An extension of the standard model in which parity is conserved at high energies

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Abstract

To be compatible with general relativity, every fundamental theory should be invariant under general coordinate transformations including spatial reflection. This paper describes an extension of the standard model in which the action is invariant under spatial reflection, and the vacuum spontaneously breaks parity by giving a mean value to a pseudoscalar field. This field and the scalar Higgs field make the gauge bosons, the known fermions, and a set of mirror fermions suitably massive while avoiding flavor-changing neutral currents. In the model, there is no strong-CP problem, there are no anomalies, fermion number (quark-plus-lepton number) is conserved, and heavy mirror fermions form heavy neutral mirror atoms which are dark-matter candidates. In models with extended gauge groups, nucleons slowly decay into pions, leptons, and neutrinos.

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I. INTRODUCTION

Invariance under general coordinate transformations is the defining principle of general relativity. Although other theories of gravity do exist [1], I will assume in this paper that particle physics is compatible with general relativity and has an action that is invariant under general coordinate transformations and in particular under reflection of the spatial coordinates. The paper describes an extension of the standard model in which the action is invariant under spatial reflection, and the vacuum spontaneously breaks parity by giving a nonzero mean value to a pseudoscalar field. This field and the Higgs field make the gauge bosons, the known fermions, and a set of mirror fermions suitably massive while avoiding flavor-changing neutral currents. In the model, gauge fields act on four-component Dirac fields, there is no strong-CP problem, there are no anomalies, fermion number (quark-plus-lepton number) is conserved, and heavy mirror fermions form heavy neutral mirror atoms which are dark-matter candidates. In simple grandly unified extensions of the model, nucleons slowly decay in processes such as $p \to \pi^+ + 3\nu$, $p \to e^+ + 4\nu$, and $n \to 3\nu$ that conserve $Q + L$ but break $B - L$.

In the standard model, the strong and electromagnetic gauge bosons act on four-component Dirac spinors, but those of $SU_w(2)$ act on two-component left-handed spinors. This awkward feature breaks parity and general-coordinate invariance at the level of the action. Actions that break parity invite anomalies, admit the pseudoscalar term $\epsilon^{\mu\nu\sigma\tau} F^a_{\mu\nu} F^a_{\sigma\tau}$, and have a strong-$CP$ problem. And in space-times of even spatial dimensions, rotational invariance implies invariance under reflection of the spatial coordinates, and in space-times of more than three spatial dimensions, rotational invariance implies invariance under reflection of the three spatial coordinates we know about. Most string theories are not chiral [2].

The left-handedness of the weak interactions led Pati and Salam [3], Georgi and Glashow [4], and their many followers to use two-component left-handed fermion fields exclusively in their theories of grand unification. They represented right-handed fermions as left-handed antifermions, put quarks and antiquarks in the same multiplet, and shifted the focus of particle physics higher in energy by 12 orders of magnitude.

A theory whose action conserves parity with gauge fields acting on four-component spinors becomes invariant under general coordinate transformations when suitably decorated with tetrads and Christoffel symbols; it is free of anomalies and avoids the problem of strong-CP

violation. In the model of this paper and in its natural grand unifications, fermion number F or the number of quarks plus the number of leptons

$$F = Q + L \tag{1}$$

is conserved. The model has primary and secondary fermions. The light, known fermions and the heavy, mirror fermions are linear combinations of the primary and secondary fermions. Heavy mirror fermions form heavy, neutral mirror atoms which are dark-matter candidates. The CP-breaking phases of the heavy fermion sector may be enough to explain the excess of matter over antimatter. These are some of the advantages of theories in which parity is spontaneously broken and gauge fields act on four-component Dirac (or Majorana) spinors.

At energies exceeding those of the heavy mirror fermions, parity is restored in these models. The lower limits on heavy quarks are 735 GeV for $t_3 = \frac{1}{2}$ and 755 GeV for $t_3 = -\frac{1}{2}$ [5]. The lower limits on heavy charged and neutral leptons respectively are 101.9 GeV and 80.5–101.5 GeV [6]. The restoration of parity may occur at tens of TeV.

The model avoids flavor-changing neutral currents because the 3×3 Yukawa matrices that couple the scalar and pseudoscalar fields to the fermions have singular-value decompositions that differ only in their singular values. For instance, the matrices that give masses to the three generations of $t_3 = 1/2$ quarks and mirror quarks $U^u \Sigma_h^u V^{u\dagger}$ and $U^u \Sigma_p^u V^{u\dagger}$ differ only in the singular values $x_j \geq 0$ and $y_j \geq 0$ of the diagonal 3×3 matrices Σ_h^u and Σ_p^u. If the mirror fermions have masses of 1 TeV, then all the Yukawa coupling constants x and y are between 3 and 5. The full action is invariant under two global $U(1) \otimes U(1)$ symmetries which block mass terms like $\bar{u}_i u_i$ and so forth.

The model implies the existence of heavy mirror fermions with interactions much like those of the known fermions, but with right-handed weak interactions. They would form very heavy positive nuclei surrounded by shells of heavy mirror electrons with $m_{e'} \geq 100.8$ GeV. These mirror atoms would be very small with Bohr radii less than $(\alpha m_{e'})^{-1} \sim 0.27$ fm—and less than 0.027 fm if $m_e' \geq 1$ TeV. The energy needed to excite these atoms would be of the order of $m_{e'} \alpha^2$, so these atoms would interact only with photons of at least an MeV. These atoms are candidates for dark matter. Because their masses would exceed a TeV, their number density would be 100 times lower than that of a 10 GeV WIMP. This low number density may be why physicists have not detected dark matter even though its mass density is 5.4 times greater than that of ordinary matter [7]. The quarks of these putative dark-matter particles interact via QCD, but their interactions are of very short range because of the high masses of the exchanged heavy pions. Theories in which dark matter consists of stealthy strongly interacting particles [8] or strongly interacting massive particles (SIMPs) [9] have been developed. Strongly interacting dark matter [10] broadens dark-matter cusps into cores [11] as suggested by some observations of galaxies in clusters [12] and of stars in nearby galaxies [13] and so may explain the apparent paucity of heavy dwarf galaxies around the Milky Way [14].

The mirror-fermion trick that makes theories that conserve parity appear chiral at low energies was invented by physicists trying to define chiral gauge theories on the lattice [15]. They doubled the number of fermion fields. A primary set of fermions $\psi = (\psi_\ell; \psi_r)$ transforms under $G \supseteq SU_c(3) \otimes SU(2) \otimes U(1)$, and a secondary set of fermions $\psi' = (\psi'_\ell; \psi'_r)$ transforms under $G' \supseteq SU_c(3) \otimes U(1)$. When certain spinless fields assume suitable mean values in the vacuum, the light fermions are $\psi_m \simeq (\psi_\ell; \psi'_r)$ and have left-handed charged-current weak interactions, while the heavy fermions are $\psi_M \simeq (\psi'_\ell; \psi_r)$ and have right-handed charged-current weak interactions. At low energies the theory looks chiral. Many physicists have used this mirror-fermion trick. Anber, Aydemir, Donohue, and Pais [16] used it to make a model in which the fermions have vector-like gauge interactions but chiral Yukawa interactions. Other physicists [17] have used the trick to add vector-like fermions to the standard model in various ways, but few have discussed the spontaneous breaking of parity, perhaps because Vafa and Witten showed [18] that "in parity-conserving vector-like theories such as QCD, parity conservation is not spontaneously broken," a no-go theorem that does not, however, apply to theories with Yukawa interactions [18, 19]. Among the few are Aoki and Gocksch who exhibited the spontaneous breakdown of parity in lattice simulations with Wilson fermions [20].

