

Noncompact lattice simulations of SU(2) gauge theory

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Wilson loops have been measured at strong coupling, $\beta=0.5$, on a 12^4 lattice in noncompact simulations of pure SU(2) without gauge fixing. There is no sign of quark confinement.

1. Introduction

In 1980 Creutz [1] displayed quark confinement at moderate coupling in lattice simulations [2] of both abelian and nonabelian gauge theories. Whether nonabelian confinement is as much an artifact of Wilson's action as is abelian confinement remains unclear.

The basic variables of Wilson's formulation are elements of a compact group and enter the action only through traces of their products. Wilson's action has extra minima [3]. Mack and Pietarinen [4] and Grady [5] have shown that these false vacua affect the string tension. In their simulations of SU(2), they placed gauge-invariant infinite potential barriers between the true vacuum and the false vacua. Mack and Pietarinen saw a sharp drop in the string tension; Grady found that it vanished.

To avoid using an action that has confinement built in, some physicists have introduced lattice actions that are noncompact discretizations of the continuum action with fields at the basic variables [3,6–11]. Patrascioiu, Seiler, Stamatescu, Wolff and Zwanziger [6] performed the first non-compact simulations of SU(2) by using simple discretizations of the classical action. They fixed the gauge and saw a force rather like Coulomb's.

It is possible to use the exact classical action in a noncompact simulation if one interpolates the gauge fields from their values on the vertices of simplices [3,7,8] or hypercubes. For U(1) these noncompact formulations are accurate for general coupling

strengths [8]. But in four dimensions and without gauge fixing, such formulations are implementable only in code that is quite slow. One may develop faster code by interpolating across plaquettes rather than throughout simplices. For SU(2) such noncompact simulations agree well with perturbation theory at very weak coupling [9].

This report relates the results of measuring Wilson loops at strong coupling, $\beta \equiv 4/g^2 = 0.5$, on a 12^4 lattice in a noncompact simulation of SU(2) gauge theory without gauge fixing or fermions. Creutz ratios of large Wilson loops provide a lattice estimate of the $q\bar{q}$ -force for heavy quarks. There is no sign of quark confinement.

2. This noncompact method

In the present simulations, the action is free of spurious zero modes, and it is not necessary to fix the gauge. The fields are constant on the links of length a , the lattice spacing, but are interpolated linearly throughout the plaquettes. In the plaquette with vertices n , $n+e_\mu$, $n+e_\nu$, and $n+e_\mu+e_\nu$, the field is

$$A_\mu^a(x) = \left(\frac{x_\nu}{a} - n_\nu\right) A_\mu^a(n+e_\nu) + \left(n_\nu + 1 - \frac{x_\nu}{a}\right) A_\mu^a(n), \quad (1)$$

and the field strength is

$$F_{\mu\nu}^a(x) = \partial_\nu A_\mu^a(x) - \partial_\mu A_\nu^a(x) + gf_{bc}^a A_\mu^b(x) A_\nu^c(x). \quad (2)$$

The action S is the sum over all plaquettes of the integral over each plaquette of the squared field strength,

$$S = \sum_{\mu\nu} \frac{a^2}{2} \int dx_\mu dx_\nu F_{\mu\nu}^c(x)^2. \quad (3)$$

The mean value in the vacuum of a euclidean time-ordered operator $W(A)$ is approximated by a normalized multiple integral over the $A_\mu^a(n)$,

$$\langle \mathcal{F}W(A) \rangle_0 \approx \frac{\int \exp[-S(A)] W(A) \prod_{\mu,a,n} dA_\mu^a(n)}{\int \exp[-S(A)] \prod_{\mu,a,n} dA_\mu^a(n)}, \quad (4)$$

which one may compute numerically [1]. I used MACSYMA to write the FORTRAN code [10].

3. Creutz ratios

The quantity normally used to study confinement in quarkless gauge theories is the Wilson loop $W(r, t)$ which is the mean value in the vacuum of the path-and-time-ordered exponential

$$W(r, t) = \frac{1}{d} \left\langle \mathcal{P} \exp \left(-ig \oint A_\mu^a T_a dx_\mu \right) \right\rangle_0, \quad (5)$$

divided by the dimension d of the matrices T_a that represent the generators of the gauge group. Although Wilson loops vanish [12] in the exact theory, Creutz ratios $\chi(r, t)$ of Wilson loops defined [1] as double differences of logarithms of Wilson loops

$$\chi(r, t) = -\log W(r, t) - \log W(r-a, t-a) + \log W(r-a, t) + \log W(r, t-a), \quad (6)$$

are finite. For large t , the Creutz ratio $\chi(r, t)$ approximates (a^2 times) the force between a quark and an antiquark separated by the distance r .

