

Gauge Invariance and Confinement in Noncompact Simulations of SU(2)

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Wilson loops have been measured at strong coupling, $\beta = 0.5$, on a 12^4 lattice in a noncompact simulation of pure SU(2) in which random compact gauge transformations impose a kind of lattice gauge invariance. The Wilson loops suggest a confining potential.

1. INTRODUCTION

In 1980 Creutz [1] displayed quark confinement at moderate coupling in lattice simulations of both abelian and nonabelian gauge theories. Whether nonabelian confinement is as much an artifact of Wilson's action as is abelian confinement remains unclear. The basic variables of Wilson's formulation [2] are elements of a compact group and enter the action only through traces of their products. The Wilson action has false vacua [3] which affect the string tension [4,5].

In simulations of SU(2) with gauge-invariant potential barriers between the true vacuum and the false vacua, the string tension has been seen to drop [4] or even vanish [5].

To examine these questions, some physicists have introduced lattice actions that are noncompact discretizations of the continuum action with fields as the basic variables. For U(1) these noncompact formulations are accurate for all coupling strengths [6]; for SU(2) they agree well with perturbation theory at very weak coupling [7].

This report relates the results of measuring Wilson loops at strong coupling, $\beta \equiv 4/g^2 = 0.5$, on a 12^4 lattice in a noncompact simulation of SU(2) gauge theory without gauge fixing or fermions. In this simulation the fields are subjected to random compact gauge transformations which restore a semblance of lattice gauge invariance.

2. NONCOMPACT METHODS

Patrascioiu *et al.* performed the first noncompact simulations of SU(2) by discretizing the classical action and fixing the gauge [8]. They saw a Coulomb force.

Later simulations [9] were carried out with an action free of spurious zero modes, for which it was not necessary to fix the gauge. The Wilson loops of these simulations showed no sign of quark confinement. A possible explanation of this negative result is that noncompact actions lack an exact lattice gauge invariance. Yet if one subjects the fields to random compact gauge transformations during each sweep, then one may be able to restore a kind of gauge invariance to the simulation [9]. Here I report the results of such a gauge-invariant simulation in which the Wilson loops fall off exponentially with the area of the loop.

In both the earlier simulations without gauge invariance and the new simulation with gauge invariance, the fields are constant on the links of length a , the lattice spacing, but are interpolated linearly throughout the plaquettes. In the plaquette with vertices n , $n+e_\mu$, $n+e_\nu$, and $n+e_\mu+e_\nu$, the field is

$$A_\mu^a(x) = \left(\frac{x_\nu}{a} - n_\nu\right)A_\mu^a(n+e_\nu) + (n_\nu + 1 - \frac{x_\nu}{a})A_\mu^a(n), \quad (1)$$

and the field strength is

$$F_{\mu\nu}^a(x) = \partial_\nu A_\mu^a(x) - \partial_\mu A_\nu^a(x) + gf_{bc}^a A_\mu^b(x)A_\nu^c(x). \quad (2)$$

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The action S is the sum over all plaquettes of the integral over each plaquette of the squared field strength,

$$S = \sum_{\rho\mu\nu} \frac{a^2}{2} \int dx_\mu dx_\nu F_{\mu\nu}^c(x)^2. \quad (3)$$

The mean value in the vacuum of a euclidean-time-ordered operator Q is approximated by a ratio of multiple integrals over the $A_\mu^a(n)$'s

$$\langle TQ(A) \rangle_0 \approx \frac{\int e^{-S(A)} Q(A) \prod_{\mu,a,n} dA_\mu^a(n)}{\int e^{-S(A)} \prod_{\mu,a,n} dA_\mu^a(n)} \quad (4)$$

which one may compute numerically. Macsyma was used to write most of the Fortran code [10] for the present simulation.

3. GAUGE INVARIANCE

To restore gauge invariance, the fields are subjected to random compact gauge transformations during every sweep, except those devoted exclusively to measurements. At each vertex n a random number r is generated uniformly on the interval $(0, 1)$; and if r is less than a fixed probability, set equal to 0.5 in this work, then a random group element $U(n)$ is picked from the group $SU(2)$. The fields on the four links coming out of the vertex n are then subjected to the compact gauge transformation

$$e^{-igaA_\mu^{ib}(n)T_b} = e^{-igaA_\mu^a(n)T_a} U(n)^\dagger \quad (5)$$

and those on the links entering the vertex to the transformation

$$e^{-igaA_\mu^{ib}(n-e_\mu)T_b} = U(n) e^{-igaA_\mu^a(n-e_\mu)T_a}. \quad (6)$$

4. WILSON LOOPS

The quantity normally used to study confinement in quarkless gauge theories is the Wilson loop $W(r, t)$ which is the mean value in the vacuum of the trace of a path-and-time-ordered exponential of a line integral of the connection around an $r \times t$ rectangle

$$W(r, t) = (1/d) \langle \text{tr } \mathcal{P} T e^{-ig \oint A_\mu^a T_a dx_\mu} \rangle_0 \quad (7)$$

where d is the dimension of the generators T_a . Although Wilson loops vanish in the exact theory [11], Creutz ratios $\chi(r, t)$ of Wilson loops defined [12] as double differences of logarithms of Wilson loops are finite. For large t , $\chi(r, t)$ approximates (a^2 times) the force between a quark and an antiquark separated by the distance r .