In models with extended gauge groups, nucleons slowly decay in processes such as $p \to \pi^+ + 3\nu$, $p \to e^+ + 4\nu$, and $n \to 3\nu$ that involve the exchange of three heavy gauge bosons. These decays conserve fermion number $F = Q + L$ but violate baryon number minus lepton number $B - L$. Their partial lifetimes rise with the 12th power of the heavy mass scale M of the mediating gauge bosons $\tau_n \sim M^{12}/(\alpha_u^6 m_p^{13})$ in which α_u is the fine-structure constant of the unified theory. The lower bounds on such partial lifetimes are 4.9×10^{26} years for $n \to 3\nu$ [21], 5.8×10^{29} years for $n \to$ invisible [22], and 2.1×10^{29} years for $p \to$ invisible [23], so the masses of the mediating gauge bosons should exceed about a PeV. Such nucleon-decay events may lurk in SNO, KamLAND, Super-Kamiokande, and JUNO data. The residual excited nucleus $^{15}N^*$ or $^{15}O^*$ emits a γ ray of 6–7 MeV [23]. The PeV energy scale is 9 or 10 orders of magnitude lower than that of traditional grand unification.

Section II outlines how Utiyama and Kibble made theories with fermions compatible with general relativity. Section III shows how a pseudoscalar field can make a vector theory look chiral at low energies. Section IV describes a model for a single generation of quarks and leptons. A model for three generations is described in section V. Models of grand unification with extended gauge groups are briefly sketched in Section VI. The paper ends with a summary in section VII.

II. SPIN-ONE-HALF FIELDS IN GENERAL RELATIVITY

Decades ago, Utiyama [24] and Kibble [25] showed how to fit spin-one-half fields into general relativity. Suppose the flat-space action density is

$$L = -\overline{\psi} \left[\gamma^a \left(\partial_a + i g A_a \right) + m \right] \psi \tag{2}$$

in which a is a flat-space index, A is a matrix of gauge fields, ψ is a four-component Dirac or Majorana field, $\overline{\psi} = \psi^\dagger \beta = i \psi^\dagger \gamma^0$, and m is a constant or a mean value of a scalar field. One first introduces tetrad fields $e_a{}^\mu(x)$ that turn flat-space indices γ^a into curved-space indices $\gamma^a e_a{}^\mu$. Derivatives and gauge fields intrinsically are generally covariant vectors. So the first step is to replace $\gamma^a \left(\partial_a + i g A_a \right)$ by $\gamma^a \, e_a{}^\mu \left(\partial_\mu + i g A_\mu \right)$. The next step is to correct for the effect of the derivative on the field ψ by making the derivative generally covariant as well as gauge covariant. The required Einstein connection is

$$E_\mu = \frac{1}{2} \sigma^{ab} \, e_a{}^\nu \, e_{b\nu;\mu} \tag{3}$$

in which the σ^{ab} are the 4×4 matrices

$$\sigma^{ab} = \frac{1}{4} \left[\gamma^a, \gamma^b \right] . \tag{4}$$

The covariant derivative $e_{b\nu;\mu}$ of the tetrad is

$$e_{b\nu;\mu} = e_{b\nu,\mu} - e_{b\sigma} \, \Gamma^\sigma_{\nu\mu} \tag{5}$$

in which the comma denotes ordinary derivatives, and $\Gamma^\sigma_{\nu\mu} = e^\sigma{}_a e^a{}_{\mu,\nu}$ is the Levi-Civita affine connection. The resulting action density [26]

$$\mathcal{L} = \, - \, \overline{\psi} \left[\gamma^a \, e_a{}^\mu \left(\partial_\mu + i g A_\mu + E_\mu \right) \right] \psi \tag{6}$$

is invariant $\mathcal{L}(x) \to \mathcal{L}(x')$ under any coordinate transformation $x \to x'$ that is one-to-one and differentiable. In particular, it is invariant under the parity transformation $Px = x' = (x^0, -\boldsymbol{x})$ which takes the field $\psi(x)$ to

$$\mathsf{P}\,\psi(x)\,\mathsf{P}^{-1} = \eta^* \beta \psi(Px) \tag{7}$$

in which η is the intrinsic parity of the particle [27].

One may use this Utiyama-Kibble recipe and the usual covariant derivatives of general relativity to make the action of any of the models of this paper invariant under general coordinate transformations.

III. HOW TO MAKE A VECTOR THEORY LOOK CHIRAL

Most applications of the mirror-fermion trick use two or more scalar fields [15–17] and have Yukawa interactions that explicitly break parity. The models of this paper use the Higgs scalar field and one pseudoscalar field and have actions that conserve parity.

In this section and the next one, the scalar field h and a pseudoscalar field p are $SU(2) \otimes U_Y(1)$ doublets

$$h = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix} \quad \text{and} \quad p = \begin{pmatrix} p^+ \\ p^0 \end{pmatrix} \tag{8}$$

in which $h^0 = (\bar{h} + h_r + ih_i)/\sqrt{2}$ and $p^0 = (\bar{p} + p_r + ip_i)/\sqrt{2}$. To keep things simple, we first consider an $SU_w(2) \otimes U_Y(1)$ doublet $q = (u, d)$ of primary quarks of a given color and two secondary quarks u' and d' of the same color that are singlets under $SU_w(2)$ but transform under $U_Y(1)$

$$q = \begin{pmatrix} u \\ d \end{pmatrix}, \quad u', \quad d'. \tag{9}$$

All the spin-one-half fields of this paper are four-component Dirac fields. In the models of this section and the next, the Yukawa interactions of the primary quarks $q = (u, d)$ and the secondary down quark d' with h and p are

$$\begin{aligned}
V_d &= \bar{q}\left(x_d h + y_d \gamma^5 p\right) d' + \bar{d}'\left(x_d^* h^\dagger - y_d^* \gamma^5 p^\dagger\right) q \\
&= u_r^\dagger\left(x_d h^+ + y_d \gamma^5 p^+\right) d'_\ell + u_\ell^\dagger\left(x_d h^+ + y_d \gamma^5 p^+\right) d'_r \\
&\quad + d_r^\dagger\left(x_d h^0 + y_d \gamma^5 p^0\right) d'_\ell + d_\ell^\dagger\left(x_d h^0 + y_d \gamma^5 p^0\right) d'_r \\
&\quad + d_r'^\dagger\left(x_d^* h^- - y_d^* \gamma^5 p^-\right) u_\ell + d_\ell'^\dagger\left(x_d^* h^- - y_d^* \gamma^5 p^-\right) u_r \\
&\quad + d_r'^\dagger\left(x_d^* h^{0\dagger} - y_d^* \gamma^5 p^{0\dagger}\right) d_\ell + d_\ell'^\dagger\left(x_d^* h^{0\dagger} - y_d^* \gamma^5 p^{0\dagger}\right) d_r .
\end{aligned} \tag{10}$$

They are invariant under space reflection. Here $d^\dagger = (d_\ell^\dagger, d_r^\dagger)$ and $\overline{d} = i d^\dagger \gamma^0 = d^\dagger \beta = (d_r^\dagger, d_\ell^\dagger)$. The Dirac matrices obey $\{\gamma^a, \gamma^b\} = 2\eta^{ab}$ with $\eta^{00} = -1$ and $\gamma_5 = \gamma^5 = (1, 0; 0, -1)$.

