NONCOMPACT SIMULATIONS OF SU(2)$_3$ ☆

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The action density and Wilson loops for SU(2) in three dimensions have been measured using both a noncompact version of lattice gauge theory and Wilson's version. As a standard of comparison, the Creutz ratio χ of a quartet of Wilson loops has been calculated in the exact theory to order $1/\beta^2$. The noncompact method gave χ's that are between 2 and 23% below the one-loop calculation for the smaller loops at $\beta = 30$ and 60 on a 24^3 lattice. Wilson's method gave χ's that are above the one-loop calculation by comparable amounts. For $\beta \leqslant 10$, the noncompact χ's are closer than the Wilson χ's to the one-loop calculation. Also some noncompact simulations were done in the temporal gauge; the results agree subtantially with those done without gauge fixing, providing evidence for the gauge independence of the noncompact method. By comparing the χ's of the 12^3 lattice at $\beta = 30$ with those of the 24^3 lattice at $\beta = 30$ and 60, evidence was found that the accuracy of the noncompact method improves both as the volume of the lattice grows and as the lattice spacing shrinks.

Wilson's lattice gauge theory [1] is a practical way to study gauge theories nonperturbatively [2,3]. But is it valid beyond weak coupling? The basic variables of Wilson's method are the group elements $\exp(igaA_\mu{}^\alpha\lambda^\alpha)$ where g is the coupling constant, a the lattice spacing, $A_\mu{}^\alpha$ the gauge field, and λ^α a generator. Wilson's action and domain of integration resemble those of the continuum theory when the angles $|gaA_\mu{}^\alpha|$ are much less than unity, which is true only for weak coupling. At stronger coupling, the extra terms in Wilson's action and the topology and curvature of his domain of integration become important. In a theory with a running coupling constant, one may try to reduce the resultant errors by working at weak coupling. But in SU(3) the physical size of the lattice spacing shrinks with g like $\exp[-1/(2b_0g^2)]$ where $b_0 = 11/16\pi^2$ [4]. If one reduces g, one must compute on a larger lattice in order to encompass the same physical phenomenon. A reduction of g from 0.5 to 0.25 requires an increase of a 10^4 lattice to one of size $(10^{38})^4$.

In earlier papers [5,6], we developed a method for approximating euclidean path integrals at arbitrary coupling. In this method one tiles spacetime with simplexes and linearly interpolates the fields throughout each simplex from their values at the vertices. The fields are defined continuously throughout spacetime. The method uses the action and domain of integration of the exact continuum theory, unaltered apart from the granularity of the simplicial lattice. In the limit in which the lattice spacing goes to zero and the lattice size to infinity, the method provides a definition of the euclidean functional integral of the continuum theory.

In ref. [6] we applied this noncompact method to U(1) in three dimensions, computing Creutz ratios for various quartets of Wilson loops. We found that for U(1)$_3$ the noncompact method is accurate at arbitrary coupling, while Wilson's method is accurate only at weak coupling.

In the present paper, we describe our use of the noncompact method to measure the action density and Wilson loops for SU(2) in three dimensions.

We derived the theoretical weak- and strong-

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coupling limits of the action density and verified that our heat-bath Monte Carlo codes gave these limits. The action density varies with the inverse coupling $\beta \equiv 4/ag^2$ much more gradually than in Wilson's method. We saw no evidence of a phase transition.

In order to judge the accuracy of our method and of Wilson's, we derived a formula valid to order g^4 for Creutz ratios of Wilson loops in the exact theory. On the 24^3 lattice, the noncompact method gave Creutz ratios $\chi(r, t)$ that are between 2 and 23% below the χ's of the one-loop calculation for the smaller loops at $\beta = 30$ and 60.

To compare the noncompact method with Wilson's, we used Steve Otto's C code to do Monte Carlo computations in Wilson's formalism. For all β the Creutz ratios of Wilson's method are higher than those of the one-loop calculation, and those of the noncompact method are lower. For $\beta = 30$ and 60, the Wilson χ's for most loops are somewhat closer than the noncompact χ's to the one-loop χ's. As the coupling increases, the two methods diverge from the one-loop calculation: for $\beta \leqslant 10$ the noncompact χ's are closer than the Wilson χ's to the one-loop χ's.

