Energy Loss by Slow Magnetic Monopoles (*).

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Summary. – Two approximate classical calculations of energy loss by very slow magnetic monopoles are described. It is shown that the energy loss per unit length to bound electrons varies as the square of the velocity of the monopole, while that to conduction electrons varies linearly.

Recently BLAS CABRERA reported (1) the tentative discovery of a magnetic monopole carrying one Dirac unit (2) of magnetic charge. Previous experiments sensitive to fast magnetic monopoles gave negative results. The monopole discovered by CABRERA is, therefore, presumably slow. In fact, the flux of slow magnetic monopoles may be much higher than previously thought.

The rate of energy loss by fast magnetic monopoles has been reported by JACKSON (3) as

$$-\frac{\mathrm{d}E}{\mathrm{d}x} = \pi N Z mc^2 \left(\frac{\hbar}{mc}\right)^2 \ln\left(\frac{2mc^2\gamma^2\beta^2}{\hbar\omega}\right), \tag{1}$$

where N is the density of atoms of atomic number Z, m is the mass of the electron, ω is an average atomic frequency and β and γ refer to the monopole. This formula

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(1) BLAS CABRERA: *Phys. Rev. Lett.*, **48**, 1378 (1982).
(2) P. A. M. DIRAC: *Proc. Roy. Soc. A*, **133**, 60 (1931) and *Phys. Rev.*, **74**, 817 (1948).
(3) J. D. JACKSON: *Classical Electrodynamics*, 2nd edition (New York, N.Y., 1975), p. 653.

is expected to be accurate for β greater than 10^{-2}, below which the logarithm is typically negative.

We describe here two approximate classical calculations of the rate of energy loss by very slow monopoles with β less than 10^{-3}. We find that the energy loss per unit length to bound electrons is proportional to β^2, while that to conduction electrons is proportional to β.

In atomic units ($e = m_e = \hbar = 1$), the time rate of energy loss of a monopole of velocity $\boldsymbol{u}$ at $\boldsymbol{r}'$ to an electron of velocity $\boldsymbol{v}$ at $\boldsymbol{r}$ is

$$-\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{\boldsymbol{v}\cdot[\boldsymbol{u}\times(\boldsymbol{r}-\boldsymbol{r}')]}{2|\boldsymbol{r}-\boldsymbol{r}'|^3}\,. \tag{2}$$

Here and in what follows, we replace the distance $|\boldsymbol{r}-\boldsymbol{r}'|$ by r'. This is a reasonable approximation for impact parameters b greater than the radius of the atom. It is also a fair approximation for smaller b's when the monopole is outside the atom. It is a bad approximation when the monople is inside the atom. The validity of any classical calculation of what then happens is questionable, however, due to quantum-mechanical effects, including nucleon decay [4].

For very slow monopoles, the energy of the monopole and the angular momentum $\boldsymbol{L}$ of the electron vary on a slower time scale than that of the fast orbital motion of the electron. By averaging over many atomic revolutions, one may relate the slow variables by the equation

$$-\,\mathrm{d}E/\mathrm{d}t = \boldsymbol{u}\cdot\boldsymbol{L}/2r'^3\,. \tag{3}$$

One may obtain a simple formula for the rate of change of the electron's angular momentum $\boldsymbol{L}$ by averaging the torque exerted on it by the monopole over a symmetric ensemble of electronic orbits and by dropping terms odd in $\boldsymbol{r}$ and $\boldsymbol{v}$; to with

$$\mathrm{d}\boldsymbol{L}/\mathrm{d}t = -[\boldsymbol{r}'\times\boldsymbol{L} - 2r^2\boldsymbol{u}]/6r'^3\,. \tag{4}$$

The first term within the brackets represents the Larmor precession of the electron's angular momentum $\boldsymbol{L}$ about the monopole's magnetic field; the second term is due to the e.m.f. induced along the electron's orbit by the moving monopole.

Within these approximations, the energy transfer to an individual atom is small compared to its binding energy. The effective radius r, of the electron's orbit therefore changes a little, and we shall take it to be a constant when integrating eq. (4). That integration is facilitated if one uses a new time variable, $\tau = \operatorname{arctg}(ut/b) + \pi/2$, and decomposes $\boldsymbol{L}$ into components parallel and perpendicular to $\boldsymbol{r}'$. One may choose as the general solution of (4), the general solution of the associated homogeneous equation plus the particular solution that vanishes as $t \to \infty$. The contribution of the honogeneous part, being linear in the initial angular momentum, then vanishes for an unpolarized ensemble of atoms. If one now substitutes the resulting expression for $\boldsymbol{L}(t)$ into (3) and integrates over time, then one finds for the energy loss by the monopole per atom as a function of the impact parameter b the result

$$-\,\Delta E(b) = \frac{6r^2\sin^2(\pi x/2)}{b^4x^2(4-x^2)^2}\,, \tag{5}$$

(4) C. CALLAN Jr.: *Dyon-Fermion Dynamics* (Princeton University preprint, May 1982).

where the dimensionless variable x is given by

$$x = 1 + (6ub)^{-2} . \tag{6}$$

By integrating this formula over all impact parameters, one may express the energy loss rate (in any units) as

$$-\frac{dE}{dx} = 6\cdot 10^5 \pi N Z m c^2 \left(\frac{\hbar}{mc}\right)^2 \left(\frac{r}{a_0}\right)^2 \beta^2 , \tag{7}$$

where $a_0 = \hbar^2/me^2$ is the Bohr radius and r is the effective radius of the atoms of the material. Because of the above-mentioned adiabatic approximations, this formula is appropriate only for β less than 10^{-3}. Our formula and Jackson's are exhibited in the figure for the case in which $r = a_0$. A recently published correction (5) to Jackson's formula suggests a smooth transition between the two approximations.