The Yukawa interaction (10) also respects both the $U(1) \otimes U(1)$ symmetry

$$\begin{aligned}
& q \to e^{i\theta}\gamma^5 q \quad \text{and} \quad d' \to e^{i(\theta-\chi)} d' \\
& h \to -e^{i\chi} p \quad \text{and} \quad p \to -e^{i\chi} h
\end{aligned} \tag{11}$$

which leaves $\bar{q} \not{D} q$, $\bar{d}' \not{D} d'$, and $\bar{d}' d'$ but not $\bar{q} q$ invariant and the $U(1) \otimes U(1)$ symmetry

$$\begin{aligned}
& q \to e^{i\theta} q \quad \text{and} \quad d' \to e^{i(\theta-\chi)}\gamma^5 d' \\
& h \to e^{i\chi} p \quad \text{and} \quad p \to e^{i\chi} h
\end{aligned} \tag{12}$$

which leaves $\bar{q} \not{D} q$, $\bar{d}' \not{D} d'$, and $\bar{q} q$ but not $\bar{d}' d'$ invariant. These global symmetries protect fermion masses by keeping mass terms like $m \bar{q} q$ and $m' \bar{d}' d'$ out of the action.

If we replace the fields h and p in the Yukawa interaction (10) by their mean values $(0, h_0)$ and $(0, p_0)$ in the vacuum, where $h_0 = \langle 0|h^0|0\rangle = \bar{h}/\sqrt{2}$ and $p_0 = \langle 0|p^0|0\rangle = \bar{p}/\sqrt{2}$, then the Yukawa interaction (10) yields the mass terms

$$\begin{aligned}
V_{d0} &= \bar{d}\left(x_d h_0 + \gamma^5 y_d p_0\right) d' + \bar{d}'\left(x_0^* h_0^* - \gamma^5 y_0^* p_0^*\right) d \\
&= d_r^\dagger (x_d h_0 + y_d p_0) d'_\ell + d_\ell^\dagger (x_0 h_0 - y_0 p_0) d'_r + d_r'^\dagger (x_0^* h_0^* - y_0^* p_0^*) d_\ell + d_\ell'^\dagger (x_0^* h_0^* + y_0^* p_0^*) d_r \\
&= \begin{pmatrix} d_\ell^\dagger & d_\ell'^\dagger \end{pmatrix} \begin{pmatrix} x_d h_0 - y_d p_0 & 0 \\ 0 & x_d^* h_0^* + y_d^* p_0^* \end{pmatrix} \begin{pmatrix} d'_r \\ d_r \end{pmatrix} + \text{ h.c.}
\end{aligned} \tag{13}$$

The main self-interactions of the spinless bosons are

$$V(h,p) = \lambda_h \left(h^\dagger h - \frac{v^2}{4}\right)^2 + \lambda_p \left(p^\dagger p - \frac{v^2}{4}\right)^2 - \lambda_{hp}\left(h^\dagger p + p^\dagger h\right)^2 \tag{14}$$

where $v = v_{\rm sm} = 246$ GeV and $\lambda_h = 0.129$. The third term in V has $0 < \lambda_{hp} \ll 1$ so that the mean values of h^0 and p^0 have the same phase (taken to be zero) or the opposite phase in which case the light fermions would have right-handed weak interactions. If $V(h,p)$ were the exact potential of the spinless fields, the mean values of the neutral components of the doublets h and p would be

$$h_0 = p_0 = \frac{v}{2} = 123\,\text{GeV}. \tag{15}$$

For simplicity, I will assume that this is the case and will set

$$|h_0|^2 + |p_0|^2 = \frac{v^2}{2} = \frac{(246)^2}{2}\ \text{GeV}^2. \tag{16}$$

The field

$$d_{\rm light} = \begin{pmatrix} d_\ell \\ d'_r \end{pmatrix} \tag{17}$$

then has left-handed $SU(2)$ interactions and a light mass $m_d = |x_d h_0 - y_d p_0| = 123\,|x_d - y_d|$ GeV. Similarly, the field

$$d_{\rm heavy} = \begin{pmatrix} d'_\ell \\ d_r \end{pmatrix} \tag{18}$$

has right-handed $SU(2)$ interactions and a larger mass $M_d = |x_d h_0 + y_d p_0| = 123\,|x_d + y_d|$ GeV, which can be as heavy as 3 TeV with $|x_d| \le 4\pi$ and $|y_d| \le 4\pi$. Thus a theory whose action conserves parity can look chiral at low energies. If we take y_d and z_d to be positive, then the Yukawa coefficients are

$$x_d = \frac{M_d + m_d}{v} \quad \text{and} \quad y_d = \frac{M_d - m_d}{v}. \tag{19}$$

Since $M_d \gtrsim 700$ GeV while $m_d = 4.8$ MeV, these Yukawa coefficients are nearly equal, $x_d \approx y_d \gtrsim 2.8$, because $M_d \gg m_d$ and because I set $h_0 = p_0$.

Before passing to a more complete model, let's note what happens when a scalar field s replaces the pseudoscalar field p. The minus sign in our Yukawa interaction (10) arose because γ^0 and γ^5 anticommute, so the minus sign goes away, and instead of V_d we have

$$V_{ds} = \bar{q}\,(x_d h + y_d s)\, d' + \bar{d}'\,(x_d^* h^\dagger + y_d^* s^\dagger)\, q \tag{20}$$

in which x_d and y_d are coupling constants. If the neutral components of the scalar fields h and s assume the mean values h_0 and s_0 in the vacuum, then the mass terms of V_{ds} are

$$V_{ds0} = \begin{pmatrix} d_\ell^\dagger & d_\ell'^\dagger \end{pmatrix} \begin{pmatrix} x_d h_0 + y_d p_0 & 0 \\ 0 & x_d^* h_0^* + y_d^* p_0^* \end{pmatrix} \begin{pmatrix} d'_r \\ d_r \end{pmatrix} + \text{ h.c.} \tag{21}$$

The vacuum values h_0 and s_0 give the same mass to (d_ℓ, d'_r) and (d'_ℓ, d_r) and so do not break parity spontaneously.

IV. A MODEL FOR ONE GENERATION

In the proposed model, the vacuum breaks parity by giving a mean value to a new pseudoscalar field p which makes the mirror-fermion trick of section III work. The action of the model is invariant under general coordinate transformations when suitably decorated with tetrads as in section II. The model has a secondary fermion for each (primary) fermion of the standard model. A gauge group $G \supseteq SU_c(3) \otimes SU_w(2) \otimes U_Y(1)$ acts on the four-component primary fermions, and a group $G' \supseteq SU_c(3) \otimes U_Y(1)$ acts on the secondary fermions.

	$q = \begin{pmatrix} u \\ d \end{pmatrix}$	$\ell = \begin{pmatrix} \nu \\ e \end{pmatrix}$	u'	d'	ν'	e'	$h = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix}$	$p = \begin{pmatrix} p^+ \\ p^0 \end{pmatrix}$
isospin t	2	2	1	1	1	1	2	2
hypercharge y	$\frac{1}{6}$	$-\frac{1}{2}$	$\frac{2}{3}$	$-\frac{1}{3}$	0	-1	$\frac{1}{2}$	$\frac{1}{2}$
color c	3	1	3	3	1	1	1	1

TABLE I. One generation of fermions and the scalar h and pseudoscalar p fields of the simplest model. The integers t and c are the dimension of the representation of $SU_w(2)$ and of $SU_c(3)$.

