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The masses of the lighter quarks

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Abstract. Simple observations are made about both the current masses and the constituent masses of the lighter quarks. The latest values of the decay constants f_π, f_K, and f_D of the pseudoscalar mesons suggest that the current-quark mass ratio $2s/(u+d)$ of the mass of strange quark to the average mass of the up and down quarks is closer to 14 than to 26 as is usually assumed. Such heavier masses for m_u and m_d are also implied by a model in which the masses of the first family of fermions arise from their electric and chromoelectric fields rather than from the Higgs mechanism. Since constituent-quark masses incorporate some of the energy of the interaction, which is different for mesons and baryons, it might be useful to consider two sets of constituent-quark masses, one for mesons and one for baryons. Plausible quark mass values in MeV are u = 303, d = 307, s = 490 and c = 1668 for the meson constituents and U = 326, D = 328, S = 510 and C = 1726 for the baryon constituents.

1. Introduction

Because quarks have not been observed as free particles, their masses remain somewhat ambiguous, particularly those of the light quarks u, d and s. Accordingly it has been a common practice for many years to distinguish the mass parameters that appear in the renormalized Lagrangian of QCD from the effective masses that work well in non-relativistic or quasi-relativistic potential models. At a mass scale of 1 GeV, the running Lagrangian or current-quark masses are typically quoted [1] as $u = 5.6 \pm 1.1$, $d = 9.9 \pm 1.1$, $s = 199 \pm 33$ and $c = 1350 \pm 50$ MeV. The effective or constituent-quark masses are greater because they incorporate some of the interaction energy; typical values [2,3] for these masses are u = 300, d = 304, s = 500, and c = 1600 MeV, but smaller values have been used by some authors [4,5].

In what follows I shall suggest that the constituent-quark masses are not unique and that we might profitably use two sets—one set for the constituents of mesons and one set for the constituents of baryons. I shall then use the decay constants of the pseudoscalar mesons to argue that the ratio $2s/(u+d)$ of the Lagrangian mass of the strange quark to the average Lagrangian mass of the up and down quarks is closer to 14 than to 26 as is usually assumed. These heavier masses for the up and down quarks fit in better than the usual lighter masses with a scheme in which the masses of the first family of fermions arise from their electric and chromoelectric fields, which are required by the Gauss law, rather than from the Higgs mechanism.

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2. Constituent-quark masses

Constituent-quark masses are much larger than current-quark masses because they incorporate some of the energy of the QCD and electroweak fields. Since the interaction between a quark and an antiquark is different from the interaction between three quarks, it follows that the interaction energy that contributes to the masses of the quark and antiquark in a meson is different from that which contributes to the masses of the quarks in a baryon. Thus one should use two sets of constituent-quark masses, one set for mesons and one set for baryons.

2.1. Constituent quarks in mesons

We may find suitable sets of masses by following procedures developed by Lichtenberg [6]. In the non-relativistic quark model, the masses of the ground-state hadrons (the pseudoscalar and vector octets of mesons and the octet and decouplet of baryons) depend on the spins of the quarks through the colour–hyperfine interaction. But in certain 'spin-averaged' linear combinations of hadron masses, the energy of the colour–hyperfine interaction cancels [6]. For the ground-state mesons, the appropriate linear combination is the mass of the pseudoscalar plus three times the mass of the vector. Thus if we assume that the kinetic and potential energies sum to a small fraction of the mass of a hadron, then we may estimate the average effective mass of the up and down quarks in mesons as

$$\tfrac{1}{2}(\mathrm{u}+\mathrm{d}) = \tfrac{1}{8}(\pi + 3\rho) = 305\ \mathrm{MeV} \tag{1}$$

where π and ρ denote the masses [1] of those mesons averaged over the charge states. A similar spin-average of the masses of the K and K^* gives the average of $(\mathrm{u}+\mathrm{d})/2$ and the mass s of the strange quark as

$$\tfrac{1}{2}\left[\tfrac{1}{2}(\mathrm{u}+\mathrm{d})+\mathrm{s}\right] = \tfrac{1}{8}(\mathrm{K}+3\mathrm{K}^*) = 397.3. \tag{2}$$

