Nuclear Physics B (Proc. Suppl.) 9 (1989) 529–532
North-Holland, Amsterdam

NONCOMPACT SIMULATIONS OF $SU(2)$*

Kevin CAHILL

Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138, USA†
Department of Physics and Astronomy, University of New Mexico, Albuquerque, New Mexico 87131, USA‡

Noncompact simulations of $SU(2)$ without gauge fixing on a 10^4 lattice are consistent with perturbation theory at very weak coupling but show no evidence of quark confinement at strong coupling.

Gribov has suggested [1] that the existence of light quarks is a necessary part of the mechanism of quark confinement in QCD. Confinement, in his view, is essentially due to light-quark pair creation and so is not a property of QCD without quarks or with quarks that are all heavier than the QCD scale Λ. The present paper describes the first noncompact simulations of quarkless $SU(2)$ that have been done without gauge fixing in four spacetime dimensions. These simulations are consistent with perturbation theory at very weak coupling but show no evidence of quark confinement at strong coupling. They support Gribov's suggestion that pure QCD does not confine and confirm earlier gauge-fixed noncompact simulations [2,3], but disagree with the usual compact Wilsonian simulations [4,5]. However the present noncompact simulations, done on a 10^4 lattice, don't exclude the possibility that noncompact simulations done on a much larger lattice might display confinement. For as we shall see, the lattice spacing $a_{NC}(\beta)$ of the noncompact method is likely to be much smaller than the lattice spacing $a_W(\beta)$ of Wilson's method.

It is generally believed that quark confinement is a property of the QCD vacuum, having little to do with the nature of the quarks themselves. This belief is partly due to the many attempts that have been made to derive confinement in the simpler context of quarkless QCD. The best evidence that pure QCD confines comes from the lattice simulations that Creutz [5] and others have carried out using Wilson's method [4]. But confinement is a nearly universal feature of Wilson's compact method, holding in the strong-coupling limit even for abelian gauge groups in spacetimes of any dimension. So it is not clear whether the confinement seen in compact simulations is a relic of the confinement built into Wilson's method or a reflection of a property of QCD.

Because of this ambiguity, physicists have developed alternative Monte Carlo methods [2,3,6–10] for approximating ratios of euclidean path integrals. These methods are called "noncompact" because their basic variables are fields rather than group elements as in Wilson's "compact" method. One difference between the two kinds of method is that the compact method has an exact lattice gauge symmetry that is different from the gauge invariance of the continuum theory while the noncompact methods have an approximate version of the continuum gauge invariance. It is not clear which side of this tradeoff is more accurate for nonabelian theories. For $U(1)$ the noncompact method is accurate at all coupling strengths [7], whereas the compact method is accurate only at weak coupling.

Prior noncompact simulations of $SU(2)$, which were done with gauge fixing, also showed no sign of quark confinement. Patrascioiu, Seiler, and Stamatescu [2] used for the field strength of a plaquette the simple discretization

*Research supported by the U.S. Department of Energy under grant DE-FG04-84ER40166.
†Sabbatical address, 1987-'88.
‡Permanent address.

$$F^a(p_{\mu\nu}) = \frac{A^a_\mu(n+ae_\nu) - A^a_\mu(n)}{a} - \frac{A^a_\nu(n+ae_\mu) - A^a_\nu(n)}{a} + g f_{abc} A^b_\mu(n) A^c_\nu(n)$$

in which the vector n labels the vertices of the lattice. Their action was the sum

$$S = \sum_{p_{\mu\nu}} \frac{a^4}{2} F^a(p_{\mu\nu})^2$$

over all plaquettes $p_{\mu\nu}$ and colors a (but not also over μ and ν). Because this action has many zero modes, they fixed the gauge, choosing the temporal gauge for its theoretical simplicity. They saw a force law rather like Coulomb's law and found agreement with asymptotic freedom.

Seiler, Stamatescu, Wolff, and Zwanziger [3] used the more symmetrical field strength obtained by replacing the A's in the quadratic part of $F^a(p_{\mu\nu})$ by $\bar{A}^a_\mu(n) = \frac{1}{2}\left(A^a_\mu(n+ae_\nu) + A^a_\mu(n)\right)$ with a similar formula for $\bar{A}^a_\nu(n)$. The resulting action S has fewer zero modes but still requires gauge fixing. They used a technique called stochastic gauge fixing developed by Zwanziger [12]. Their simulations were also consistent with asymptotic freedom but not with quark confinement.