In the model, the pseudoscalar field p is a doublet that transforms under $SU_w(2) \otimes U_Y(1)$ like the Higgs doublet. So for $g \in SU_w(2) \otimes U_Y(1)$ and $g' \in U_Y(1)$, the fields transform as $q \to gq$, $\ell \to g\ell$, $h \to gh$, $p \to gp$, $u' \to g'u'$, $d' \to g'd'$, $\nu' \to g'\nu'$, and $e' \to g'e'$. The covariant derivatives of the primary quark and lepton doublets of the first generation are

$$\begin{aligned} D_\mu q &= \left(\partial_\mu + igT_a A^a_\mu + ig'VB_\mu\right) q = \left(\partial_\mu + ig\frac{\sigma_a}{2}A^a_\mu + ig'\frac{1}{6}B_\mu\right) q \\ D_\mu \ell &= \left(\partial_\mu + igT_a A^a_\mu + ig'VB_\mu\right) \ell = \left(\partial_\mu + ig\frac{\sigma_a}{2}A^a_\mu - ig'\frac{1}{2}B_\mu\right) \ell, \end{aligned} \tag{22}$$

while those of the secondary quarks u' and d' and leptons ν' and e' are

$$\begin{aligned} D_\mu u' &= (\partial_\mu + ig' V B_\mu)\, u' = \left(\partial_\mu + ig'\frac{2}{3} B_\mu\right) u' \\ D_\mu d' &= (\partial_\mu + ig' V B_\mu)\, d' = \left(\partial_\mu - ig'\frac{1}{3} B_\mu\right) d' \\ D_\mu \nu' &= (\partial_\mu + ig' V B_\mu)\, \nu' = \partial_\mu \nu' \\ D_\mu e' &= (\partial_\mu + ig' V B_\mu)\, e' = (\partial_\mu - ig' B_\mu)\, e'. \end{aligned} \tag{23}$$

The first-generation Yukawa terms are

$$\begin{aligned} V = {} & \bar{q}\, i\tau_2 (x_u h^* + y_u \gamma^5 p^*) u' - \bar{u}' (x_u^* h^\intercal - y_u^* \gamma^5 p^\intercal)\, i\tau_2 q \\ & + \bar{q}(x_d h + y_d \gamma^5 p) d' + \bar{d}' (x_d^* h^\dagger - y_d^* \gamma^5 p^\dagger) q \\ & + \bar{\ell}\, i\tau_2 (x_\nu h^* + y_\nu \gamma^5 p^*) \nu' - \bar{\nu}' (x_\nu^* h^\intercal - y_\nu^* \gamma^5 p^\intercal)\, i\tau_2 \ell \\ & + \bar{\ell}(x_e h + y_e \gamma^5 p) e' + \bar{e}' (x_e^* h^\dagger - y_e^* \gamma^5 p^\dagger) \ell \end{aligned} \tag{24}$$

in which τ_2 is the second Pauli matrix. The Yukawa terms (24) and the kinetic terms $\bar{q} \not{D} q$, $\bar{\ell} \not{D} \ell$, $\bar{u}' \not{D} u'$, $\bar{d}' \not{D} d'$, $\bar{\nu}' \not{D} \nu'$, and $\bar{e}' \not{D} e'$ in the action of the model are invariant under the gauged symmetry $G \otimes G'$ and under the two global $U(1) \otimes U(1)$ symmetries

$$\begin{aligned} & q \to e^{i\theta} \gamma^5 q \quad \text{and} \quad \ell \to e^{i\theta} \gamma^5 \ell \\ & u' \to e^{i(\theta+\chi)} u' \quad \text{and} \quad \nu' \to e^{i(\theta+\chi)} \nu' \\ & d' \to -\, e^{i(\theta-\chi)} d' \quad \text{and} \quad e' \to -e^{i(\theta-\chi)} e' \\ & h \to -\, e^{i\chi} p \quad \text{and} \quad p \to -e^{i\chi} h \end{aligned} \tag{25}$$

and

$$\begin{aligned} & q \to e^{i\theta} q \quad \text{and} \quad \ell \to e^{i\theta} \ell \\ & u' \to e^{i(\theta+\chi)} \gamma^5 u' \quad \text{and} \quad \nu' \to e^{i(\theta+\chi)} \gamma^5 \nu' \\ & d' \to e^{i(\theta-\chi)} \gamma^5 d' \quad \text{and} \quad e' \to e^{i(\theta-\chi)} \gamma^5 e' \\ & h \to e^{i\chi} p \quad \text{and} \quad p \to e^{i\chi} h. \end{aligned} \tag{26}$$

But the global symmetry (25) changes $\bar{u}u$, ..., $\bar{e}e$, and the other global symmetry (26) changes $\bar{u}'u'$, ..., $\bar{e}'e'$. So these global symmetries keep mass terms like $m_u \bar{u}u$, $m_{u'} \bar{u}'u'$, and so forth out of an action that consists only of the Yukawa interaction (24), the kinetic terms $\bar{q} \not{D} q$, and so forth.

Replacing the spinless bosons in the Yukawa interaction (24) by their mean values in the vacuum and grouping fields of the same handedness together, we get

$$\begin{aligned}
V_0 = {} & \begin{pmatrix} u_\ell^\dagger & u_\ell'^\dagger \end{pmatrix} \begin{pmatrix} x_u^* h_0^* - y_u^* p_0^* & 0 \\ 0 & x_u^* h_0 + y_u^* p_0 \end{pmatrix} \begin{pmatrix} u_r' \\ u_r \end{pmatrix} \\
& + \begin{pmatrix} d_\ell^\dagger & d_\ell'^\dagger \end{pmatrix} \begin{pmatrix} x_d h_0 - y_d p_0 & 0 \\ 0 & x_d^* h_0^* + y_d^* p_0^*) \end{pmatrix} \begin{pmatrix} d_r' \\ d_r \end{pmatrix} \\
& + \begin{pmatrix} \nu_\ell^\dagger & \nu_\ell'^\dagger \end{pmatrix} \begin{pmatrix} x_{\nu_e} h_0^* - y_{\nu_e}^* p_0^*) & 0 \\ 0 & x_{\nu_e}^* h_0 + y_{\nu_e}^* p_0) \end{pmatrix} \begin{pmatrix} \nu_r' \\ \nu_r \end{pmatrix} \\
& + \begin{pmatrix} e_\ell^\dagger & e_\ell'^\dagger \end{pmatrix} \begin{pmatrix} x_e h_0 - y_e p_0) & 0 \\ 0 & x_e^* h_0^* + y_e^* p_0^*) \end{pmatrix} \begin{pmatrix} e_r' \\ e_r \end{pmatrix} + \text{ h.c.}
\end{aligned} \tag{27}$$

We set

$$|h_0|^2 + |p_0|^2 = \tfrac{1}{2}v^2 = \tfrac{1}{2}(246 \text{ GeV})^2 \tag{28}$$

and keep $h_0 = p_0 = 123$ GeV as in (15). The light-mass fields of the first generation are

$$u_m = \begin{pmatrix} u_\ell \\ u_r' \end{pmatrix}, \quad d_m = \begin{pmatrix} d_\ell \\ d_r' \end{pmatrix}, \quad \nu_m = \begin{pmatrix} \nu_\ell \\ \nu_r' \end{pmatrix}, \quad e_m = \begin{pmatrix} e_\ell \\ e_r' \end{pmatrix}. \tag{29}$$

Their masses are

$$\begin{aligned}
m_u &= |x_u h_0 - y_u p_0| \qquad & m_d &= |x_d h_0 - y_d p_0| \\
m_{\nu_e} &= |x_{\nu_e} h_0 - y_{\nu_e} p_0| \qquad & m_e &= |x_e h_0 - y_e p_0|.
\end{aligned} \tag{30}$$

The heavy-mass fields are

$$u_M = \begin{pmatrix} u_\ell' \\ u_r \end{pmatrix}, \quad d_M = \begin{pmatrix} d_\ell' \\ d_r \end{pmatrix}, \quad \nu_M = \begin{pmatrix} \nu_\ell' \\ \nu_r \end{pmatrix}, \quad e_M = \begin{pmatrix} e_\ell' \\ e_r \end{pmatrix}. \tag{31}$$

Their masses are

$$\begin{aligned}
M_u &= |x_u h_0 + y_u p_0| \qquad & M_d &= |x_d h_0 + y_d p_0| \\
M_{\nu_e} &= |x_{\nu_e} h_0 + y_{\nu_e} p_0| \qquad & M_e &= |x_e h_0 + y_e p_0|.
\end{aligned} \tag{32}$$

The Yukawa coefficients are

$$x_u = \frac{M_u + m_u}{2h_0} \quad \text{and} \quad y_u = \frac{M_u - m_u}{2p_0} \tag{33}$$

with similar formulas for d, ν_e, and e. So we assume the equality (15) of the mean values h_0 and p_0, then the x's and the y's are nearly equal and less than 4π. For instance, if $M_u = 750$ GeV, then $x_u = 750/246 = 3.05$ and $y_u \approx x_u$.

