New dates for the eras of the universe from the Planck data

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Abstract

Data from the Planck satellite imply new dates for the major eras of the universe. The era of radiation ended 50,150 years after inflation and the era of matter 10.31 billion years after inflation or 3.51 billion years ago.

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I. THE PLANCK DATA

The Planck Collaboration and the European Space Agency have published their remarkable data in a paper on the cosmological parameters [1] and in 31 other arXiv articles. These data imply that the era of radiation ended 15 thousand years earlier and the era of matter ended 1.5 Gyr later than the WMAP data had suggested.

The Planck Collaboration used $T_0 = 2.7255 \pm 0.0006$ K [2] as the current temperature of the (known) universe and listed in column 7 of Table 2 of their paper on cosmological parameters [1] these data: The Hubble constant is $H_0 = 67.3 \pm 1.2$ km/sMpc; the fraction of the critical energy density $\rho_c = 3H_0^2/8\pi G = 8.506 \times 10^{-30}$ g cm$^{-3} = [0.002461$ eV$]^4$ that is due to dark energy is $\Omega_\Lambda = 0.685^{+0.018}_{-0.016}$; the fraction due to matter is $\Omega_m = 0.315^{+0.016}_{-0.018}$; and the fraction due to ordinary matter is $\Omega_b = 0.0487 \pm .0006$ or 15.5% of the total matter fraction. The ratio of the energy density to the critical energy density is $\Omega = 1.000 \pm 0.036$. The age of the known universe is 13.817 ± 0.048 Gyr.

I use these data and some standard formulas to estimate first in section II the time at which the energy density of matter was the same as that of dark energy and then in section III the time at which the energy density of matter was the same as that of radiation. Both computations are straightforward exercises. The paper ends with a display and brief discussion of two remarkable Planck CMB figures.

II. THE END OF THE ERA OF MATTER

Baryonic matter, mostly hydrogen and helium, is stable on a time scales that are much longer than the age of the universe. To estimate the end of the era of matter, I will assume that dark matter also is stable on time scales of tens of billions of years (or that it decays into other forms of matter). In this case, the energy density ρ_m of all matter and the scale factor a at any given time (after inflation) are related to their current values $\rho_{m,0}$ and a_0 by

$$a^3 \, \rho_m = a_0^3 \, \rho_{m,0}. \tag{1}$$

I assume that the density of dark energy is constant

$$\rho_v = \rho_{v,0}. \tag{2}$$

The era of matter ended when $\rho_m = \rho_v$, which occurred when the scale factor satsified

$$\rho_m = a_0^3 \, \rho_{m,0}/a^3 = \rho_v \tag{3}$$

or

$$\frac{a}{a_0} = \left(\frac{\rho_m}{\rho_v}\right)^{1/3} = \left(\frac{\Omega_m}{\Omega_\Lambda}\right)^{1/3} = \left(\frac{0.315}{0.685}\right)^{1/3} = 0.7719. \tag{4}$$

This is a redshift of

$$z = a_0/a - 1 = 0.2956. \tag{5}$$

To convert this redshift into a time, we use Eq.(1.5.42) of Weinberg's *Cosmology* [3]

$$t(z) = \frac{1}{H_0} \int_0^{a/a_0} \frac{1}{\sqrt{\Omega_\Lambda + \Omega_k x^{-2} + \Omega_m x^{-3} + \Omega_r x^{-4}}} \frac{dx}{x} \tag{6}$$

in which $\Omega_k = -K/(a_0 \, H_0)^2$ (with $K = -1$, 0, or 1) is effectively zero. By Eq.(2.1.10) of his *Cosmology*, the fraction Ω_r of the energy density that is due to radiation is

$$\Omega_r = 4.15 \times 10^{-5} \, h^{-2} = 4.15 \times 10^{-5}/(0.673)^2 = 9.1626 \times 10^{-5}. \tag{7}$$

Doing the integral (6), we find that the era of matter ended at about

$$t(0.2956) = 10.309 \text{ Gyr} \tag{8}$$

after inflation, give or take a few million years. Thus the era of dark energy began about 3.508 billion years ago.