Thus the mass of the strange constituent quark in mesons is

$$\mathrm{s} = 490\ \mathrm{MeV}. \tag{3}$$

A spin-average of the masses of the charmed mesons D and D^* gives the average of $(\mathrm{u}+\mathrm{d})/2$ and the mass c of the charmed quark as

$$\tfrac{1}{2}\left[\tfrac{1}{2}(\mathrm{u}+\mathrm{d})+\mathrm{c}\right] = \tfrac{1}{8}(\mathrm{D}+3\mathrm{D}^*) = 986.6 \tag{4}$$

so the mass of the constituent charmed quark in mesons is

$$\mathrm{c} = 1668\ \mathrm{MeV}. \tag{5}$$

Similarly the difference between the constituent masses of the down and up quarks is the spin-averaged difference

$$\mathrm{d}-\mathrm{u} = \tfrac{1}{4}\left[\mathrm{K}^0 - \mathrm{K}^+ + 3(\mathrm{K}^{0*} - \mathrm{K}^{+*})\right] = 4\ \mathrm{MeV}. \tag{6}$$

Thus we find for the masses of the constituent quarks in mesons the values

$$\mathrm{u} = 303 \qquad \mathrm{d} = 307 \qquad \mathrm{s} = 490 \qquad \mathrm{c} = 1668\ \mathrm{MeV}. \tag{7}$$

Because the value of a confining potential is not easily inferred, we are actually free to add a common mass to the masses of all these constituents.

2.2. Constituent quarks in baryons

For the baryons we may perform a similar analysis by using the spin averages [6]

$$\tfrac{1}{2}(\mathrm{U}+\mathrm{D}) = \tfrac{1}{6}(\mathrm{N}+\Delta) = 362 \text{ MeV} \tag{8}$$

and

$$\mathrm{U}+\mathrm{D}+\mathrm{S} = \tfrac{1}{4}\left(\Lambda + \Sigma^0 + 2\Sigma^{0*}\right) = 1269 \text{ MeV} \tag{9}$$

which imply for the mass S of the strange quark in a baryon the value

$$\mathrm{S} = 545 \text{ MeV}. \tag{10}$$

The difference between the constituent masses of the down and up quarks is harder to estimate; I shall take the partially spin-averaged difference

$$\mathrm{D}-\mathrm{U} = \tfrac{1}{5}\left[\mathrm{N}-\mathrm{P}+\tfrac{1}{3}\left(\Delta^- - \Delta^{++}\right)+\tfrac{1}{2}\left(\Sigma^- - \Sigma^+\right)+\Sigma^{*-}-\Sigma^{*+}\right] = 2 \text{ MeV}. \tag{11}$$

The masses of the charmed baryons are not known well enough for us to perform spin-averages, but we may infer approximately the mass C of the charmed quark in baryons by averaging differences between the ordinary and the charmed hyperons

$$\mathrm{C}-\mathrm{S} = \tfrac{1}{2}\left[\Lambda_c^+ - \Lambda + \Sigma_c^{++} - \Sigma^+\right] = 1216 \text{ MeV} \tag{12}$$

obtaining

$$\mathrm{C} = 1761 \text{ MeV}. \tag{13}$$

Thus our values for the masses of the constituent quarks in baryons are U = 361, D = 363, S = 545 and C = 1761 MeV to within a common additive term.

By using Lichtenberg's technique of spin-averaging [6], we have found for the masses of the constituent quarks in mesons the values

$$\mathrm{u} = 303+\delta m \qquad \mathrm{d} = 307+\delta m \qquad \mathrm{s} = 490+\delta m \qquad \mathrm{c} = 1668+\delta m \ \text{ MeV} \tag{14}$$

and for those in baryons the values

$$\mathrm{U} = 361+\delta M \qquad \mathrm{D} = 363+\delta M \qquad \mathrm{S} = 545+\delta M \qquad \mathrm{C} = 1761+\delta M \ \text{ MeV}. \tag{15}$$

Both sets of masses contain additive unknown masses δm and δM which reflect ambiguities in the confining potentials. Apart from δm and δM, the meson constituents are somewhat lighter than the baryon constituents. We are free to choose δm and δM separately so as to fit various properties of the mesons and baryons.

This distinction between the values of δm and δM, or equivalently between the masses of the constituent quarks in mesons and the masses of those in baryons, is a natural one that should lead to better agreement between the non-relativistic quark model and experiment. Thus there may be particular values of δm that lead to good fits for various properties of mesons, such as the masses of the tensor mesons. For instance, the choice $\delta m \approx -80$ MeV brings the masses of our meson constituents closer to those used by Godfrey and Isgur in their calculations of the properties of mesons [4]. Similarly, one may adjust δM to fit the magnetic moments of the baryons [1,7–9]† or the masses of their excited states.

