Statistical Enhancement of Gauge Invariance (*).

K. CAHILL and P. DENES

Department of Physics and Astronomy
University of New Mexico - Albuquerque, N.M. 87131

(ricevuto il 2 Dicembre 1981)

Summary. – It is suggested that theories may acquire apparent gauge invariance from statistical effects.

The four fundamental interactions of Nature seem to be described by theories possessing local gauge symmetry. Various explanations for this prevalence of local symmetry may be offered. One explanation follows from the application of the principle of relativity to both internal and external co-ordinate systems. A second explanation is that the gauge invariance of contemporary physics is a reflection of the redundancy of its description of Nature. A more fundamental theory presumably would require fewer variables.

The purpose of the present paper is to suggest a statistical mechanism that may give the appearance of gauge invariance to some theories that are not fully gauge invariant. We have in mind theories whose gauge invariance is spoiled by interactions with fields φ that have not yet been detected, due perhaps to their large masses or weak couplings. Processes for which the spoiler fields φ vanish possess gauge copies of equal action. They therefore enter the Feynman path integral with a statistical weight equal to the (infinite) number of their gauge copies. Processes for which the spoiler fields do not vanish enter with unit statistical weight. In this way the generating functional for the observed fields, A_μ etc., may have the same form as if the underlying gauge invariance were not broken by the spoiler fields. This restoration of gauge invariance is not dependent at all upon the masses or couplings of the spoiler fields (1). It is rather that gauge invariance itself seems to have a certain inherent resiliency.

We shall illustrate this statistical mechanism by means of an example in which the photon field A_μ is coupled to a scalar field φ and to a conserved external current j_μ. The system is described by the Euclidean Lagrange density

$$\mathscr{L} = \tfrac{1}{4}F_{\mu\nu}^2 + \tfrac{1}{2}(\partial_\nu\varphi)^2 + \tfrac{1}{2}m^2\varphi^2 + \mu\varphi\eta_\nu A_\nu + j_\nu A_\nu\,,$$

(*) Research supported in part by the U.S. Department of Energy under contract DE-AC04-81ER40042.

(1) The proposed statistical suppression of the spoiler fields would tend to reduce their experimental accessibility.

where η_ν stands for ∂_ν or $(0, \nabla)$ or a constant vector. We shall see that the generating functional for the response of the photon field to the external current j_μ

$$Z(j) = \int \mathrm{D}A_\mu \, \mathrm{D}\varphi \exp\left[-\int \mathrm{d}^4x \, \mathscr{L}\right] = \int \mathrm{D}A_\mu \, \mathrm{D}\varphi \exp\left[-\int \mathrm{d}^4x[\tfrac{1}{4}F_{\mu\nu}^2 + \tfrac{1}{2}\varphi(-\partial^2 + m^2)\varphi + \mu\varphi\eta\cdot A + j\cdot A]\right]$$

recovers from the breakdown of gauge invariance induced by the spoiler field φ (2).

By shifting φ to $\varphi - \mu K^{-1}\eta\cdot A$, where $K = -\partial^2 + m^2$, we may write the integral over φ as

$$\int \mathrm{D}\varphi \exp\left[-\tfrac{1}{2}\int \mathrm{d}^4x \, \varphi(x)K\varphi(x) + \tfrac{1}{2}\mu^2\int \mathrm{d}^4x \, \mathrm{d}^4x' \, \eta\cdot A(x)K^{-1}(x, x')\eta\cdot A(x')\right].$$

The first term is a constant C

$$C = \int \mathrm{D}\varphi \exp\left[-\tfrac{1}{2}\int \mathrm{d}^4x \, \varphi K\varphi\right],$$

so the functional Z becomes

$$Z(j) = C\int \mathrm{D}A_\mu \exp\left[-\int \mathrm{d}^4x(\tfrac{1}{4}F_{\mu\nu}^2 + j\cdot A) + \tfrac{1}{2}\mu^2\int \mathrm{d}^4x \, \mathrm{d}^4x' \, \eta\cdot A(x)K^{-1}(x, x')\eta\cdot A(x')\right].$$

The integral over A_μ is simpler in momentum space. We put

$$A_\mu(x) = \int \mathrm{d}^4k(2\pi)^{-4}\alpha_\mu(k)\exp[-ikx],$$

where the reality of A_μ is ensured by the condition $\alpha_\mu^*(k) = \alpha_\mu(-k)$. With a similar definition for $\tilde{\jmath}^\mu(k)$, the functional Z is

$$Z(j) = C\int \mathrm{D}\alpha_\mu \exp\left[-\int \mathrm{d}^4k(2\pi)^{-4}(\tfrac{1}{2}\alpha_\mu^* M_{\mu\nu}\alpha_\nu + \tilde{\jmath}_\nu^*\alpha_\nu)\right],$$

where

$$M_{\mu\nu}(k) = \delta_{\mu\nu}k^2 - k_\mu k_\nu - \mu^2(k^2 + m^2)^{-1}\tilde{\eta}_\mu\tilde{\eta}_\nu.$$

Here $\tilde{\eta}_\mu$ is k_μ if $\eta_\mu = \partial_\mu$, $(0, \boldsymbol{k})$ if $\eta = (0, \nabla)$, or η_μ if η_μ is a constant vector. The integration over α_μ is performed by shifting α_μ to $\alpha_\mu - M_{\mu\nu}^{-1}j_\nu$, where the inverse of $M_{\mu\nu}$ is

$$M_{\mu\nu}^{-1} = \frac{\delta_{\mu\nu}}{k^2} - \left[\frac{\mu^{-2}(k^2 + m^2) - k^{-2}\tilde{\eta}^2}{(\tilde{\eta}\cdot k)^2}\right]k_\mu k_\nu - \frac{k_\mu\tilde{\eta}_\nu + \tilde{\eta}_\mu k_\nu}{k^2\tilde{\eta}\cdot k}.$$

One then finds for the functional Z

$$Z(j) = C'\exp\left[\tfrac{1}{2}\int \mathrm{d}^4k(2\pi)^{-4}\tilde{\jmath}_\mu M_{\mu\nu}^{-1}\tilde{\jmath}_\nu^*\right],$$

where C' is the constant

$$C' = C\int \mathrm{D}\alpha_\mu \exp\left[-\tfrac{1}{2}\int \mathrm{d}^4k(2\pi)^{-4}\alpha_\mu^* M_{\mu\nu}\alpha_\nu\right].$$

---

(2) The recovery would be equally complete if the breakdown of gauge invariance due to φ were augmented by the replacement of $\frac{1}{4}F_{\mu\nu}^2$ by $\frac{1}{2}(\partial_\mu A_\nu)^2$.

All the terms in $M^{-1}_{\mu\nu}$, except the first. contain at least one factor of k_μ. They do not contribute to the product $\tilde{j}_\mu M^{-1}_{\mu\nu}\tilde{j}^*_\mu$, therefore, since $k_\mu \tilde{j}_\mu = 0$ because of current conservation. This the generating functional Z is

$$Z(j) = C' \exp\left[\tfrac{1}{2}\int \mathrm{d}^4k(2\pi)^{-4} j^*_\mu j_\mu/k^2\right],$$

which is also the result ([3]) of the gauge-invariant theory in which

$$\mathscr{L} = \tfrac{1}{4}F^2_{\mu\nu} + j_\nu A_\nu .$$

* * *

We are grateful to the U.S. Department of Energy for its support of this work through contract DE-AC04-81ER40042.

([3]) R. J. GLAUBER: *Phys. Rev.*, **84**, 395 (1951).