III.   THE END OF THE ERA OF RADIATION

I will take the energy of radiation as due to photons and nearly massless neutrinos whose wavelengths are stretched by the scale factor a. Their energy density ρ_r then is related to its current value $\rho_{r,0}$ by

$$a^4 \, \rho_r = a_0^4 \, \rho_{r,0}. \tag{9}$$

The era of radiation ended when

$$\rho_r = a_0^4 \, \rho_{r,0}/a^4 = \rho_m = a_0^3 \, \rho_{m,0}/a^3 \tag{10}$$

or when

$$\frac{a}{a_0} = \frac{\rho_{r,0}}{\rho_{m,0}} = \frac{\Omega_r}{\Omega_m} = \frac{9.1626 \times 10^{-5}}{0.315} = 2.9088 \times 10^{-4} \tag{11}$$
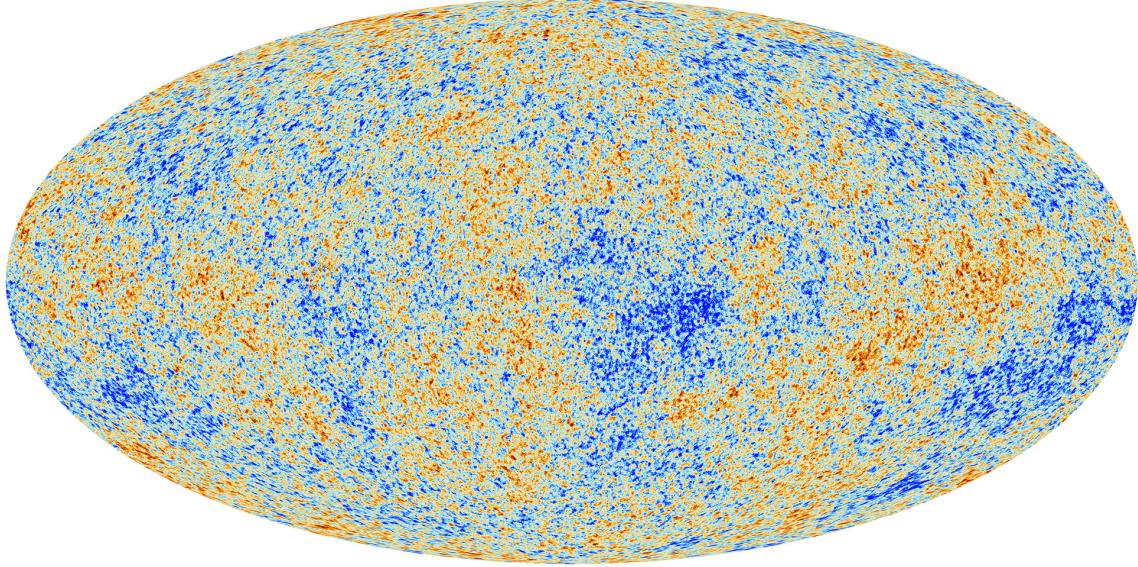
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FIG. 1. CMB temperature fluctuations over the celestial sphere as measured by the Planck satellite. The average temperature is 2.7255 K. Red regions are warmer and blue ones colder by $\lesssim 500\,\mu$K. © ESA and the Planck Collaboration.

which is a redshift of

$$z = a_0/a - 1 = 3436.9. \tag{12}$$

Using the value (11) as the upper limit of the integral (6), we get as the end of the era of radiation

$$t(3436.9) = 50,152 \text{ yr} \tag{13}$$

after inflation, give or take a few decades.

IV. TWO IMAGES

I cannot resist displaying two extraordinary figures published by the Planck Collaboration. The first is their full-sky image of the tiny fluctuations in the temperature of the cosmic microwave background radiation. In this Fig. 1, red is hotter and blue colder by $\lesssim 500\,\mu$K.

The second remarkable figure is their graph of the angular correlations of the full-sky, CMB temperature data shown in Fig. 2. They expanded the temperature $T(\theta, \phi)$ in spherical harmonics as [4]