The action of the noncompact method is exactly gauge invariant, although it is possible, but unwise, to fix the gauge. However the functional-integration space of the method, being sparse, is only approximately gauge invariant. To test the effective gauge independence of the method, we computed some Creutz ratios in the temporal gauge and compared them with those computed in the usual way, i.e. without gauge fixing. For $r > t$, the temporal-gauge $\chi(r, t)$'s agree with those computed without gauge fixing to within about 10% for $\beta = 6$ and about 20% for $\beta = 2$.

If the noncompact method is valid, then its accuracy should improve both as the volume of the lattice grows and as the lattice spacing shrinks. We found evidence that this was so by comparing the χ's of a 12^3 lattice at $\beta = 30$ with those of the 24^3 lattice at $\beta = 30$ and 60.

In the noncompact method, spacetime is limited to a periodic cubic lattice each cube of which is tiled with six (tetrahedral) simplexes, as shown in fig. 1. Each spacetime point x lying in a simplex with vertices v_i can be uniquely expressed in the form $x = \Sigma_i \rho_i v_i$ in which the four nonnegative weights ρ_i sum to one. We use this formula linearly to interpolate the field $A_\mu{}^\alpha(x)$ at x from its values $A(\mu, \alpha, v_i)$ at the vertices v_i: $A_\mu{}^\alpha(x) = \Sigma_i \rho_i A(\mu, \alpha, v_i)$. Since the interpolated fields are defined throughout spacetime, we use the euclidean action of the continuum theory, $S[A] = \int d^3x \frac{1}{4} F_{\mu\nu}{}^\alpha(x)^2$, where $F_{\mu\nu}{}^\alpha(x)$ is defined in terms of the field $A_\mu{}^\alpha(x)$ as in the continuum theory. Because the interpolation is linear in the field variables $A(\nu, \beta, v)$, the jacobian $\det[\partial A_\mu{}^\alpha(x)/\partial A(\nu, \beta, v)]$ cancels in ratios of path integrals. Thus we approximate the mean value in the vacuum of a euclidean-time-ordered operator $Q(A)$ by a normalized multiple integral over the $A(\mu, \alpha, v)$'s:

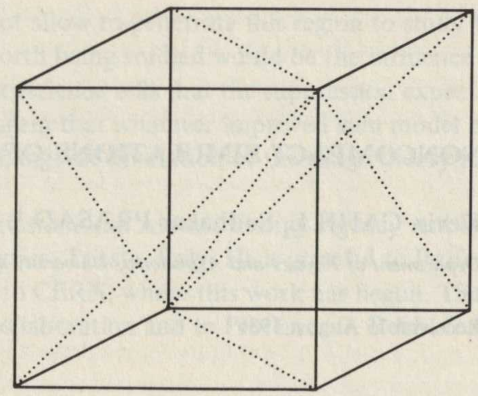

Fig. 1. A cube of the lattice divided into six tetrahedral simplexes.

$$\langle 0|\mathrm{T}\, Q(A)|0\rangle$$

$$\approx \frac{\int \Pi\, dA(\mu, \alpha, v) \exp\{-S[A]\} Q(A)}{\int \Pi\, dA(\mu, \alpha, v) \exp\{-S[A]\}}, \qquad (1)$$

where $Q(A)$ is obtained from $Q(A)$ by replacing the operator $A_\mu{}^\alpha(x)$ with the interpolated field $A_\mu{}^\alpha(x)$. We use Monte Carlo techniques to evaluate such multiple integrals.

For the 24^3 lattice, the action is a quartic polynomial in 124 416 variables, the $A(\mu, \alpha, v)$'s. Each A occurs only quadratically in the action, due to the antisymmetry of $F_{\mu\nu}{}^\alpha(x)$, and is coupled to only 135 other A's in 24 simplexes. We used these facts to write a heat-bath algorithm [7] that needs only the first and second derivatives of the action with respect to each A. As described in ref. [8], we used the symbol manipulator MACSYMA to calculate these derivatives and to write them in FORTRAN.

In the weak-coupling limit, the field variables A are small, and the action reduces to a quadratic form.

Apart from zero modes, the mean value $\langle S\rangle_c$ of the action per cube is half the number of field variables per cube. Since there are nine A's per cube, we expect $\langle S\rangle_c = 4.5$. A run on the 16^3 lattice at $\beta = 400$ gave $\langle S\rangle_c = 4.474$. Already for $\beta \geqslant 2$, $\langle S\rangle_c$ is within 10% of its weak-coupling limit, and by $\beta = 9$ it has gone 90% of its way from its strong-coupling limit to its weak-coupling limit.