In this simulation the data are not yet sufficient to allow one to determine the Creutz ratios beyond the 3×4 loop. The Wilson loops therefore have been fitted to an expression involving Coulomb, perimeter, scale, and area terms.

5. MEASUREMENTS AND RESULTS

It will be useful to compare this simulation with an earlier one [9] in which the fields were not subjected to random gauge transformations. Both simulations were done on a 12^4 periodic lattice with a heat bath. The earlier simulation consisted of 20 independent runs with cold starts. The first run had 25,000 thermalizing sweeps at inverse coupling $\beta = 2$ followed by 5000 at $\beta = 0.5$; the other nineteen runs began at $\beta = 0.5$ with 20,000 thermalizing sweeps. There were 59,640 Parisi-assisted [13] measurements, 20 sweeps apart.

Noncompact Wilson loops at $\beta = 0.5$

| $\frac{r}{a} \times \frac{t}{a}$ | Not invariant | Invariant |
|----------------------------------|---------------|-------------|
| 1×1 | 0.402330(6) | 0.254564(8) |
| 2×2 | 0.085426(4) | 0.018711(6) |
| 3×3 | 0.018080(2) | 0.001429(3) |
| 4×4 | 0.003993(1) | 0.000117(3) |
| 5×5 | 0.000893(1) | 0.000014(6) |
| 6×6 | 0.000201(0) | 0.000004(2) |

The present simulation with random gauge transformations is very noisy. So far it consists of 13 runs, all with cold starts and 20,000 thermalizing sweeps. Wilson loops have been measured every five sweeps for a total of 689,684 measurements. The values of the diagonal Wilson loops so obtained are listed in the table. The errors have been estimated by the jackknife method, with all measurements in bins of 100 considered to be independent.

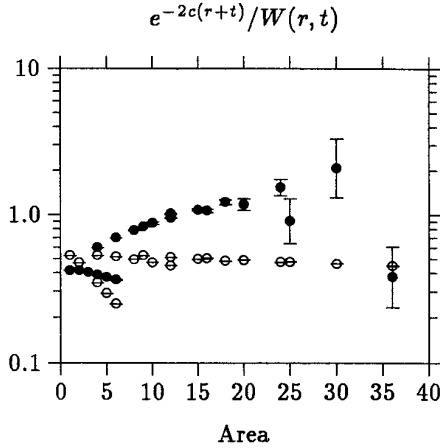


Figure 1. The negative logarithms of Wilson loops with the perimeter factor canceled are plotted against the area rt of the loop. The loops of the gauge-invariant simulation are represented by bullets; those of the earlier simulation by circles.

The Wilson loops of the gauge-invariant simulation fall off much faster with increasing loop size than do those of the earlier simulation. Because the data do not accurately determine all the Creutz ratios, I have fitted both sets of loops, including the non-diagonal loops, to the formula

$$W(r, t) \approx e^{a+b(r/t+t/r)-2c(r+t)-drt} \quad (8)$$

in which a is a scale factor, b a Coulomb term, c a perimeter term, and d an area term. For the simulation without random gauge transformations, I found $a \approx 0.25$, $b \approx 0.20$, $c \approx 0.39$, and $d \approx 0.00$. For the simulation with random gauge transformations, I found $a \approx 0.60$, $b \approx 0.20$, $c \approx 0.56$, and $d \approx 0.11$. In the gauge-invariant simulation, the coefficient of the area-law term is about two orders of magnitude larger than in the earlier simulation which lacked gauge invariance.

To exhibit the renormalized quark-antiquark potential, I have plotted in the figure the negative logarithms $-\log_{10}(e^{2c(r+t)}W(r, t))$ of the Wilson loops with the perimeter term removed. Apart from the uncertain value of $W(6, 6)$, the bigger loops of the gauge-invariant simulation, repre-

sented by bullets, display an area law; whereas the larger loops of the earlier simulation, represented by circles, show an essentially flat potential. The smallest loops reflect the symmetrized Coulomb term $\propto (t/r + r/t)$.

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