a volume at weak coupling. The noncompact lattice spacing $a_{NC}(\beta)$ is probably smaller than the compact one $a_C(\beta)$. Thus confinement might appear in noncompact simulations done on much larger lattices or at stronger coupling. Both possibilities would be expensive to test.

Perhaps SU(3), but not SU(2), confines.

Possibly as Gribov has suggested [15], pure SU(3) does not confine, confinement being a feature only of QCD with light quarks.

Perhaps the textbook quantization of QCD, which this noncompact method emulates, does not correctly implement Gauss's law. Polonyi has argued [16] that Gauss's law should be enforced by an integration over the group manifold weighted by the Haar measure, rather than by the usual integration over copies of the real line. In a preliminary test of these ideas, I measured the Polyakov line of length $12a$ at $\beta = 0.5$ to be 0.00853(67) with the Haar measure which is to be compared with 0.01131(3) without it.

Confinement is a robust and striking phenomenon. Maybe the true continuum theory is one like Wilson's that can directly account for it. The hybrid measure

$$e^{-\int DCf(C,L)\,\mathrm{Tr}(1-\Re\mathcal{P}e^{ig\oint_C A_\mu^a T_a dx_\mu})/(kg^2)}d\mu(A) \quad (9)$$

reduces to Wilson's prescription if the weight functional $f(C,L)$ of the path integration over closed curves C is a delta functional with support on the plaquettes and if $d\mu(A)$ incorporates the Haar measure. A weight functional like $f(C,L) \sim \exp[-(||C||/L)^4]$ where $||C||$ is the length of the curve might give confinement for distances much longer than L and perturbative QCD for much shorter distances.

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