To fit the baryon magnetic moments, we assume Dirac magnetic moments for the quarks. The magnetic moment μ_Λ of the Λ is then $\mu_\Lambda = \mu_S = -P/(3S) = 0.613$ nuclear magnetons. We may fit this value exactly by letting $\delta M = -35$ MeV so that $S = 510$ MeV. With this change, the masses of the constituent quarks in baryons are

$$U = 326 \qquad D = 328 \qquad S = 510 \qquad C = 1726 \text{ MeV} \tag{16}$$

and the magnetic moments of the proton and neutron are $\mu_P = 2.88$ nm, which is 2.8% too high, and $\mu_N = -1.91$ nm, which is correct to three significant figures. The resulting magnetic moments in nuclear magnetons for other baryons (with the measured values [1, 10] in parentheses) are $\mu_{\Sigma^+} = 2.76(2.42)$, $\mu_{\Sigma^-} = -1.07(-1.16)$, $\mu_{\Sigma^0} = 0.85$, $\mu_{\Sigma^0 \to \Lambda} = -1.66(-1.61)$, $\mu_{\Xi^0} = -1.46(-1.25)$, $\mu_{\Xi^-} = -0.50(-0.65)$ and $\mu_{\Omega^-} = -1.84(-2.0)$. These values are closer than those that follow from the masses u, d and s of the meson-constituent quarks, even when they are increased by 20 MeV. They are almost as good as those obtained by adjusting the masses of all three quarks to get the exact values for μ_P, μ_N and μ_Λ. The values of the μ_Σ and μ_Ξ are sensitive to configuration mixing [7–9].

The main point of the first half of this paper is that it makes sense to use two sets of masses for constituent quarks, one set for the constituents of mesons and one set for the constituents of baryons. The values of the masses quoted in (14)–(16) are meant to illustrate of the freedom of using two such sets of constituent-quark masses, with additive constants δm and δM. These quark masses are not advertised as new or definitive. In computing them I have neglected many electromagnetic and some QCD effects as well as configuration mixing in the Λ hyperon. The reader is invited to do better.

3. Current-quark masses

The basis of my discussion of Lagrangian or current-quark masses is the decay constants f_π, f_K and f_D of the charged pseudoscalar mesons. For a meson of type a, the decay constant f_a is defined by

$$\langle 0 | j^\mu_{5a}(x) | a(p) \rangle = \mathrm{i} f_a p^\mu \mathrm{e}^{-\mathrm{i}px} \tag{17}$$

in which $j^\mu_{5a} = \bar{q}\gamma^\mu\gamma_5 q'$ is the part of the charged axial-vector weak current that is bilinear in the valence quarks q and q′ of the meson a, with the Cabbibo–Kobayashi–Maskawa mixing matrix element $V_{qq'}$ removed. The state vector $|a(p)\rangle$ is normalized as

$$\langle a(p') | a(p) \rangle = (2\pi)^3 2p^0 \delta^{(3)}(p' - p) \tag{18}$$

† I am grateful to B-K Luk of Fermilab E-756 for providing the preliminary values of -2.0 ± 0.3 for the magnetic moment of the Ω^- and -0.65 for that of the Ξ^-.

and its phase is chosen so that f_a is real and positive. The measured values [1]† of the decay constants are

$$\begin{aligned} f_\pi &= 131.74 \pm 0.15 \ \text{MeV} \\ f_{\mathrm{K}} &= 160.6 \pm 1.4 \ \text{MeV} \\ f_{\mathrm{D}} &< 310 \ \text{MeV (CL = 90\%)}. \end{aligned} \tag{19}$$

There are two ways to compute the matrix elements of the divergence $\partial_\mu j^\mu_{5a}$ of the axial-vector current. One is to use the definition (17) of f_a

$$\langle 0|\partial_\mu j^\mu_{5a}(x)|a(p)\rangle = f_a \mu_a^2 \mathrm{e}^{-\mathrm{i}px}. \tag{20}$$