Gauge fixing is undesirable because the integrations over all gauge copies enforce Gauss's law and because it is not possible to transform an arbitrary gauge configuration into a particular gauge, such as the temporal gauge, while preserving periodic boundary conditions in a finite spacetime [10]. To be able to integrate over all gauge configurations, it is necessary to use an action that is quite free of zero modes. One way to do this is to interpolate the fields throughout spacetime from their values at the vertices of a lattice tiled with simplices [6–10]. In four dimensions, however, the Fortran source code is over 600 K bytes and the program is slow.

One may shorten the code and increase its speed by adopting Wilson's structure of links and plaquettes and linearly interpolating the fields throughout the plaquettes. The fields are then constant on the links of length a, the lattice spacing, but are interpolated linearly throughout the six transverse plaquettes. In the plaquette bounded by the vertices n, $n+ae_\mu$, $n+ae_\nu$, and $n+ae_\mu+ae_\nu$, the field is

$$A^a_\mu(x) = [(x_\nu - n_\nu)A^a_\mu(n+ae_\nu) + (n_\nu + a - x_\nu)A^a_\mu(n)]/a,$$

and the field strength is given by the continuum formula

$$F^a_{\mu\nu}(x) = \partial_\nu A^a_\mu(x) - \partial_\mu A^a_\nu(x) + g f_{abc} A^b_\mu(x) A^c_\nu(x).$$

The action is then the sum over all plaquettes of the integrals over each plaquette of the square of the field strength

$$S = \sum_{p_{\mu\nu}} \frac{a^2}{2} \int dx_\mu dx_\nu F^c_{\mu\nu}(x)^2.$$

Finally the mean-value in the vacuum of a euclidean-time-ordered operator $Q(A)$ is approximated by a normalized multiple integral over the $A^a_\mu(n)$'s —

$$\langle\Omega|\mathcal{T}Q(A)|\Omega\rangle \approx \frac{\int e^{-S(A)} Q(A) \prod_{\mu,a,n} dA^a_\mu(n)}{\int e^{-S(A)} \prod_{\mu,a,n} dA^a_\mu(n)}.$$

The quantity normally used to study confinement in quarkless gauge theories is the Wilson loop, which is the mean-value in the vacuum of the path-and-time-ordered exponential

$$W(r,t) = \langle\Omega|\mathcal{P}\mathcal{T} e^{-ig\oint A^a_\mu T_a dx_\mu}|\Omega\rangle/d,$$

where d is the dimension of the matrices T_a that represent the generators of the algebra of the gauge group and g is the coupling constant. Although Wilson loops vanish [13] in the exact theory, Creutz ratios [5] of Wilson loops defined as

$$\chi(r,t) \equiv -\log\left[\frac{W(r,t)W(r-a,t-a)}{W(r,t-a)W(r-a,t)}\right]$$

are finite and for large t provide an estimate of the static $q\bar{q}$ force.

For a representation of a group with N generators that satisfy the trace rule $\mathrm{Tr}(T_aT_b) = k\delta_{ab}$, the value of the Creutz ratio $\chi(r,t)$ in tree-level perturbation theory may be expressed in terms of the function

$$U(r,t) = (r/t)\arctan(r/t) + (t/r)\arctan(t/r) - \log(r^{-2} + t^{-2})$$

as

$$\chi(r,t) = \frac{kNg^2}{\pi^2 d}\left[-U(r,t) - U(r-a,t-a) + U(r,t-a) + U(r-a,t)\right].$$

Some values of this formula are listed in the table.

To measure Wilson loops and their Creutz ratios

$\chi(r,t)$ by means of the noncompact method. I used a 10^4 periodic lattice, began with cold starts, in which all fields were initialized to zero, and allowed 1000 sweeps for thermalization. I measured Wilson loops every 10 sweeps, using all the different r-by-t loops that occur in a 10^4 lattice, including periodic translations and rotations by $\pi/2$. I made 100 measurements at $\beta = 1$, fewer at $\beta = 30$ and 400. Some of the resulting values for the Creutz ratios $\chi(r,r)$ are listed in the table. These values are in approximate agreement both with perturbation theory and with the χ's obtained by Patrascioiu *et al.* [2], who found, for instance, at $\beta = 1$: $\chi(2,2) = 0.1$ and $\chi(3,3) = 0.02$.