At strong coupling, it is the quartic terms in the action that dominate. If the quadratic and cubic terms are neglected, there is no coupling between fields at different points. The classical action is then an integral of a sum of squares of cross-products of the fields $A_\mu(x)$:

$$S \approx \frac{g^2}{2} \int d^3x\,[(A_1 \times A_2)^2 + (A_2 \times A_3)^2 + (A_3 \times A_1)^2]\,. \tag{2}$$

In this case it is reasonable to approximate the interpolated fields in each cube by A_μ's that are independent of x. Then the mean value of the action in a single cube is the logarithmic derivative $\langle S\rangle_c \approx -\beta d \log Z/d\beta$ of the partition function

$$Z = \int d^3a_1\, d^3a_2\, d^3a_3 \exp\left(-\beta \sum_{\mu<\nu}^{3} (a_\mu \times a_\nu)^2\right), \tag{3}$$

where $a_\mu \equiv agA_\mu$. By scaling the a_μ by $\beta^{1/4}$, one finds that the strong-coupling limit of the action per cube is just $\langle S\rangle_c = 2.25$. It is only for $\beta \leqslant 0.002$ that $\langle S\rangle_c$ gets to within 10% of this limit. A short run at $\beta = 10^{-8}$ gave $\langle S\rangle_c = 2.249$. This result illustrates a quartic equipartition in which the mean value of the action per field variable is $\frac{1}{4}$. It also suggests that the noncompact method makes sense even at very strong coupling.

On the 10^3 lattice, we measured $\langle S\rangle_c$ in the temporal gauge from $\beta = 10$ to $\beta = 0.225$ and back. No sign of hysteresis was evident, indicating the absence of a first-order phase transition over this range of coupling. The action density of Wilson's method also exhibits no phase transition; but it varies more abruptly, vanishing like β in the limit of strong coupling [9].

For the fundamental representation of SU(2), the Wilson loop is the mean value

$$W(r, t) = \langle 0| \mathrm{Tr\, P\, T} \exp\left(ig \oint A_\mu{}^\alpha(x)\sigma^\alpha/2 dx_\mu\right) |0\rangle/2 \tag{4}$$

in which the operators are time-ordered in the euclidean sense, the matrices are path ordered, and the contour of integration is an r-by-t rectangle. Wilson loops vanish in more than two dimensions due to the singular electric string generated by the Wilson-loop operator [10]. Creutz introduced the practice of measuring ratios of products of Wilson loops in a way that separates the physically important area term from this singularity [2]. The Creutz ratio of a quartet of Wilson loops is the logarithm of a ratio of their products:

$$\chi(R, T) \equiv \chi(r/a, t/a) = -\ln\left(\frac{W(r, t)W(r-a, t-a)}{W(r-a, t)W(r, t-a)}\right). \tag{5}$$

Using perturbative methods [11,12], we derived a formula for Creutz ratios in $SU(2)_3$ valid to order g^4. Our formula is obtained by substituting for $\ln[W(r, t)]$ in the definition of $\chi(R, T)$ the expression $U(r, t) + U(t, r)$ where

$$\begin{aligned} U(r, t) = {} & (3g^2/8\pi)[r\,\mathrm{ash}(r/t) - (r^2 + t^2)^{1/2}] \\ & + (3g^4/256\pi^2)[-2(4r^2 + t^2)\,\mathrm{ash}^2(r/t) \\ & + 4[5r(r^2 + t^2)^{1/2} - 2rt - 2r^2]\,\mathrm{ash}(r/t) \\ & - 8rt\,\mathrm{ash}(r/t)\,\mathrm{ash}(t/r) \\ & + 8t[(r^2 + t^2)^{1/2} - t]\,\mathrm{ash}(t/r) \\ & + 8(r + 2t)(r^2 + t^2)^{1/2} - 8rt + 8r^2 I(r, t)]\,, \end{aligned} \tag{6}$$

ash $\equiv$ arcsinh, and I is the integral

$$I(r, t) = \int_0^1 dx\,[x\,\mathrm{ash}(rx/t) - (x^2 + t^2/r^2)^{1/2} - 2x \ln x] \times [\mathrm{ash}(rx/t) + \mathrm{ash}(r(1-x)/t)]\,. \tag{7}$$

We have suppressed in $U(r, t)$ all terms that are independent of r or of t because Creutz ratios do not depend on such terms.