The other is to use the field equations of QCD (neglecting electroweak effects)

$$\partial_\mu j^\mu_{5a}(x) = \partial_\mu(\bar{\mathrm{q}}(x)\gamma^\mu\gamma_5 \mathrm{q}'(x)) = \mathrm{i}(m_{\mathrm{q}} + m_{\mathrm{q}'})\bar{\mathrm{q}}(x)\gamma_5\mathrm{q}'(x). \tag{21}$$

By combining the two results, we have at $x = 0$

$$f_a\mu_a^2 = \mathrm{i}(m_{\mathrm{q}} + m_{\mathrm{q}'})\langle 0|\bar{\mathrm{q}}(0)\gamma_5\mathrm{q}'(0)|a(p)\rangle. \tag{22}$$

One might be tempted to conclude that the ratio $f_a\mu_a^2/(m_{\mathrm{q}} + m_{\mathrm{q}'})$ is independent of the meson type a. But that would be a mistake for two reasons. The first is that the normalization of the meson states contributes a factor of $\sqrt{p^0}$, which for mesons at rest is $\sqrt{\mu_a}$. The second is that the matrix element in (22) is approximately proportional to the meson wavefunction of the valence quarks at zero relative separation. There is some evidence [10] that the wavefunctions of the pseudoscalar mesons at zero relative separation are in turn proportional to the square-roots of the reduced masses of the pairs of constituent quarks. If we use M_{u}, M_{d}, etc, to denote the constituent-quark masses, then we have for instance $\psi_{\pi^+}(0) \propto \langle 0|\bar{\mathrm{d}}(0)\gamma_5\mathrm{u}(0)|\pi^+\rangle \propto \sqrt{M_{\mathrm{u}}M_{\mathrm{d}}/(M_{\mathrm{u}} + M_{\mathrm{d}})}$. Thus it is the ratio

$$\frac{f_a\mu_a^{3/2}\sqrt{M_{\mathrm{q}} + M_{\mathrm{q}'}}}{(m_{\mathrm{q}} + m_{\mathrm{q}'})\sqrt{M_{\mathrm{q}}M_{\mathrm{q}'}}} \tag{23}$$

that is approximately independent of the meson type a. So in terms of the reduced masses of the pairs of constituent quarks

$$\mu(\mathrm{q}, \mathrm{q}') = \frac{M_{\mathrm{q}}M_{\mathrm{q}'}}{M_{\mathrm{q}} + M_{\mathrm{q}'}} \tag{24}$$

we have the relations

$$\frac{f_\pi(\pi^+)^{3/2}}{(m_{\mathrm{u}} + m_{\mathrm{d}})\sqrt{\mu(\mathrm{u},\mathrm{d})}} \approx \frac{f_{\mathrm{K}}(\mathrm{K}^+)^{3/2}}{(m_{\mathrm{u}} + m_{\mathrm{s}})\sqrt{\mu(\mathrm{u},\mathrm{s})}} \approx \frac{f_{\mathrm{D}}(\mathrm{D}^+)^{3/2}}{(m_{\mathrm{c}} + m_{\mathrm{d}})\sqrt{\mu(\mathrm{c},\mathrm{d})}} \tag{25}$$

where π^+, K^+ and D^+ denote the masses of those mesons.

† I am grateful to G Wagman of the Particle Data Group for providing me with these latest values of the decay constants of the pseudoscalar mesons.

We may now solve these equations for the masses m_u, m_d and m_s of the three light quarks in terms of the mass m_c of the charmed quark, the decay constant f_D, the mass difference $m_d - m_u$ and the masses of the constituent quarks of mesons. After a little algebra, we find for the ratio of the mass of the strange quark to the average of the masses of the up and down quarks

$$\frac{2m_s}{m_u + m_d} = 2\frac{f_K}{f_\pi}\left(\frac{K^+}{\pi^+}\right)^{3/2}\left(\frac{\mu(u,d)}{\mu(u,s)}\right)^{1/2} - 1 + \frac{m_d - m_u}{m_c}\left[\frac{f_D}{f_\pi}\left(\frac{D^+}{\pi^+}\right)^{3/2}\left(\frac{\mu(u,d)}{\mu(c,d)}\right)^{1/2} - 1\right]. \tag{26}$$