With $L = (1+\gamma^5)/2$ and $R = (1-\gamma^5)/2$, the light and heavy quark doublets are

$$q_m = Lq + Rq' \quad \text{and} \quad q_M = Rq + Lq'. \tag{34}$$

Thus $Lq = Lq_m$ and $Rq = Rq_M$, so that $q = (L+R)q = Lq_m + Rq_M$. The covariant derivative $\left(\partial_\mu + \frac{ig}{2}\sigma_a A^a_\mu + \frac{ig'}{6}B_\mu\right) q$ of the primary quarks (22) therefore acts on the left-handed light quarks Lq_m and on the right-handed heavy quarks Rq_M

$$\bar{q}\not{D}q = \overline{(Lq_m + Rq_M)}\,\not{D}(Lq_m + Rq_M) = \overline{Lq_m}\,\not{D}Lq_m + \overline{Rq_M}\,\not{D}Rq_M \tag{35}$$

which accordingly interact with all the $SU_w(2) \otimes U_Y(1)$ gauge bosons. An analogous rule applies to the primary leptons.

Similarly, $Rq' = Rq_m$ and $Lq' = Lq_M$, so that $q' = Rq_m + Lq_M$. The covariant derivative $\left(\partial_\mu + \frac{2ig'}{3}B_\mu\right)$ of the secondary up quark (23) therefore acts on the right-handed light up quark Ru_m and on the left-handed heavy up quark LU_M

$$\bar{u}'\not{D}u' = \overline{(Ru_m + Lu_M)}\,\not{D}(Ru_m + Lu_M) = \overline{Ru_m}\,\not{D}Ru_m + \overline{Lu_M}\,\not{D}Lu_M \tag{36}$$

which accordingly interacts only with the $U_Y(1)$ gauge boson. Analogous rules apply to the other secondary fermions.

To see if we can keep the W and Z gauge bosons and the photon at their physical masses, we note that the covariant derivative acting on h and on p, which have $y = \frac{1}{2}$, is

$$D_\mu = \partial_\mu + igT_a A^a_\mu + ig'VB_\mu = \partial_\mu + ig\frac{\sigma_a}{2}A^a_\mu + ig'\frac{1}{2}B_\mu. \tag{37}$$

The mean value in the vacuum of $D_\mu h$ is

$$\langle D_\mu h\rangle_0 = \left(ig\frac{\sigma_a}{2}A^a_\mu + ig'\frac{1}{2}B_\mu\right)\begin{pmatrix}0\\ h_0\end{pmatrix} = i\frac{h_0}{2}\begin{pmatrix}g(A^1_\mu - iA^2_\mu)\\ -gA^3_\mu + g'B_\mu\end{pmatrix}, \tag{38}$$

and that of $D_\mu p$ is

$$\langle D_\mu p\rangle_0 = \left(ig\frac{\sigma_a}{2}A^a_\mu + ig'\frac{1}{2}B_\mu\right)\begin{pmatrix}0\\ p_0\end{pmatrix} = i\frac{p_0}{2}\begin{pmatrix}g(A^1_\mu - iA^2_\mu)\\ -gA^3_\mu + g'B_\mu\end{pmatrix}. \tag{39}$$

The mass terms of the gauge bosons are

$$\langle\,(D_\mu h)^\dagger D^\mu h\,\rangle_0 + \langle\,(D_\mu p)^\dagger D^\mu p\,\rangle_0 = \frac{|h_0|^2 + |p_0|^2}{4}\Big[g^2\left(A^1_\mu A^{1\mu} + A^2_\mu A^{2\mu}\right) + \left(-gA^3_\mu + g' B_\mu\right)\left(-gA^{3\mu} + g' B^\mu\right)\Big]. \tag{40}$$

The photon $g' A^3_\mu + g B_\mu$ is massless, and the $W^\pm$ and Z

$$W^\pm_\mu = \frac{A^1_\mu \mp i A^2_\mu}{\sqrt{2}} \quad \text{and} \quad Z_\mu = \frac{g A^3_\mu - g' B_\mu}{\sqrt{g^2 + g'^2}} \tag{41}$$

get their physical masses

$$m_W = \frac{g\,v}{2} \quad \text{and} \quad m_Z = \frac{\sqrt{g^2 + g'^2}\,v}{2}. \tag{42}$$

So the electro-weak gauge bosons have their usual masses as long as the squares of their mean values (28) add up to $h_0^2 + p_0^2 = v^2/2 = (246)^2/2$ GeV^2. The choice (15) of equal mean values $h_0 = p_0 = 123$ GeV is a simple way to satisfy this constraint.

V. A MODEL FOR THREE GENERATIONS

The model for three generations is essentially three copies of the model for one generation. Its Yukawa sector has three doublets of primary quarks

$$q_1 = \begin{pmatrix} u \\ d \end{pmatrix} \qquad q_2 = \begin{pmatrix} c \\ s \end{pmatrix} \qquad q_3 = \begin{pmatrix} t \\ b \end{pmatrix} \tag{43}$$

and three doublets of primary leptons

$$\ell_1 = \begin{pmatrix} \nu_e \\ e \end{pmatrix} \qquad \ell_2 = \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \qquad \ell_3 = \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} \tag{44}$$

as well as secondary quarks and leptons $u'_1 = u'$, $u'_2 = c'$, $u'_3 = t'$, $\ldots e'_1 = e$, $e'_2 = \mu'$, and $e'_3 = \tau'$ that are singlets under $SU(2) \otimes U(1)$.

The model avoids flavor-changing neutral currents by means of a new form of Yukawa alignment in which the 3×3 matrices that couple the fields h and p to the fermions have singular-value decompositions that differ only in their singular values. For instance, the Yukawa matrices $U^u \Sigma^u_h V^{u\dagger}$ and $U^u \Sigma^u_p V^{u\dagger}$ that give masses to the three generations of up quarks and mirror up quarks differ only in the 3×3 diagonal matrices Σ^u_h and Σ^u_p of singular values $x_j \ge 0$ and $y_j \ge 0$. If $q_i = (u_i, d_i)$ for $i = 1, 2, 3$ are the primary quarks, u'_k and d'_k the secondary quarks, $\ell_i = (\nu_i, e_i)$ the primary leptons, and ν'_k and e'_k the secondary leptons, then the Yukawa interactions are

$$
\begin{aligned}
V = & \sum_{i,j,k=1}^{3} \bar{q}_i \, i\tau_2 \, U^u_{ij} \left(x^u_j h + y^u_j \gamma^5 p \right) V^{u\dagger}_{jk} \, u'_k + \sum_{i,j,k=1}^{3} \bar{q}_i \, U^d_{ij} \left(x^d_j h + y^d_j \gamma^5 p \right) V^{d\dagger}_{jk} \, d'_k \\
& + \sum_{i,j,k=1}^{3} \bar{\ell}_i \, i\tau_2 \, U^\nu_{ij} \left(x^\nu_j h + y^\nu_j \gamma^5 p \right) V^{\nu\dagger}_{jk} \, \nu'_k + \sum_{i,j,k=1}^{3} \bar{\ell}_i \, U^e_{ij} \left(x^e_j h + y^e_j \gamma^5 p \right) V^{d\dagger}_{jk} \, e'_k + \text{h.c.}
\end{aligned} \tag{45}
$$