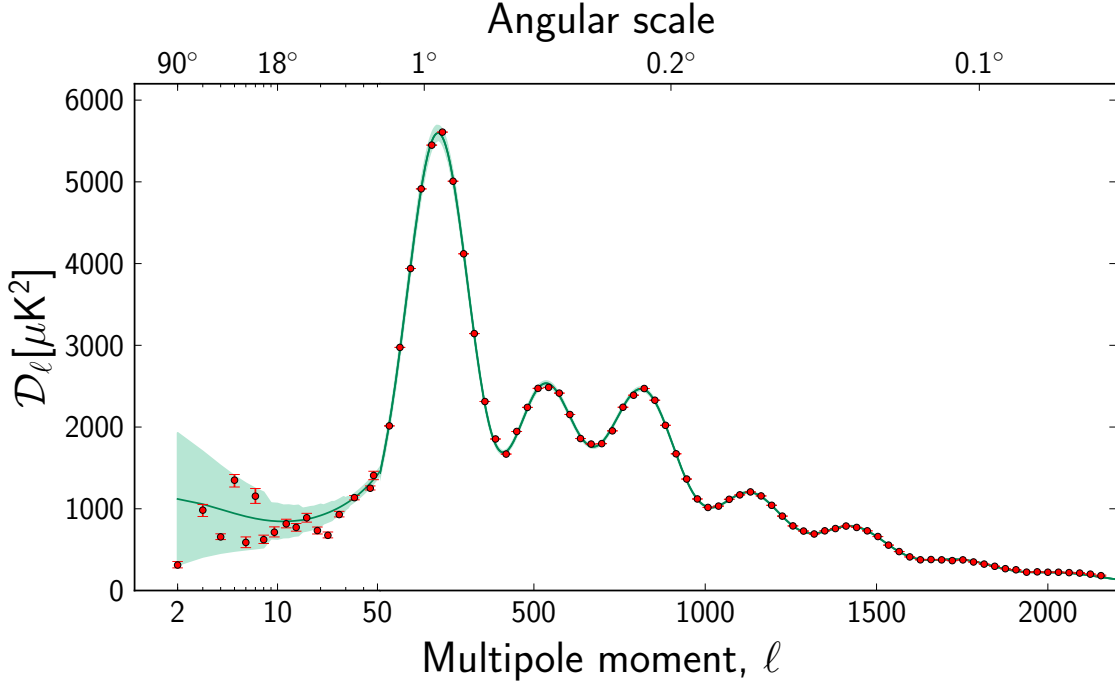$$T(\theta, \phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell,m} Y_{\ell,m}(\theta, \phi) \tag{14}$$

FIG. 2. The power spectrum $\mathcal{D}_\ell = \ell(\ell+1)C_\ell/2\pi$ of the CMB temperature fluctuations in μK^2 as measured by the Planck Collaboration (arXiv:1303.5062) is plotted against the angular size and the multipole moment ℓ. The solid curve is the ΛCDM prediction.

in which the coefficients are

$$a_{\ell,m} = \int_0^{2\pi} d\phi \int_0^\pi \sin\theta\, d\theta\ Y^*_{\ell,m}(\theta,\phi)\, T(\theta,\phi). \tag{15}$$

The average temperature $\overline{T}$ contributes only to $a_{0,0} = \overline{T} = 2.7255$ K. The other coefficients describe the difference $\Delta T(\theta,\phi) = T(\theta,\phi) - \overline{T}$. The angular power spectrum is

$$C_\ell = \frac{1}{2\ell+1} \sum_{m=-\ell}^{\ell} |a_{\ell,m}|^2. \tag{16}$$

They displayed these coefficients scaled by $\ell(\ell+1)/2\pi$ in Fig. 2.

This power spectrum is a snapshot at the moment of initial transparency (380,000 years after inflation) of the temperature distribution of the plasma of photons, electrons, and nuclei undergoing acoustic oscillations. In these oscillations, gravity opposes radiation pressure, and $|\Delta T(\theta,\phi)|$ is maximal both when the oscillations are most compressed and when they are most rarefied. Regions that gravity has been squeezing since inflation and that reach maximum compression at the moment of initial transparency form the first and highest peak.

Regions that have bounced off their first maximal compression and that radiation pressure has expanded to minimum density at initial transparency form the second peak. Regions that reach their second maximal compression at transparency form the third peak, and so forth. The agreement of the ΛCDM model with the Planck data is excellent for $\ell > 40$. Discrepancies below $\ell = 40$ may point to new physics.

ACKNOWLEDGMENTS

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[3] S. Weinberg, *Cosmology* (Oxford University Press, 2010).

[4] K. Cahill, *Physical Mathematics* (Cambridge University Press, 2013) Chap. 8.