To extract the one-loop quark–antiquark static potential $V(r)$ from our formula for $\ln[W(r, t)]$ we took the $t \to \infty$ limit [10] of $(\partial/\partial t)\ln[W(r, t)]$, obtaining $V(r) = (3g^2/8\pi)\ln r$ to within an additive constant. In this formula the term proportional to g^4 vanishes.

The main result of this paper is that the noncompact method gives Creutz ratios that at weak coupling agree quantitatively with those of the one-loop approximation to the exact theory. In this method, since the fields are interpolated throughout space-time, one may approximate the path-ordering in the definition (4) of the Wilson loop to arbitrary precision. We used an ordered product of exponentials, one for each half lattice spacing for $\beta = 30$ and 60 and one for each whole lattice spacing [‡1] for $\beta \leqslant 10$. To compute Creutz ratios at $\beta = 60$, we made 64 sweeps on the 24^3 lattice, with measurements every sweep, starting from fields thermalized by 230 sweeps. For $R = r/a = 2$–4 and $T = t/a = 2$–4, these $\chi(R, T)$'s are between 2% and 16% below the χ's of the one-loop calculation, with an average error of 8%. For R and $T \leqslant 5$, the average error is 10%. The $\chi(R, T)$ with $R \leqslant 6$ and $T \leqslant 3$ (or vice versa) are between 2% and 10% below the one-loop χ's. At $\beta = 30$, we made 154 measurements, starting from fields thermalized by 300 sweeps. For R and $T = 2$–4, these $\chi(R, T)$'s are between 8% and 30% below the one-loop values, with an average error of 17%. For R and $T \leqslant 5$, the average error is 21%. The $\chi(R, T)$'s with $R \leqslant 6$ and $T \leqslant 3$ are between 8% and 23% below the one-loop χ's.

Wilson's method gave comparable results at $\beta = 30$ and 60 in the simulations we ran using Otto's code on the 24^3 lattice: its χ's are between 5 and 13% above the one-loop values for the smaller loops.

At $\beta = 10$ we ran 206 sweeps of the noncompact method on the 24^3 lattice with measurements every other sweep after 190 thermalizing sweeps. For R and $T \leqslant 4$, these $\chi(R, T)$'s average 32% below the one-loop values. For R and $T \leqslant 5$, they average 37% below. For all R and T, they are closer than the Wilson χ's to the one-loop results.

As the coupling grows stronger, the χ's of the two methods diverge from those of the one-loop calculation. The noncompact χ's remain below the one-loop χ's; the Wilson χ's rise higher above the one-loop χ's. For $\beta \leqslant 10$, the noncompact χ's are closer than the Wilson χ's to the one-loop χ's. This behavior is illustrated in figs. 2 and 3 which display $\chi(2, 2)$ and $\chi(3, 3)$ for $\beta \leqslant 10$. The Wilsonian χ's are represented by asterisks, the noncompact χ's by crosses, and the one-loop calculation by curves. The error bars, had

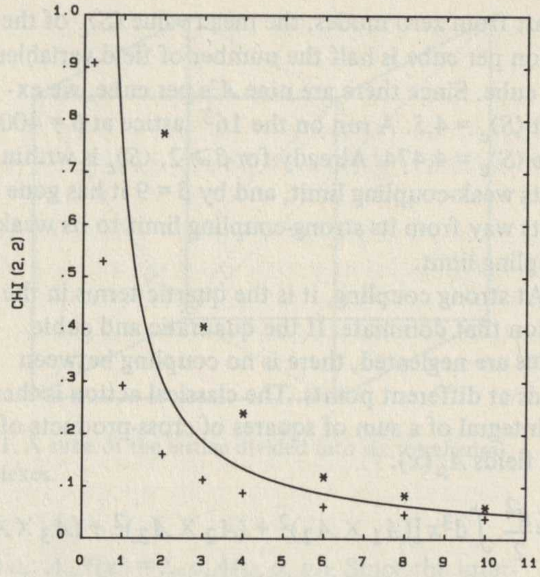

Fig. 2. The Creutz ratio $\chi(2, 2)$ from Wilson's method, asterisks; from the one-loop calculation, curve; and from the noncompact method, crosses.