The left and right singular vectors are

$$
\begin{aligned}
\bar{u}^s_j &= \sum_{i=1}^{3} \bar{u}_i \, U^u_{ij} \qquad \bar{d}^s_j = \sum_{i=1}^{3} \bar{d}_i \, U^d_{ij} \qquad u'^s_j = \sum_{k=1}^{3} V^{u\dagger}_{jk} \, u'_k \qquad d'^s_j = \sum_{k=1}^{3} V^{d\dagger}_{jk} \, d'_k \\
\bar{\nu}^s_j &= \sum_{i=1}^{3} \bar{\nu}_i \, U^\nu_{ij} \qquad \bar{e}^s_j = \sum_{i=1}^{3} \bar{e}_i \, U^e_{ij} \qquad \nu'^s_j = \sum_{k=1}^{3} V^{\nu\dagger}_{jk} \, \nu'_k \qquad e'^s_j = \sum_{k=1}^{3} V^{e\dagger}_{jk} \, e'_k.
\end{aligned} \tag{46}
$$

Replacing the scalar and pseudoscalar doublets by their neutral components h^0 and p^0, we see that the Yukawa interactions V make no neutral currents

$$
\begin{aligned}
V = & \sum_{j=1}^{3} \bar{u}^s_j \left(x^u_j h^0 + y^u_j \gamma^5 p^0 \right) u'^s_j + \sum_{j=1}^{3} \bar{d}^s_j \left(x^d_j h^0 + y^d_j \gamma^5 p^0 \right) d'^s_j \\
& + \sum_{j=1}^{3} \bar{\nu}^s_j \left(x^\nu_j h^0 + y^\nu_j \gamma^5 p^0 \right) \nu'^s_j + \sum_{j=1}^{3} \bar{e}^s_j \left(x^e_j h^0 + y^e_j \gamma^5 p^0 \right) e'^s_j + \text{h.c.}
\end{aligned} \tag{47}
$$

The CKM matrices of the quarks and leptons are

$$
W^q_{\text{CKM}} = U^{u\dagger} U^d \quad \text{and} \quad W^\ell_{\text{CKM}} = U^{\nu\dagger} U^e. \tag{48}
$$

If the secondary fermions have their own $SU(2)'$, then their quark and lepton CKM matrices are

$$
W^{q'}_{\text{CKM}} = V^{u'\dagger} V^{d'} \quad \text{and} \quad W^{\ell'}_{\text{CKM}} = V^{\nu'\dagger} V^{e'}. \tag{49}
$$

The CKM matrices W^ℓ_{CKM}, $W^{q'}_{\text{CKM}}$, and $W^{\ell'}_{\text{CKM}}$ may break CP enough to explain why there's so much more matter than antimatter.

Replacing h^0 and p^0 in the Yukawa potential (47) by their mean values in the vacuum h_0 and p_0, we get the mass terms

$$
\begin{aligned}
V_0 = & \sum_{j=1}^{3} \bar{u}^s_j \left(x^u_j h_0 + y^u_j \gamma^5 p_0 \right) u'^s_j + \sum_{j=1}^{3} \bar{d}^s_j \left(x^d_j h_0 + y^d_j \gamma^5 p_0 \right) d'^s_j \\
& + \sum_{j=1}^{3} \bar{\nu}^s_j \left(x^\nu_j h_0 + y^\nu_j \gamma^5 p_0 \right) \nu'^s_j + \sum_{j=1}^{3} \bar{e}^s_j \left(x^e_j h_0 + y^e_j \gamma^5 p_0 \right) e'^s_j + \text{h.c.}
\end{aligned} \tag{50}
$$

Thus by analogy with the one-generation case (29–33), the light-mass fields are

$$u_{mj} = \begin{pmatrix} u^s_{\ell j} \\ u'^s_{rj} \end{pmatrix}, \quad d_{mj} = \begin{pmatrix} d^s_{\ell j} \\ d'^s_{rj} \end{pmatrix}, \quad \nu_{mj} = \begin{pmatrix} \nu^s_{\ell j} \\ \nu'^s_{rj} \end{pmatrix}, \quad e_{mj} = \begin{pmatrix} e^s_{\ell j} \\ e'^s_{rj} \end{pmatrix}. \tag{51}$$

The singular values $x^u_j, y^u_j \dots x^e_j, y^e_j$ are nonnegative, so we take h_0 and p_0 to be positive. Thus the masses of the light particles are

$$\begin{aligned} m_{uj} &= x^u_j h_0 - y^u_j p_0 \qquad m_{dj} = x^d_j h_0 - y^d_j p_0 \\ m_{\nu_e j} &= x^{\nu_e}_j h_0 - y^{\nu_e}_j p_0 \qquad m_{ej} = x^e_j h_0 - y^e_j p_0. \end{aligned} \tag{52}$$

The heavy-mass fields are

$$u_{Mj} = \begin{pmatrix} u'^s_{\ell j} \\ u^s_{rj} \end{pmatrix}, \quad d_{Mj} = \begin{pmatrix} d'^s_{\ell j} \\ d^s_{rj} \end{pmatrix}, \quad \nu_{Mj} = \begin{pmatrix} \nu'^s_{\ell j} \\ \nu^s_{rj} \end{pmatrix}, \quad e_{Mj} = \begin{pmatrix} e'^s_{\ell j} \\ e^s_{rj} \end{pmatrix}. \tag{53}$$

Their masses are

$$\begin{aligned} M_{uj} &= x^u_j h_0 + y^u_j p_0 \qquad M_{dj} = x^d_j h_0 + y^d_j p_0 \\ M_{\nu_e j} &= x^{\nu_e}_j h_0 + y^{\nu_e}_j p_0 \qquad M_{ej} = x^e_j h_0 + y^e_j p_0. \end{aligned} \tag{54}$$

The Yukawa coefficients are

$$\begin{aligned} x^u_j &= \frac{M_{u_j} + m_{u_j}}{2h_0} \quad y^u_j = \frac{M_{u_j} - m_{u_j}}{2p_0} \qquad x^d_j = \frac{M_{d_j} + m_{d_j}}{2h_0} \quad y^d_j = \frac{M_{d_j} - m_{d_j}}{2p_0} \\ x^\nu_j &= \frac{M_{\nu_j} + m_{\nu_j}}{2h_0} \quad y^\nu_j = \frac{M_{\nu_j} - m_{\nu_j}}{2p_0} \qquad x^e_j = \frac{M_{e_j} + m_{e_j}}{2h_0} \quad y^e_j = \frac{M_{e_j} - m_{e_j}}{2p_0}. \end{aligned} \tag{55}$$

