The Gravitational Constant (*).

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Summary. – Gravitational theories can be written in terms of rescaled fields without the Planck mass. The rescaled tetrads acquire the dimension of mass. The actual distribution of energy throughout space-time causes the tetrads to assume vacuum expected values of the order of the Planck mass, m_P. Thus the gravitational constant, $G = \hbar c/m_P^2$, may be viewed not as a fundamental constant, but as a mass scale that is dynamically determined by the large-scale structure of the Universe.

How and why does Nature choose such vastly different mass scales? It is possible to use the Higgs mechanism to set arbitrary mass scales and to use supersymmetry to keep them fixed at their tree-level values. But this leaves open the question of why the mass scales have the values they do. For scalar fields have no reason to assume nonzero vacuum expected values. In fact the only fields that naturally do so are the tetrads.

Tetrad vacuum expected values, therefore, provide a natural way to establish mass scales and break symmetries [1,2]. It will be shown here that tetrads spontaneously break the general co-ordinate invariance and local Lorentz symmetry of gravitational theories by assuming vacuum expected values.

It will also be pointed out that gravitational theories can be written in terms of rescaled fields in such a way that the Planck mass never appears. The rescaled fields are dimensionless except for gauge fields and tetrads, both of which acquire the dimension of mass.

The v.e.v.'s of the tetrads are determined by the distribution of energy throughout space-time. It is, therefore, not possible to calculate tetrad v.e.v.'s without a knowledge of the actual energy distribution. However, to the extent that cosmological models plausibly yield Robertson-Walker metrics with $g_{00} = 1$, these same models yield v.e.v.'s of the rescaled tetrads of the order of the Planck mass m_P. Thus

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Newton's constant $G = \hbar c/m_P^2$ may be viewed not as a fundamental constant, but as a mass scale that is dynamically determined by the actual large-scale structure of the universe.

Theories of a Weyl spinor $\psi(x)$ interacting with a tetrad $e^\mu(x)$ have a two-fold symmetry. The action is invariant under a transformation U that is both a local SL_{2C} transformation $T(x)$ of the field $\psi(x)$ and a general co-ordinate transformation of the space-time co-ordinates x^μ. U transforms the two-component Weyl spinor $\psi(x)$ and the tetrad $e^\mu(x)$ into

$$\psi(x)' = U^{-1}\psi(x)\,U = T(x)\,\psi(x') \tag{1}$$

and

$$e^\mu(x)' = U^{-1}e^\mu(x)\,U = \partial'_\nu x^\mu\,T^{-1\dagger}(x)\,e^\nu(x')\,T^{-1}(x)\,, \tag{2}$$

in which $\partial'_\nu = \partial/\partial x'^\nu$ and x' is the image of x under the associated general co-ordinate transformation. The action is invariant [1,2] even if there is no correlation whatsoever between the local Lorentz transformation $T(x)$ and the general co-ordinate transformation $x \to x'$.

Let us suppose that the tetrad $e^\mu(x)$ assumes an expected value in the « vacuum » state $|\Omega\rangle$ that represents the universe:

$$e^\mu(x)_c = \langle\Omega|e^\mu(x)|\Omega\rangle\,. \tag{3}$$

By eqs. (2)-(3) any transformation U that leaves the state $|\Omega\rangle$ invariant must satisfy [2]

$$e^\mu(x)_c = \partial'_\nu x^\mu\,T^{-1\dagger}(x)\,e^\nu(x')_c\,T^{-1}(x)\,. \tag{4}$$

Its local Lorentz transformation $T(x)$ and its general co-ordinate transformation $x \to x'$ must be so synchronized as to leave the tetrad v.e.v.'s invariant.

The general co-ordinate transformation itself must be an isometry [2] of the space-time metric $g_{\nu\mu}(x)_c$,

$$g_{\mu\nu}(x)_c = \partial_\mu x'^\sigma\,\partial_\nu x'^\lambda g_{\sigma\lambda}(x')_c\,. \tag{5}$$

The isometries of the flat-space metric, $g_{\mu\nu}(x)_c = \eta_{\mu\nu}$, are just the global Poincaré transformations. So to the extent that empty space is flat over laboratory distances we may say that tetrad v.e.v.'s reduce the symmetry of the « vacuum » state $|\Omega\rangle$ to global Poincaré invariance.

The action for a Weyl spinor ψ interacting with the gravitational field may be written as

$$S = \int d^4x(-g)^{\frac{1}{2}}\left[(m_P/32\pi)\,\mathrm{tr}(e^\mu F_{\mu\nu}h^\nu) + \frac{i}{2}\,\psi^\dagger e^\mu D_\mu\psi + \text{h.c.}\right], \tag{6}$$

where g is the determinant of $g_{\mu\nu}$, h^ν is the contragredient tetrad $h^\nu = \sigma_2 e^{\nu t}\sigma_2$, D_μ is the covariant derivative $D_\mu = \partial_\mu + A_\mu$ and $F_{\mu\nu}$ is the curvature tensor $F_{\mu\nu} = [D_\mu, D_\nu]$. The action is invariant when U transforms ψ and e^μ according to eqs. (1), (2), provided the connection A_μ transforms suitably [1].