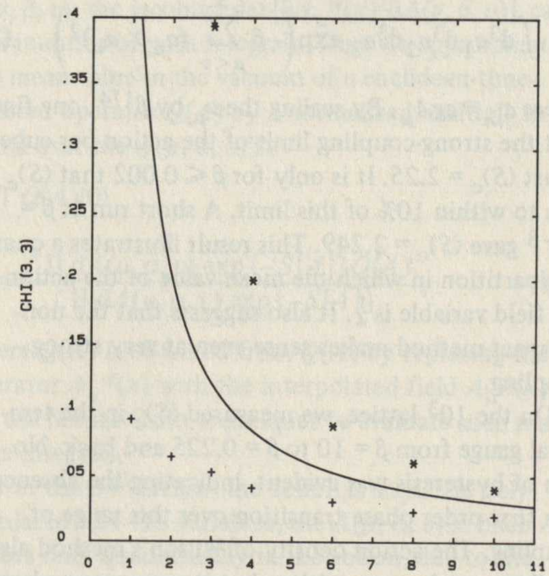

Fig. 3. The Creutz ratio $\chi(3, 3)$ from Wilson's method, asterisks; from the one-loop calculation, curve; and from the noncompact method, crosses.

‡1 After the runs of this paper, we tried much finer approximations, such as using one exponential for each 1/100th of a lattice spacing. The χ's moved closer to the one-loop values. At $\beta = 1$, $\chi(2,2)$ and $\chi(3,3)$ increased by about 10% and 45%, respectively. The improvement was much less, however, for $\beta \geqslant 6$.

we shown them, would have been smaller than the plotted symbols, except for the left-most point which is uncertain by about 10% in each figure. The Monte Carlo data were obtained on the 16^3 lattice in runs of several hundred sweeps. The range of validity of the one-loop calculation is unknown. However, the fact that in the noncompact simulations the action is within 10% of its weak-coupling limit for $\beta \geqslant 2$ suggests that the one-loop results may have some validity in that region. If so, then the noncompact method may be more accurate then Wilson's method for $\beta \leqslant 10$, as it was [6] for $U(1)_3$. In this range of β, the angles $|gaA_\mu{}^\alpha|$ may not be sufficiently small for Wilson's method to be accurate. Our simulations indicate that at $\beta = 2.5$, for example, the mean value of these angles is about 0.7.

Of the Wilson loops that we measured on the 16^3 lattice for β between 0.05 and 10, only the 1×1, ..., 1×6, 2×2, and 2×3 loops exhibit an area law. It is possible that the bigger loops had not yet thermalized, since none of the runs had more than 1400 sweeps. However, our data on the action density suggest that it is not until β is less than about 0.002 that the theory enters the region of strong coupling. We do not have data on Wilson loops for such β and are unable to decide whether the theory confines [13]. The Wilson loops of Wilson's method do display an area law [9,14], at least for β less than about 5.

Some of the discrepancy between the noncompact χ's and the Wilson χ's may be due to differences in coupling-constant renormalization. The Wilson $\chi(2, 2)$ at $\beta_W \approx \beta + \beta^{1/2} + 1.6$ is approximately equal to the noncompact $\chi(2, 2)$ at β. For $\chi(3, 3)$ the shift is $\beta_W \approx \beta + 1.5\beta^{1/2} + 3.5$.

The action $S[A]$ of the noncompact method, being that of the continuum theory, is invariant under an arbitrary continuum gauge transformation $A \to A'$. In this sense the gauge invariance of the method is exact. However, the functional-integration space of the method, being the space of linearly interpolated fields, is uniformly sparse in the space of all gauge fields. For every gauge field A, there is a unique linearly interpolated field $P(A)$, but the correspondence is infinitely many to one. The space of all linearly interpolated fields $P(A)$ is therefore not gauge invariant, but for reasonably smooth gauge transformations the action $S[P(A')]$ differs only slightly from $S[P(A)]$. The Wilson method, on the other hand, has an exact lattice gauge invariance, which converges to the gauge invariance of the continuum theory as the coupling and lattice spacing go to zero.