Table: *Noncompact and perturbative Creutz ratios.*

β	$\frac{r}{a}, \frac{t}{a}$	Noncompact	Perturb.
1	2,2	0.1254(14)	
	3,3	0.0233(39)	
30	2,2	0.00619(10)	0.00661
	3,3	0.00191(25)	0.00218
	4,4	0.00067(47)	0.00109
400	2,2	0.000511(12)	0.000496
	3,3	0.000175(28)	0.000164
	4,4	0.000071(44)	0.000082

If the static force between heavy quarks is independent of distance, corresponding to a linear confining potential, then the Creutz ratios $\chi(r,t)$ should be independent of r and t at least for large t. In his compact simulations, Creutz found at $\beta = 3.25$: $\chi(3a,3a) = 0.048$ and $\chi(2a,2a) = 0.12$; at $\beta = 2.5$: $\chi(3a,3a) = 0.10$ and $\chi(2a,2a) = 0.21$; at $\beta = 2.25$: $\chi(3a,3a) = 0.22$ and $\chi(2a,2a) = 0.33$; and at $\beta = 2.0$: $\chi(2a,2a) = 0.60$. These data show a clumping of the χ's in that as $\beta \to 2$, $\chi(3,3)$ approaches $\chi(2,2)$, an effect verified in other Wilsonian simulations. In the present non-compact simulations, however, $\chi(3,3)$ is nearly six times smaller than $\chi(2,2)$ even at $\beta = 1$. So there is no sign of confinement in these noncompact simulations.

These results and those of the earlier gauge-fixed noncompact simulations of $SU(2)$ may present us with a clear conflict between the two methods. But the ratio of the energy scale of the continuum theory to that of Wilson's lattice theory is quite large, reaching $\Lambda^{MOM}_{(\alpha=1)}/\Lambda_W = 57.5$ for $SU(2)$ and 83.5 for $SU(3)$ [11]. And because of the closeness of the noncompact method to the continuum theory, it is reasonable to expect that the scale of the noncompact method is approximately that of the continuum theory, $\Lambda_{NC} \sim \Lambda^{MOM}_{(\alpha=1)}$. So for $SU(2)$, the lattice spacing of the Wilson theory $a_W(\beta)$ should be about 57.5 times as big as the lattice spacing $a_{NC}(\beta)$ of the noncompact method at the same value of β. Thus to display with the noncompact method the clumping of the χ's seen with the compact method on a distance scale of about $2a_W(2)$, one would have to run either at stronger coupling or on a much larger lattice.

It is probably necessary to run on a bigger lattice, because if the lattice spacing $a_{NC}(\beta)$ for $\beta \leq 2$ varies either as $a_W(\beta)$ or perturbatively, then it is not possible to enlarge $a_{NC}(\beta)$ by a factor of 57.5 merely by running at stronger coupling. For if $a_{NC}(\beta)$ for $\beta \leq 2$ varies like $a_W(\beta)$, *i.e.* as $\log(4/\beta)$ as reported by Creutz [5], then even at $\beta = 0.1$ the lattice spacing $a_{NC}(\beta)$ would swell by only a factor of 2.3. If a_{NC} varied perturbatively, *i.e.* as $a_{NC}(\beta) = \Lambda_{NC}^{-1}(\beta/2N\gamma_0)^{\gamma_1/2\gamma_0^2}\exp(-\beta/4N\gamma_0)$, then it would scale by about 49 at $\beta = 0.16$. If the energy scale Λ_{NC} really is the same as that of the continuum theory, and if the noncompact method does display confinement at the same distance scale as the compact method, then one would have to run simulations on a huge 200^2-by-360^2 lattice to see it.