The last term, with the factor $(m_d - m_u)/m_c$, is small; if we ignore it, then this formula gives for strange-to-up–down ratio the value

$$\frac{2m_s}{m_u + m_d} \approx 2\frac{f_K}{f_\pi}\left(\frac{K^+}{\pi^+}\right)^{3/2}\left(\frac{\mu(u,d)}{\mu(u,s)}\right)^{1/2} - 1 \approx 13.6 \tag{27}$$

independently of the charmed-quark sector and of the mass difference $m_d - m_u$. Similarly the ratio of the mass of the strange quark to that of the charmed quark is

$$\frac{m_s}{m_c} = \left[2\frac{f_K(K^+)^{3/2}}{\sqrt{\mu(u,s)}} - \frac{f_\pi(\pi^+)^{3/2}}{\sqrt{\mu(u,d)}} + \frac{m_d - m_u}{m_c}\left(\frac{f_K(K^+)^{3/2}}{\sqrt{\mu(u,s)}} + \frac{f_D(D^+)^{3/2}}{\sqrt{\mu(c,d)}} - \frac{f_\pi(\pi^+)^{3/2}}{\sqrt{\mu(u,d)}}\right)\right]\left(2\frac{f_D(D^+)^{3/2}}{\sqrt{\mu(c,d)}} - \frac{f_\pi(\pi^+)^{3/2}}{\sqrt{\mu(u,d)}}\right)^{-1} \tag{28}$$

or since the term with the factor $(m_d - m_u)/m_c$ is small

$$\frac{m_s}{m_c} \approx \left(2\frac{f_K(K^+)^{3/2}}{\sqrt{\mu(u,s)}} - \frac{f_\pi(\pi^+)^{3/2}}{\sqrt{\mu(u,d)}}\right)\left(2\frac{f_D(D^+)^{3/2}}{\sqrt{\mu(c,d)}} - \frac{f_\pi(\pi^+)^{3/2}}{\sqrt{\mu(u,d)}}\right)^{-1}. \tag{29}$$

The Lagrangian mass of the charmed quark probably lies in the interval

$$1300 < m_c < 1600 \text{ MeV}. \tag{30}$$

The lower limit is that given as the lower limit by the Particle Data Group [1]; the upper limit is 66 MeV above the constituent-quark mass that results from a spin-average of the masses of the J/Ψ and the η_c.

The value of f_D is less than 310 MeV [1] and probably lies in the range

$$f_D = 207 \pm 60 \text{ MeV} \tag{31}$$

suggested by Rosner in his recent analysis [11] of weak B-meson decays.

One way of estimating the mass difference $m_d - m_u$ is to examine the masses of the K and D mesons. The mass of the K^0 is that of the K^+ plus the mass difference $m_d - m_u$ minus an electromagnetic contribution: $K^0 = K^+ + m_d - m_u - \gamma_K$. For the charmed mesons, the analogous mass relation is $D^+ = D^0 + m_d - m_u + \gamma_D$. Thus from the measured [1] values of these masses and the positivity of the electromagnetic contributions, we find

$$4.0 < m_d - m_u < 4.8 \text{ MeV}. \tag{32}$$

In fact, since the D^+ meson is more compact than the K^+, it is reasonable to assume that γ_D is slightly greater than γ_K, which further narrows the range to

$$4.0 < m_d - m_u < 4.4 \text{ MeV} \tag{33}$$

if we neglect the experimental errors. In what follows I shall use

$$m_d - m_u \approx 4.3 \text{ MeV}. \tag{34}$$

In fact both the mass m_s of the strange quark and the average mass of the up and down quarks $(m_d + m_u)/2$ are insensitive to the mass difference $m_d - m_u$.

For the masses of the constituent quarks of mesons, I shall use the values given by (7) above. Actually the masses of the three light quarks u, d and s depend only on the square-roots of the ratios of these masses and thus are fairly insensitive to them.

The masses of the light quarks u, d and s that result for various values of the mass of the charmed quark in the interval $1300 < m_c < 1600$ MeV, for various values of the decay constant f_D of the charmed meson $D^\pm$ in the interval $147 < f_D < 267$ MeV, for the mass splitting $m_d - m_u = 4.3$, and for the masses (7) of the constituent quarks of mesons are given in the table.