To test the effective gauge independence of the noncompact method, we computed some Creutz ratios in the temporal gauge and compated them with those computed without gauge fixing. In temporal-gauge quantization, the field A_0 is missing and Gauss's law must be imposed upon the physical states as the constraint that they be gauge invariant. The vacuum state automatically satisfies this constraint because it is the state of lowest energy [15]. However, in the temporal-gauge formalism, the vacuum state is defined in terms of the hamiltonian H as the approximate projection operator $\exp(-aN_t H/2)$, where aN_t is the temporal extent of the lattice. Since this projection becomes exact only as $N_t \to \infty$, finite-size effects affect temporal-gauge $\chi(r, t)$'s, particularly for r less than t. For example, $\chi(2, 6)$ can be much less than $\chi(6, 2)$, rather than equal to it as required by euclidean symmetry. Thus it is better not to fix the gauge.

If the noncompact method is gauge independent, then for $r > t$ the temporal-gauge $\chi(r, t)$'s should agree with those computed without gauge fixing. At $\beta = 6$ on the 16^3 lattice, we got the following pairs of χ's in the temporal gauge and without gauge fixing, respectively: $\chi(6, 2) = 0.047$ and 0.043, $\chi(5, 2) = 0.048$ and 0.044, $\chi(4. 2) = 0.048$ and 0.046, $\chi(3. 2) = 0.050$ and 0.051, $\chi(6, 3) = 0.024$ and 0.021, $\chi(5, 3) = 0.024$ and 0.021, and $\chi(4, 3) = 0.025$ and 0.024. The analogous χ pairs at $\beta = 2$ are: $\chi(6, 2) = 0.13$ and 0.10, $\chi(5, 2) = 0.13$ and 0.11, $\chi(4, 2) = 0.13$ and 0.11, $\chi(3, 2) = 0.13$ and 0.13, $\chi(6, 3) = 0.05$ and 0.04, $\chi(5, 3) = 0.06$ and 0.04, and $\chi(4, 3) = 0.06$ and 0.05. All the pairs agree to within the statistical errors. This agreement illustrates the approximate gauge independence of the noncompact method.

If the noncompact method is valid, its accuracy should improve as the volume of the lattice increases. To test this, we compared runs on the 12^3 lattice with ones on the 24^3 lattice. At $\beta = 30$ and for R and $T \leqslant 5$, the $\chi(R, T)$'s of the smaller lattice averaged 24% below the one-loop χ's while those of the bigger lattice were only 21% below. These results suggest that the noncompact method becomes more accurate as the volume of the lattice increases.

The accuracy of the method should also improve as the lattice spacing decreases with the volume of the lattice and the coupling constant held fixed. Since $\beta = 4/ag^2$, the Wilson loop $W(R, T)$ on the 12^3 lattice with lattice spacing a at $\beta = 30$ corresponds to the loop $W(2R, 2T)$ on the 24^3 lattice with lattice spacing $a' = a/2$ at $\beta = 60$. Thus the Creutz ratio corresponding to $\chi(R, T)$ on the coarser lattice is $\chi'(2R, 2T)$ on the finer lattice defined as

$$\chi'(2R, 2T) = -\ln\left(\frac{W(2R, 2T)\,W(2R-2, 2T-2)}{W(2R, 2T-2)\,W(2R-2, 2T)}\right).$$

On the coarser lattice we found $\chi(2, 2) = 0.0140 \pm 0.0001$, which is 13.2% below the one-loop value of 0.0160; on the finer lattice we found $\chi'(4, 4) = 0.0145 \pm 0.0002$, which is 9.6% below the one-loop value. This improvement suggests that the noncompact method becomes more accurate as the lattice spacing shrinks.

From this work we draw the following conclusions: (1) Both the noncompact method and Wilson's method give Creutz ratios χ that are close to our one-loop perturbative formula for very weak coupling, i.e. for $\beta \geqslant 30$. (2) For $\beta \leqslant 10$, the χ's of the noncompact method are closer than the Wilson χ's to the one-loop χ's. To the extent that the one-loop result may be reliable for $\beta \leqslant 10$, the noncompact method may be more accurate there than Wilson's. (3) The χ of the noncompact method are always lower than the one-loop χ's. This effect may be evidence for infrared softening [11]. (4) The region of strong coupling in the noncompact method does not set in until β is less than about 0.002. We do not have data on χ's of the such β and are unable to decide whether the theory confines [13]. (5) The noncompact method seems approximately gauge independent inasmuch as for $r > t$ its temporal-gauge $\chi(r, t)$'s agree substantially with those computed without gauge fixing. (6) The accuracy of the method seems to improve both as the volume of the lattice increases and as the lattice spacing decreases.

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