We may, however, get some idea from presently available data as to whether the two methods are compatible by comparing them at a smaller value of the lattice spacing, for instance at $a_W(\beta) = a_{NC}(1)$. The lattice spacing $a_{NC}(1)$ is likely to be about 57.5 times smaller than $a_W(1)$ if the two methods represent the same physics apart from the energy scale Λ. Since [5] $a_W(\beta)$ varies as $\log(4/\beta)$ for $1 \leq \beta \leq 2$, $a_W(2)$ should be twice as small as $a_W(1)$. For $\beta \geq 2$, $a_W(\beta)$ varies approximately perturbatively. So by $\beta = 3.33$ it will have dropped by another factor of 28.75, for a total shrinkage by a factor of 57.5. Thus we expect that $\chi_{NC}(r,t)$ at $\beta = 1$ should resemble $\chi_W(r,t)$ at $\beta = 3.33$. We have compact data [5] at $\beta = 3.25$, which should be close enough for this comparison with my noncompact data at $\beta = 1$. For the $2a$-by-$2a$ ratios, we have approximate agreement: $\chi_{NC}(2,2) = 0.125$ and $\chi_W(2,2) = 0.12$. But for the $3a$-by-$3a$ ratios, we have a conflict: $\chi_{NC}(3,3) = 0.023$ and $\chi_W(3,3) = 0.048$. The compact method assigns to the static force between heavy quarks greater strength at $r = 3a_{NC}(1)$ than does the noncompact method. This discrepancy suggests that the two methods may describe different physics.

I have also run noncompact simulations of $SU(2)$ on a 5^4 lattice to see whether the Creutz ratios of the noncompact method exhibit asymptotic freedom, according to the criterion introduced by Creutz [14]. My preliminary results suggest that the χ_{NC}'s do display asymptotic freedom at $\beta = 400$, which is very weak coupling, $g_0 = 0.1$, but not at $\beta = 30$ or $\beta = 1$.

In conclusion, both the present noncompact simulations of $SU(2)$, which were done without gauge fixing, and earlier gauge-fixed noncompact simulations [2,3] show no sign of quark confinement on a 10^4 lattice. This absence of confinement offers some support for Gribov's [1] view that confinement occurs only in QCD with light quarks and not in pure or heavy-quark QCD. However, because the lattice spacing $a_W(\beta)$ of Wilson's method is probably much bigger than the lattice spacing $a_{NC}(\beta)$ of the noncompact method, it is possible that noncompact simulations on much larger lattices might exhibit confinement on the scale of $a_W(\beta)$. It would be interesting to see if that is, in fact, the case.

ACKNOWLEDGMENTS

I am grateful to S. Adler, M. Creutz, V. N. Gribov, and J. Smit for useful conversations and to F. Pipkin for hospitality.

REFERENCES

1. V. N. Gribov, *Physica Scripta* **T15** (1987) 164 and *Phys. Lett.* **194B** (1987) 119.
2. A. Patrascioiu, E. Seiler, and I. Stamatescu, *Phys. Lett.* **107B** (1981) 364.
3. I. Stamatescu, U. Wolff, and D. Zwanziger, *Nucl. Phys. B* **225** [FS9] (1983) 377 and E. Seiler, I. Stamatescu, and D. Zwanziger, *Nucl. Phys. B* **239** (1984) 177 and 201.
4. K. Wilson, *Phys. Rev. D* **10** (1974) 2445.
5. M. Creutz, *Phys. Rev. D* **21** (1980) 2308, *Phys. Rev. Letters* **45** (1980) 313, and M. Creutz and K. Moriarty, *Phys. Rev. D* **26** (1982) 2166.
6. K. Cahill, S. Prasad, and R. Reeder, *Phys. Lett.* **149B** (1984) 377 and K. Cahill and R. Reeder, in *Advances in Lattice Gauge Theory* (World Scientific, Singapore, 1985), p. 424.
7. K. Cahill and R. Reeder, *Phys. Lett.* **168B** (1986) 381 and *J. Stat. Phys.* **43** (1986) 1043.
8. K. Cahill, S. Prasad, R. Reeder, and B. Richert, *Phys. Lett.* **181B** (1986) 333.
9. K. Cahill, M. Hebert, and S. Prasad, *Phys. Lett.* **210B** (1988) 198.
10. K. Cahill and S. Prasad, "Wilson Loops in Three Dimensions," Harvard preprint HUTP-88/A025.
11. A. Hasenfratz and P. Hasenfratz, *Phys. Lett.* **93B** (1980) 165, and R. Dashen and D. Gross, *Phys. Rev. D* **23** (1981) 2340.
12. D. Zwanziger, *Nucl. Phys. B* **192** (1981) 259.
13. K. Cahill and D. Stump, *Phys. Rev. D* **20** (1979) 2096.
14. M. Creutz, *Phys. Rev. D* **23** (1981) 1815.