Let us now rescale the fields by means of the following definitions:

$$m_{\mu\nu} = m_P^2 g_{\mu\nu}\,, \tag{7}$$

$$\hat\psi = m_P^{-\frac{3}{2}}\psi\,, \tag{8}$$

according to which the new metric $m_{\mu\nu}$ has the dimension of mass squared and the Fermi field $\hat{\psi}$ is dimensionless. These rescaling imply that the inverse metric is $m^{\mu\nu} = m_{\rm P}^{-2} g^{\mu\nu}$, and that the rescaled tetrads are $k_\mu = m_{\rm P} e_\mu$, $n_\mu = m_{\rm P} h_\mu$, $k^\mu = m_{\rm P}^{-1} e^\mu$ and $n^\mu = m_{\rm P}^{-1} h^\mu$. In terms of the rescaled fields, the action is

$$S = \int d^4x (-m)^{\frac{1}{2}} \left[(1/32\pi)\, \mathrm{tr}(k^\mu F_{\mu\nu} n^\nu) + \frac{i}{2} \hat{\psi}^\dagger k^\mu D_\mu \hat{\psi} + \mathrm{h.c.} \right], \tag{9}$$

where m is the determinant of $m_{\mu\nu}$. The Planck mass does not appear in the expression. Can a constant that can be scaled away be fundamental? One may recall that it is not possible to scale away the coupling constant in a Yang-Mills theory.

This rescaling (7)-(8) may be generalized so as to remove the Planck mass from an arbitrary theory of gravity. The new volume element $d^4x(-m)^{\frac{1}{2}}$ is dimensionless. Let us make all matter fields dimensionless by multiplying them by an appropriate power of the Planck mass. Then the only source of dimension will be the covariant derivatives, D_μ, each of which is always accompanied by a tetrad, k^μ or n^μ, so as to form an invariant combination. But such combinations are also dimensionless. Since the action is dimensionless, every term in the action must be dimensionless. Thus neither the Planck mass nor any other mass can appear in a rescaled theory.

The dimension of a rescaled field is that of mass raised to the power d, where d is the number of its covariant indices minus the number of its contravariant indices. The summation convention yields quantities that are both invariant and dimensionless.

The scaled metric $m_{\mu\nu}$ is determined by the distribution of energy in the universe. If an unscaled model cosmology gives a metric tensor $g_{\mu\nu}$, then the same cosmological model with scaled fields will yield as the scaled metric $m_{\mu\nu} = m_{\rm P}^2 g_{\mu\nu}$. The Riemann tensor $R_{\mu\nu}$ is unchanged by scaling. The scaled cosmological equation is

$$R_{\mu\nu} - \tfrac{1}{2} m_{\mu\nu} m^{\sigma\tau} R_{\sigma\tau} = \hat{T}_{\mu\nu} \tag{10}$$

in which the scaled energy-momentum tensor $\hat{T}_{\mu\nu}$ is

$$\hat{T}_{\mu\nu} = (-m)^{-\frac{1}{2}} \frac{\partial (-m)^{\frac{1}{2}} \hat{I}}{\partial m^{\mu\nu}} = m_{\mu\nu} \hat{I} + \dots, \tag{11}$$

where $\hat{I}$ is the scalar through which matter contributes to the action.

Let us make a very rough estimate of the magnitude $|m_{\mu\nu}|$ of the components of the scaled metric $m_{\mu\nu}$. Since $R_{\mu\nu}$ does not scale, the left-hand side of eq. (10) has the approximate value, $\approx H^2$, where H is Hubble's constant. So $H^2 \approx |m_{\mu\nu}|\hat{I}$. The scalar $\hat{I}$, when multiplied by $(-m)^{\frac{1}{2}}$ and integrated over space-time, gives matter's contribution to the action S of the universe. Since that action S is, very roughly, the energy density ϱ multiplied by $L^3 t \approx L^4 \approx H^{-4}$, we have

$$S \approx |m_{\mu\nu}|^2 \hat{I} L^4 \approx \varrho L^4 \tag{12}$$

or $\hat{I} \approx \varrho/|m_{\mu\nu}|^2$, whence $|m_{\mu\nu}| \approx \varrho H^{-2}$. Thus, since the density ϱ has the approximate empirical value $\varrho \approx H^2/G$, we find for the magnitude of the components of the scaled metric the value

$$|m_{\mu\nu}| \approx 1/G = m_{\rm P}^2 . \tag{13}$$

The actual distribution of energy throughout the universe causes the scaled metric $m_{\mu\nu}$ to assume v.e.v.'s of the order of the square of the Planck mass. One may regard the presence of Newton's constant G in gravitational theories as a consequence of the arbitrary requirement that $g_{\mu\nu}$ assumes v.e.v.'s of the order of unity.

After this work was completed, its author became acquainted with several interesting articles ([3,4]) by DE ALFARO, FUBINI and FURLAN in which they showed that the v.e.v.'s of the gravitational field break general co-ordinate invariance and give rise to Newton's constant. These authors also noted that generally covariant actions contain no dimensional parameters if the fields are suitably scaled. The reader's attention is also called to a recent independent unpublished work by WARD that is similar in spirit to some of the views advocated here. Ward's work extends his earlier paper ([5]), which is of considerable significance.

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([3]) V. DE ALFARO, S. FUBINI and G. FURLAN: *Nuovo Cimento A*, **50**, 523 (1979); *Nuovo Cimento B*, **57**, 227 (1980); *Phys. Lett. B*, **97**, 67 (1980); in *Unification of the Fundamental Interactions, Erice, 1981*, edited by ELLIS and FERRARA (New York, N.Y., 1983); *Z. Phys. C*, **18**, 349 (1983).
([4]) V. DE ALFARO: in *The High-Energy Limit, Erice, 1980*, edited by A. ZICHICHI (New York, N.Y., 1981), p. 149.
([5]) J. C. WARD: *Proc. Natl. Acad. Sci. USA*, **75**, No. 6, 2568 (1978).