The Particle Data Group [28] quark masses are $m_u = 2.3$, $m_d = 4.8$, and $m_s = 95$ MeV; and $m_c = 1.275$, $m_b = 4.66$, and $m_t = 173.1$ GeV. The PDG masses of the charged leptons are $m_e = 0.511$, $m_\mu = 105.66$, and $m_\tau = 1776.82$ MeV. The neutrino masses are unknown, but the PDG estimates are $m_{\nu_e} < 2$ eV, and $m_{\nu_\mu} < 0.19$ and $m_{\nu_\tau} < 18.2$ MeV. The PDG lower limits on the masses of heavy fermions are $m'_t > 700$ GeV, $m'_b > 675$ GeV, and $m'_\tau > 100.8$ GeV. The lower limits on the mass of a fourth generation t' quark run from 350 to 782 GeV [29]. If we choose $h_0 = p_0 = 123$ GeV and assume that the masses of the heavy particles are 1 TeV, then the largest Yukawa coefficient is

$$x^u_3 = \frac{M_{u_3} + m_{u_3}}{2h_0} = \frac{1173}{246} = 4.77 \tag{56}$$

which is less than 4π. The smallest coefficient is

$$y^u_3 = \frac{M_{u_3} - m_{u_3}}{2p_0} = \frac{827}{246} = 3.36. \tag{57}$$

Under the same assumptions, $M_{u1} = 1$ TeV and $h_0 = p_0 = 123$ GeV, the x and y coefficients of the first generation are much closer together:

$$x_1^u = \frac{M_{u_1} + m_{u_1}}{2h_0} = 4.06505 \quad \text{and} \quad y_1^u = \frac{M_{u_1} - m_{u_1}}{2p_0} = 4.06503. \tag{58}$$

Inasmuch as the mass M_{u_1} is unknown, the extra digits are meant only to suggest how close x_1^u is to y_1^u and not what their actual values are. Under the same assumptions, $M_{e1} = M_{e3} = 1$ TeV and $h_0 = p_0 = 123$ GeV, the coefficients of the charged leptons of the first generation are

$$x_1^e = \frac{M_{e_1} + m_{e_1}}{2h_0} = 4.065043 \quad \text{and} \quad y_1^e = \frac{M_{e_1} - m_{e_1}}{2p_0} = 4.065039 \tag{59}$$

and those of the third generation are

$$x_3^e = \frac{M_{e_3} + m_{e_3}}{2h_0} = 4.072 \quad \text{and} \quad y_3^e = \frac{M_{e_3} - m_{e_3}}{2p_0} = 4.058. \tag{60}$$

These Yukawa coefficients are bigger than the ones computed from the standard model and the observed fermion masses, but they all are less than 4π, and the mass mechanism (52) is different from that of the standard model.

The clustering of the Yukawa coefficients of all three generations about values near 4 is due to the simplifying assumptions that $h_0 = p_0$ and that all the heavy masses are 1 TeV. Future measurements of Higgs decays will determine whether the Yukawa coefficients look at all like (56–60). Models with two doublets h_i and p_i for each generation $i = 1, 2, 3$ also are possible.

VI. MODELS WITH EXTENDED GAUGE GROUPS

In most models of grand unification, a Higgs mechanism breaks a simple gauge group G_u into the group of the standard model at a unification energy E_u. If this energy lies somewhat above 10^{15} GeV, then proton decay, proceeding through the exchange of a single heavy gauge boson, is slow enough not to have been seen in current experiments [30]. A unification energy that high also lets the coupling parameters of the subgoups $SU_c(3)$, $SU_w(2)$, and $U_Y(1)$ of G_u run to values close to those observed at TeV energies—at least if there's no new relevant physics between 10^3 and 10^{15} GeV.

Grand unification takes a different form when the action of the model conserves parity as in the models of sections IV & V. On the one hand, fermion fields and antifermion fields do not occur in the same multiplets. Thus nucleon decay is intrinsically slow, proceeding through the exchange of three gauge bosons, and so the unification energy E_u can be much lower than 10^{15} GeV, perhaps as low as a PeV. On the other hand, the energy of unification E_u must be high enough that the three coupling parameters run long enough to unify. Two possibilities come to mind depending upon whether there's new physics between a TeV and E_u.

Without new physics in that grand desert, the energy of unification E_u and the mass M of the heavy gauge bosons would be of the order of $M \sim 10^{15}$ GeV or higher. As we'll see presently, in simple extensions of the model of this paper, nucleon decay proceeds via the exchange of three heavy gauge bosons. The lifetime of the nucleon therefore rises with the twelfth power of the ratio of the mass of the heavy gauge boson to that of the proton, $\tau_n \sim M^{12}/(\alpha_u^4 m_p^{13})$, where α_u is the fine-structure constant of the unified theory. The resulting nucleon lifetime of more than 10^{150} years would be too long for nucleon decay to be seen.

If there is new physics below 10^{15} GeV, then the simple group G_u might break twice. For instance, it might break at an energy E_u to $SU(n) \otimes SU_w(2) \otimes \tilde{G}$ and then break again to the group of the standard model $SU_c(3) \otimes SU_w(2) \otimes U_Y(1)$ at a lower energy E_s. The larger the integer n, the faster the coupling parameter of $SU(n)$ runs between E_u and E_s.

The existence of three generations of fermions below a TeV may be a sign of new physics below 10^{15} GeV. One can imagine that at an energy E_u the simple group breaks down to $SU(9) \otimes SU_w(2) \otimes G_1$, and then at E_s this group breaks to $SU_c(3) \otimes SU_w(2) \otimes U_Y(1)$. Between E_u and E_s, the coupling parameters g_9 and g_2 run as [31]

$$\mu \frac{dg(\mu)}{d\mu} = -\frac{g^3(\mu)}{4\pi^2} \left(\frac{11}{12} C_1 - \frac{1}{3} C_2 \right) \tag{61}$$

in which $C_1 = n$ for $SU(n)$ and $C_2 = n_f/2$ where n_f is the number of fermions in the representation of $SU(n)$. For $SU(9)$ there are two primary multiplets U and D of three generations of three colors for a total of nine quarks and similarly two secondary nonets U' and D' of quarks, so $n_{f,9} = 4$. For $SU_w(2)$, there are nine quark doublets, and three lepton doublets, so $n_{f,2} = 12$. Thus between E_u and E_s, the coupling parameters g_9 and g_2 run as

$$\begin{aligned} \mu \frac{dg_9(\mu)}{d\mu} &= -\frac{g_9^3(\mu)}{4\pi^2} \left(\frac{11}{12} 9 - \frac{1}{3}\frac{4}{2} \right) = -\frac{g_9^3(\mu)}{4\pi^2} \left(\frac{33}{4} - \frac{2}{3} \right) = -\frac{91\, g_9^3(\mu)}{48\pi^2} \\ \mu \frac{dg_2(\mu)}{d\mu} &= -\frac{g_2^3(\mu)}{4\pi^2} \left(\frac{11}{12} 2 - \frac{1}{3}\frac{12}{2} \right) = -\frac{g_2^3(\mu)}{4\pi^2} \left(\frac{11}{6} - 2 \right) = \frac{g_2^3(\mu)}{24\pi^2} \end{aligned} \tag{62}$$

which shows that $SU_w(2)$ is not asymptotically free in this model. Integrating, we get

$$\begin{aligned} \frac{1}{g_9^2(E_s)} &= \frac{1}{g_9^2(E_u)} - \frac{91}{24\pi^2} \log\left(\frac{E_u}{E_s}\right) \\ \frac{1}{g_2^2(E_s)} &= \frac{1}{g_2^2(E_u)} + \frac{1}{12\pi^2} \log\left(\frac{E_u}{E_s}\right). \end{aligned} \tag{63}$$

The traces are $\mathrm{Tr}\left[(\frac{1}{2}g_9\lambda_3)^2\right] = 6g_9^2$ and $\mathrm{Tr}\left[(\frac{1}{2}g_2\sigma_3)^2\right] = 6g_2^2$. So setting $g_9^2(E_u) = g_2^2(E_u)$ and subtracting, we find

$$\frac{1}{\alpha_2(E_s)} - \frac{1}{\alpha_9(E_s)} = \frac{31}{2\pi} \log\left(\frac{E_u}{E_s}\right). \tag{64}$$

Thus

$$E_u = E_s \exp\left[\frac{2\pi}{31}\left(\frac{1}{\alpha_2(E_s)} - \frac{1}{\alpha_9(E_s)}\right)\right]. \tag{65}$$

If $E_s = 100$ TeV where $1/\alpha_2(E_s) - 1/\alpha_9(E_s) \sim 33 - 17 = 16$ [28], then the higher energy scale is $E_u = 100 \exp(32\pi/31) = 2561$ TeV $= 2.56$ PeV.