The second formula describes the energy loss rate in media of high specific conductivity σ. The classical force on a nonrelativistic conduction electron may be written as

$$m\, d\boldsymbol{v}/dt = e(\boldsymbol{E} + (\boldsymbol{v}/c)\times\boldsymbol{B}) - (m/\tau)\boldsymbol{v} , \tag{8}$$

where τ is an effective relaxation time. The conductivity σ is related to τ and to the density n of conduction electrons by the formula $\sigma = \tau n e^2/m$. By solving this equation for the velocity $\boldsymbol{v}$ under steady-state conditions $\dot{\boldsymbol{v}} = 0$, one may express the current density, $\boldsymbol{j} = en\boldsymbol{v}$, as

$$\boldsymbol{j} = \sigma(\boldsymbol{E} + y\boldsymbol{E}\times\hat{\boldsymbol{B}} + y^2\boldsymbol{E}\cdot\hat{\boldsymbol{B}}\hat{\boldsymbol{B}})/(1 + y^2) , \tag{9}$$

where $y = e\tau|\boldsymbol{B}|/mc$.

The electric field due to the moving monopole is $\boldsymbol{E} = \beta\times\boldsymbol{B}$, which implies $\boldsymbol{E}\cdot\boldsymbol{B} = 0$. The rate of energy loss by the monopole to the conduction electrons is the volume integral

$$-dE/dt = \int d^3x\, \boldsymbol{j}\cdot\boldsymbol{E} = \sigma\int d^3x\, \boldsymbol{E}^2/(1 + y^2) . \tag{10}$$

Since the magnetic field of the monopole is $\boldsymbol{B} = g(\boldsymbol{r} - \boldsymbol{r}')/|\boldsymbol{r} - \boldsymbol{r}'|^3$, where $g = \hbar c/2e$ is its magnetic charge, the integration gives

$$-dE/dt = (2\pi^2/3) gc\sqrt{\sigma\hbar n}\, \beta^2 . \tag{11}$$

The monopole's energy loss rate is thus

$$-dE/dx = (\pi^2/3)(\hbar c/e)\sqrt{\sigma\hbar n}\, \beta . \tag{12}$$

This formula takes into account the effect of the electromagnetic field of the moving monopole on the conduction electrons, but neglects the screening of the monopole's field by the induced currents. In copper this energy loss rate (12) to the conduction

(5) K. HAYASHI: *Lett. Nuovo Cimento*, **33**, 324 (1982).

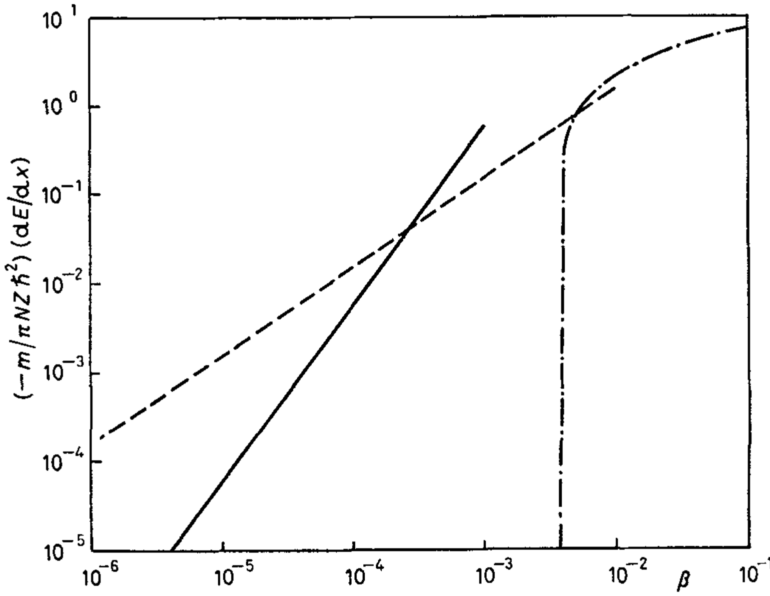

Fig. 1. – The energy loss rate $-dE/dx$ divided by the product $\pi N Z \hbar^2/m$ as given by Jackson's formula (1) (—·—·—), by our formula (7) (———) for $r = a_0$, and by our formula (12) (– – –) for Cu

electrons dominates that (7) to the bound electrons for $\beta < 10^{-\frac{1}{2}}$, as shown in the figure. A related calculation ([6]) shows substantial agreement with eq. (12).

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([6]) V. P. Marten'yanov and S. Kh. Khakimov: *Sov. Phys. JETP*, **35**, 20 (1972).