Table 1. Current-quark masses (in MeV) of the light quarks for various values of the mass of the charmed quark and of the decay constant f_D of the $D^\pm$ mesons, with $m_d - m_u = 4.3$ MeV.

m_c	f_D	m_u	m_d	m_s	$2m_s/(m_u + m_d)$
1300	147	13.6	17.9	216.4	13.8
1300	177	10.9	15.2	179.7	13.8
1300	207	9.0	13.3	153.8	13.8
1300	237	7.6	11.9	134.4	13.9
1300	267	6.5	10.8	119.5	13.9
1400	147	14.8	19.1	232.8	13.8
1400	177	11.9	16.2	193.3	13.8
1400	207	9.8	14.1	165.4	13.8
1400	237	8.3	12.6	144.6	13.8
1400	267	7.1	11.4	128.5	13.9
1500	147	16.0	20.3	249.3	13.8
1500	177	12.9	17.2	207.0	13.8
1500	207	10.7	15.0	177.0	13.8
1500	237	9.0	13.3	154.7	13.8
1500	267	7.8	12.1	137.5	13.9
1600	147	17.2	21.5	265.7	13.7
1600	177	13.9	18.2	220.6	13.8
1600	207	11.5	15.8	188.7	13.8
1600	237	9.8	14.1	164.9	13.8
1600	267	8.4	12.7	146.5	13.8

The average values of the resulting light-quark masses in the table are

$$m_u = 10.8 \pm 5.4 \qquad m_d = 15.1 \pm 5.4 \qquad m_s = 179.0 \pm 73 \ \text{MeV}. \tag{35}$$

The uncertainties in these masses would decrease if a more precise value for f_D were available through a better measurement of the rate of the decay $D^+ \to \mu^+ \nu_\mu$. As the value of m_c ranges from 1300 to 1600 MeV and as that of f_D ranges from 160 to 310 MeV, the mass ratio $2m_s/(m_u + m_d)$ remains close to the value 13.6 given by the approximation (27)

$$\frac{2m_s}{(m_u + m_d)} = 13.81 \pm 0.11 \tag{36}$$

which is about half of the value $2m_s/(m_u + m_d) \approx 25.7$ usually quoted [1, 12].

There is independent evidence for even smaller values of this mass ratio. For instance, Anselmino and Scadron [13] have argued that this ratio may be more like 5 or 6 on the basis of their infinite-momentum-frame analysis and SLAC scaling data, as have for other reasons Sazdjian and Stern [14]; Fuchs *et al* [15]; Gunion *et al* [16]; Fuchs and Scadron [17]; and Scadron [18].

4. Fermion masses and Gauss's law

We have found that the average mass of the two lightest quarks is given by (35) as

$$\tfrac{1}{2}(m_u + m_d) \approx 13.0 \pm 5.4 \ \text{MeV}. \tag{37}$$

Even larger masses have been advanced in [13–18]. It is amusing to note that we may find some support for such large masses from recent work in lattice gauge theory, which indicates that the Higgs sector is trivial, i.e. consistent only when free [19]. A trivial Higgs sector can only be an effective mass mechanism, good at low energies. The only non-trivial theories we know of are the non-Abelian gauge theories. Within the context of such theories, the most conservative mechanism for the generation of fermion masses is Gauss's law which requires charges to be surrounded by their electric, chromoelectric and weak-electric fields. Let us assume that the physical fermion masses are given by the energies of these fields, regularized at short distances by some new physics. If we employ a cutoff to represent the scale of the onset of the new physics, then the field energies will depend only logarithmically upon the cutoff. The masses of the up and down quarks will arise mainly from their chromoelectric fields and that of the electron from its electric field. The mass of the electron neutrino ν_e would presumably arise from the damped weak-electric field and thus be very small. The masses of the second and third families, being much larger, would in the present scheme arise from other, stronger non-Abelian gauge fields.

The QCD charge density is $g\psi^\dagger \frac{1}{2}\lambda_a \psi$ while the QED charge density is $e\psi^\dagger\psi$. Since the sum of the squares of Gell-Mann's SU(3) generators $\frac{1}{2}\lambda_a$ is

$$\sum_{a=1}^{8} \left(\frac{\lambda_a}{2}\right)^2 = \tfrac{4}{3} I \tag{38}$$

the ratio of the average mass $\frac{1}{2}(m_u + m_d)$ of the two lightest quarks to that of the electron should be given approximately for weak coupling by 4/3 times the ratio of the squares of the coupling constants:

$$\frac{\frac{1}{2}(m_u + m_d)}{m_e} \approx \frac{\langle q | \int \boldsymbol{E}_a^2 \, d^3x | q \rangle}{\langle e | \int \boldsymbol{E}^2 \, d^3x | e \rangle} \approx \frac{\frac{4}{3}\alpha_s}{\alpha}. \tag{39}$$

The ratio of the mass of the electron neutrino to that of the electron would be given by a similar expression with the chromoelectric field $\boldsymbol{E}_a$ replaced by the weak-electric field $\boldsymbol{E}_i$. The mass ratio m_{ν_e}/m_e would then be very small because the weak field $\boldsymbol{E}_i$ has a very short range, of the order of $\hbar/M_Z c$.