We can imagine putting the nonets U and D of primary fermions and the triplets $E = (e, \mu, \tau)$ and $N = (\nu_e, nu_\mu, \nu_\tau)$ into the a multiplet F of dimension 24

$$F = \begin{pmatrix} U \\ D \\ E \\ N \end{pmatrix}. \tag{66}$$

One would put the secondary fermions into a similar 24-plet F'.

I will assume that the group G_u has colored gauge bosons $\vec{J} = (J, J, J)$ that mediate $u \to \nu + J^{2/3}$ and other colored gauge bosons $\vec{K} = (K, K, K)$ that mediate $d \to \nu + K^{-1/3}$ and that a Higgs mechanism gives them masses of at least a PeV. I also assume that the cubic Yang-Mills coupling allows the process $J^{2/3} + K^{-1/3} \to \bar{K}^{1/3}$. In such grand unifications of the model of section V, the proton is unstable to decays like $p \to \pi^+ + 3\nu$ and $p \to e^+ + 4\nu$ since the proton and the combinations $\pi^+ + 3\nu$ and $e^+ + 4\nu$ all have $F = 3$ and electric charge $+1$. These decays tend to be slow because they involve three heavy gauge bosons as in the process

$$\begin{aligned} (u, u, d) &\to (u, d) + \nu + J^{2/3} \to u + \nu + J^{2/3} + \nu + K^{-1/3} \\ &\to u + \nu + \nu + \bar{K}^{1/3} \to u + \bar{d} + \nu + \nu + \nu \to \pi^+ + \nu + \nu + \nu. \end{aligned} \tag{67}$$

The neutron has $F = 3$ and charge zero. Any state of three light neutrinos also has $F = 3$ and charge zero. So a neutron inside a nucleus can decay into three neutrinos, $n \to 3\nu$.

This decay also tends to be slow because it also involves three heavy gauge bosons as in the process

$$\begin{aligned}(u, d, d) &\to (d, d) + \nu + J^{2/3} \to d + \nu + \nu + J^{2/3} + K^{-1/3} \\ &\to d + \nu + \nu + \bar{K}^{1/3} \to \nu + \nu + \nu.\end{aligned} \tag{68}$$

If the masses of these heavy gauge bosons are M_J and M_K, then the lifetimes of the proton and of the nuclear neutron in these models are proportional to $m_J^8 m_K^4/(\alpha_u^6 m_p^{13})$ in which α_u is the fine-structure constant of the unified theory.

The lower bounds on nucleon partial lifetimes are 4.9×10^{26} years for $n \to 3\nu$ [21], 5.8×10^{29} years for $n \to$ invisible [22], and 2.1×10^{29} years for $p \to$ invisible [23]. So the masses of the mediating gauge bosons J and K should be a PeV or more. This energy scale is 9 or 10 orders of magnitude lower than the scale of traditional grand unification because these models don't put fermions and antifermions in the same multiplets. The running of masses and coupling constants poses less of a fine-tuning problem between 1 TeV and 10^4 TeV than between 1 TeV and 10^{12} TeV.

Neutron decay might be seen in the SNO, KamLAND, Super-Kamiokande, and JUNO detectors and may lurk in their recorded data. The decay of a nucleon in an ^{16}O nucleus would leave behind an excited ^{15}N* or ^{15}O* nucleus which 45% of the time emits a γ ray of 6–7 MeV [23]. In its ground state, an ^{15}O nucleus has a half-life of 122 s and decays into a stable nucleus of ^{15}N, a positron e^+, and a neutrino ν_e. The energy of the positron can be as high as 1.732 MeV with a mean energy of 735.28 keV [32].

The present model requires the existence of heavy mirror fermions with interactions much like those of the known fermions. They would form heavy positive nuclei surrounded by shells of heavy mirror electrons with $m_{e'} \gg m_e$. These mirror atoms would be very small with Bohr radii of the order of $(\alpha_u m_{e'})^{-1}$. The photon energy needed to excite these atoms would be of the order of $m_{e'}\alpha^2$, and so a photon would need an energy in excess of 100 MeV to excite one of these atoms. These atoms are candidates for dark matter. Because their masses would exceed 10 TeV, their number density would be about 1000 times lower than that of a 10 GeV WIMP. This low number density may be why physicists have not detected dark matter despite its energy density being 5.4 times greater than that of ordinary matter [7].

The primary and secondary fermions both interact through the gluons of $SU_c(3)$, so one might think that the nuclei of the heavy neutral atoms of the present model would interact strongly with those of ordinary matter. But at low energies nuclei scatter off other nuclei by exchanging pions, and the analog of a pion in heavy-nucleus–light-nucleus scattering is a WIMPy pion consisting of a light quark and a heavy antiquark or a light antiquark and a heavy quark. A WIMPy pion would have a mass in excess of 1 TeV and so the cross-section for heavy-nucleus–light-nucleus scattering would be like that of a weak interaction. The quarks of these putative dark-matter particles interact with QCD, but their interactions are weak because the exchanged heavy pions are so massive. Theories in which dark matter consists of stealthy strongly interacting particles [8] or strongly interacting massive particles (SIMPs) [9] have been developed. Strongly interacting dark matter [10] broadens dark-matter cusps into cores [11], as suggested by some observations, and so may explain the apparent paucity of heavy dwarf galaxies around our galaxy [14].

Some mechanism—perhaps initial conditions or CP-violation—has created an excess of matter over antimatter and possibly of dark matter over dark antimatter. The standard model does not have enough CP-violation, but the CKM matrices (49) of the heavy fermions might. If dark matter is composed of heavy quarks and leptons, then the symmetry between the fermions and the mirror fermions may explain why the two excesses differ only by a factor of 5.4.

VII. SUMMARY

Although current research may change our understanding of gravity, I have assumed in this paper that the action of a fundamental theory should be invariant under general coordinate transformations so as to be compatible with general relativity. If this is so, then the standard model should be extended to one whose action is invariant under spatial reflection, which is a simple coordinate transformation. This paper describes such a model. In the model, the mean value in the vacuum of a pseudoscalar field breaks parity. This field and a scalar Higgs field make the gauge bosons, the known fermions, and a set of mirror fermions suitably massive while avoiding flavor-changing neutral currents due to a novel kind of Yukawa alignment. Because the action of the model is invariant under spatial reflection, the theory conserves quark-plus-lepton number and has no anomalies and no strong-CP problem. The restoration of parity could occur at energies as low as 10 TeV. The model predicts heavy mirror fermions which form heavy neutral mirror atoms which are dark-matter candidates. In some grandly unified extensions of the model, the scale of grand unification can be as low as 2.5 PeV, which reduces the fine-tuning problem, and nucleons slowly decay into pions, antileptons, and neutrinos in processes like $p \to \pi^+ + 3\nu$, $p \to e^+ + 4\nu$, and $n \to 3\nu$ that conserve fermion number but violate $B - L$.

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