For a compact Lie group with N generators T_a normalized as $\text{Tr}(T_a T_b) = k\delta_{ab}$, the lowest-order perturbative formula for the Creutz ratio is

$$\chi(r, t) = \frac{N}{2\pi^2\beta} [-f(r, t) - f(r-a, t-a) + f(r, t-a) + f(r-a, t)], \quad (7)$$

where the function $f(r, t)$ is

$$f(r, t) = \frac{r}{t} \arctan \frac{r}{t} + \frac{t}{r} \arctan \frac{t}{r} - \log \left(\frac{a^2}{r^2} + \frac{a^2}{t^2} \right), \quad (8)$$

and β is the inverse coupling $\beta = d/kg^2$.

4. Measurements and results

To measure Wilson loops and their Creutz ratios, I used a 12^4 periodic lattice, a heat bath, and 20 independent runs with cold starts. The first run began with 25 000 thermalizing sweeps at $\beta=2$ followed by 5000 at $\beta=0.5$; the other nineteen runs began at $\beta=0.5$ with 20 000 thermalizing sweeps. In all I made 59 640 measurements, 20 sweeps apart. I used a version of Parisi's trick [13] that respects the dependencies that occur in corners of loops and between lines separated by a single lattice spacing. The values of the Creutz ratios so obtained are listed in table 1 along with the tree-level theoretical values as given by eqs. (7), (8). I estimated the errors by the jackknife method [14], assuming that all measurements were independent. Binning in small groups made little difference.

If the static force between heavy quarks is independent of distance, then the Creutz ratios $\chi(r, t)$ for large t should be independent of r and t . The measured $\chi(r, t)$'s are smaller than their tree-level perturbative counterparts. The measured $\chi(r, t)$'s also fall faster with increasing loop size. There is no sign of confinement.

5. Plausible interpretations

Why don't noncompact simulations display quark

Table 1
Noncompact Creutz ratios at $\beta=0.5$.

$r/a \times t/a$	Monte Carlo	Order $1/\beta$
2×2	0.23107(4)	0.39648
3×3	0.03576(11)	0.13092
4×4	0.00485(29)	0.06529
5×5	0.00094(82)	0.03913
6×6	-0.00149(226)	0.02608

confinement? Here are some possible answers:

(1) Although noncompact methods have approximate forms of all continuum symmetries, including gauge invariance, they lack an exact lattice gauge invariance. It may be possible to impose a kind of lattice gauge invariance by having the fields randomly make suitably weighted compact

$$\exp[-igaA_\mu^b(n)T_b] \\ = U(n+e_\mu) \exp[-igaA_\mu^b(n)T_b] U(n)^\dagger, \quad (9)$$

or noncompact gauge transformations.

(2) The Maxwell–Yang–Mills action contains squares of (covariant) curls of gauge fields, not squares of derivatives of gauge fields. Thus the gauge fields may vary markedly from one link to the next. In fact in these noncompact simulations, the ratio of differences of adjacent gauge fields to their moduli

$$\frac{\langle |A_\mu^b(n+e_\nu) - A_\mu^b(n)| \rangle}{\langle |A_\mu^b(n)| \rangle} \quad (10)$$

exceeds unity for all couplings from $\beta=0.5$ to $\beta=60$ and all μ and ν . It is not obvious that noncompact methods can cope with such choppiness.

(3) The noncompact lattice spacing $a_{\text{NC}}(\beta)$ is probably smaller than the compact one $a_{\text{C}}(\beta)$. Thus noncompact methods may accommodate too small a volume at weak coupling; confinement might appear in noncompact simulations done on much larger lattices or at stronger coupling. Both possibilities would be expensive to test.

(4) Perhaps $SU(3)$, but not $SU(2)$, confines.

(5) Possibly as Gribov has suggested [15], quark confinement is due to the lightness of the up and down quarks and not a feature of pure QCD.

(6) As Polonyi [16] has suggested, it may be necessary in lattice definitions of path integrals in gauge theories to enforce Gauss's law by integrations over the group manifold weighted by the Haar measure. In a preliminary test of this idea, I measured the Polyakov line of length $12a$ at $\beta=0.5$ to be $0.00853(67)$ with the Haar measure and $0.01135(1)$ without it.

(7) Confinement is a robust and striking phenomenon. Maybe the true continuum theory is one like Wilson's that can directly account for it. The hybrid measure

$$\exp\left\{-\int \frac{Dc}{kg^2} f(c, l)\right\} \\ \times \text{Tr}\left[1 - \mathcal{R}\mathcal{P} \exp\left(ig \oint_c A_\mu^a T_a dx_\mu\right)\right] d\mu(A), \quad (11)$$

reduces to Wilson's prescription if the weight functional $f(c, l)$ of the path integration over closed curves c is a delta functional with support on the plaquettes and if $d\mu(A)$ incorporates the Haar measure. A weight functional like $f(c, l) \sim \exp[-(\|c\|/l)^4]$, where $\|c\|$ is the length of the curve, might give confinement for distances much longer than l and perturbative QCD for much shorter distances.

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