The value of Λ computed by the BCDMS collaboration [1, 20] is

$$\Lambda^{(4)}_{\overline{\rm MS}} = 230 \pm 20 \ \text{(stat.)} \ \pm 60 \ \text{(sys.)} \ \text{MeV} \tag{40}$$

which implies that for an energy scale of 5 GeV the squared coupling constant α_s is about $\alpha_s \approx 0.191$. If we use this value for α_s, then the product of the ratio (39) with the mass of the electron gives for the average of the masses of the up and down quarks

$$\tfrac{1}{2}(m_u + m_d) \approx 17.9 \pm 1.8 \ \text{MeV} \tag{41}$$

which is even higher than the average mass of (37). If we evolve α_s down to 2 GeV, where $\alpha_s \approx 0.261$, then the predicted average mass increases further to

$$\tfrac{1}{2}(m_u + m_d) \approx 24.4 \pm 3.6 \ \text{MeV}. \tag{42}$$

These considerations suggest that the masses of the lightest quarks as given by (37) and by [15–20] may be more accurate than the much lower average $\frac{1}{2}(m_u + m_d) \approx 7.75 \pm 1.1$ MeV usually quoted [1, 8].

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References

[1] Particle Data Group, Yost G P *et al* 1990 *Phys. Lett.* **239B** 1
[2] Lichtenberg D B 1987 *Int. J. Mod. Phys.* A **2** 1669 and private communication
[3] Elias V, Mong Tong and Scadron M D 1989 *Phys. Rev.* D **40** 3670
[4] Godfrey S and Isgur N 1985 *Phys. Rev.* D **32** 189
[5] Capstick S and Isgur N 1986 *Phys. Rev.* D **34** 2809
[6] Lichtenberg D B 1989 *Phys. Rev.* D **40** 3675; 1987 *Phys. Rev.* D **35** 2183
[7] Brekke L and Rosner J L 1988 *Comment. Nucl. Part. Phys.* **18** 83
[8] Franklin J 1989 *High-Energy Spin Physics (AIP Conf. Proc. No. 187)* ed K Heller (New York: American Institute of Physics) p 298
[9] Lichtenberg D B and Namgung W 1989 *Phys. Rev.* D **40** 1606
[10] Frank M and O'Donnell P J 1985 *Phys. Lett.* **159B** 174
Lichtenberg D B 1987 *Phys. Lett.* **193B** 95

[11] Rosner J L 1990 *Phys. Rev.* D **42** 3732

[12] Dominguez C A and de Rafael E 1987 *Ann. Phys., NY* **174** 372
Gasser J and Leutwyler H 1982 *Phys. Rep.* **87** 77

[13] Anselmino M and Scadron M D 1989 *Phys. Lett.* **229B** 117

[14] Sazdjian H and Stern J 1975 *Nucl. Phys.* B **94** 163

[15] Fuchs N H, Sazdjian H and Stern J 1990 *Phys. Lett.* **238B** 380

[16] Gunion J F, McNamee P C and Scadron M D 1976 *Phys. Lett.* **63B** 81; 1977 *Nucl. Phys.* B **123** 445

[17] Fuchs N H and Scadron M D 1979 *Phys. Rev.* D **20** 2421; 1989 *J. Phys. G: Nucl. Part. Phys.* **15** 943, 957

[18] Scadron M D 1981 *Rep. Prog. Phys.* **44** 213

[19] Shigemitsu J 1991 *Lattice 90: Proc. 1990 Symp. on Lattice Field Theory (Tallahassee, Florida, 8–12 October 1990) Nucl. Phys.* B (Proc. Suppl.) (in press)

[20] Benvenuti A C *et al* (BCDMS collaboration) 1987 *Phys. Lett.* **195B** 97