DOES PURE SU(2) GAUGE THEORY CONFINE? ☆

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Received 14 August 1989

Noncompact simulations of pure SU(2) without gauge fixing on a 10^4 lattice are consistent with perturbation theory at very weak coupling but show no evidence of quark confinement at strong coupling.

It is generally believed that quarks are confined by a nearly linear potential that is a property of the QCD vacuum, having little to do with the nature of the quarks beyond their color charges. This belief may be partly due to the many fruitless theoretical attempts made during the 1970's to derive confinement in pure or quarkless QCD. The best evidence that pure QCD confines comes from the impressive lattice simulations that Creutz [1] and others have carried out using Wilson's formalism [2].

While this belief in a linearly confining potential is widespread, it is not quite unanimous. Some doubts have arisen because confinement is a nearly universal feature of Wilson's formalism, holding in the strong-coupling limit even for abelian gauge groups in spacetimes of any dimension. Thus it is not clear whether the confinement seen in Creutz simulations is an artifact of Wilson's formalism or a property of QCD.

Confinement is built into Wilson's method because its basic variables are elements of a compact gauge group rather than real numbers as in continuum gauge theory and because the Wilson action is a function of the product of the group elements of the links around the elementary squares of the lattice. The Wilson action consequently possesses multiple minima, some of which correspond to false vacua not present in the continuum theory [3,4]. These extra vacua make a major contribution to the string tension, as has been shown by Mack and Pietarinen [5] and by Grady [6]. In their simulations of SU(2), they modified Wilson's action gauge invariantly by erecting, in effect, infinite potential barriers between the true vacuum and the false vacua. Mack and Pietarinen found a sharp drop in the string tension; Grady found no string tension at all.

Wilsonian simulations of abelian theories exhibit deconfining phase transitions at weak coupling. It has recently been suggested that this is also the case for nonabelian theories. Thus Grady [6,7] has found that the SU(2) theory in four dimensions is consistent with a correlation-length exponent of $\nu \approx \frac{2}{3}$ and a critical point of $\beta_c \approx 2.47$. For SU(3) he found $\nu \approx 1$ and $\beta_c \approx 6.69$. Patrascioiu, Seiler, Linke, and Stamatescu [8] argue that all zero-temperature lattice theories in three or more dimensions must undergo a phase transition that is the limit of the finite-temperature deconfining transition. For SU(2) on a small lattice, they found $\beta_c \approx 2.3$.

To avoid the artifacts of the Wilson action, some physicists have developed Monte Carlo simulation methods [3,4,9–13] that do not have confinement built in. These methods are called "noncompact" because their basic variables are fields rather than group elements as in Wilson's "compact" method. One difference between the two kinds of method is that the compact method has an exact lattice gauge symmetry

☆ Research supported by the US Department of Energy under grant DE-FG04-84ER40166 and contract W-31-109-ENG-38.
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that is different from the gauge invariance of the continuum theory, while the noncompact methods have an approximate version of continuum gauge invariance. It is not clear which side of this tradeoff is more accurate for nonabelian theories. For $U(1)$ the noncompact method is accurate at all coupling strengths [12], whereas the compact method is accurate only at weak coupling. Noncompact simulations have shown no sign of quark confinement.

How are quarks confined, if not by the vacuum of pure QCD? Gribov has suggested [14] that the existence of light quarks is a necessary part of the mechanism of quark confinement and that confinement arises because the asymptotically free QCD force between a separating quark–antiquark pair eventually becomes strong enough to pair-produce up and down quarks. In his view confinement is not a property of QCD without quarks or with quarks that are all heavier than the QCD scale Λ.

The present paper describes the first noncompact simulations of pure SU(2) that have been done without gauge fixing in four dimensions [15]. These simulations are consistent with perturbation theory at very weak coupling but show no evidence of quark confinement at strong coupling. They support Gribov's suggestion that pure QCD does not confine and confirm earlier gauge-fixed noncompact simulations [9,10], but disagree with the usual compact wilsonian simulations [1,2]. Yet the present noncompact simulations, done on a 10^4 lattice, do not exclude the possibility that noncompact simulations done on a much larger lattice might display some sort of confinement – a possibility that may be relevant because the lattice spacing $a_{\rm NC}(\beta)$ of the noncompact method is likely to be much smaller than that of Wilson's method.

The first noncompact simulations of SU(2) which were done by Patrascioiu, Seiler, and Stamatescu [9] who used for the field strength of a plaquette the simple discretization

$$F^a(p_{\mu\nu}) = \frac{A^a_\mu(n+ae_\nu) - A^a_\mu(n)}{a} - \frac{A^a_\nu(n+ae_\mu) - A^a_\nu(n)}{a} + g f_{abc} A^b_\mu(n) A^c_\nu(n) , \qquad (1)$$

in which the vector n labels the vertices of the lattice. Their action was the sum

$$S = \sum_{p_{\mu\nu}} \frac{a^4}{2} F^a(p_{\mu\nu})^2 \qquad (2)$$

over all plaquettes $p_{\mu\nu}$ and colors a (but not also over μ and ν). Because this action has many zero modes, they fixed the gauge choosing the temporal gauge for its theoretical simplicity. They saw a force law rather like Coulomb's law and found agreement with asymptotic freedom.

Seiler, Stamatescu, Wolff, and Zwanziger [10] used the more symmetrical field strength obtained by replacing the A's in the quadratic part of $F^a(p_{\mu\nu})$ by $\bar{A}^a_\mu(n) = \frac{1}{2}[A^a_\mu(n+ae_\nu) + A^a_\mu(n)]$ with a similar expression for $A^a_\nu(n)$. The resulting action S has fewer zero modes but still requires gauge fixing. They used a technique called stochastic gauge fixing developed by Zwanziger [16]. Their simulations were also consistent with asymptotic freedom but not with quark confinement.

Gauge fixing is undesirable because of the Gribov ambiguity [17], because Faddeev–Popov ghost fields are fermionic and so are hard to handle, because the integrations over the gauge copies help enforce Gauss' law, and because it is not possible to transform an arbitrary gauge configuration into some gauges, such as the temporal gauge, while preserving periodic boundary conditions in a finite spacetime [3]. To be able to integrate over all gauge configurations, it is necessary to use an action that is quite free of zero modes. One way to do this is to interpolate the fields throughout spacetime from their values at the vertices of a lattice tiled with simplices [3,4,11–13]. In four dimensions, however, the Fortran source code is over 600 K bytes and the program is slow.

One may shorten the code and increase its speed by adopting Wilson's structure of links and plaquettes and by linearly interpolating the fields throughout the plaquettes. The fields are then constant on the links of length a, the lattice spacing, but are interpolated linearly throughout the six transverse plaquettes. In the plaquette with vertices n, $n+e_\mu$, $n+e_\nu$ and $n+e_\mu+e_\nu$, the field is

$$A^a_\mu(x) = (x_\nu/a - n_\nu) A^a_\mu(n+e_\nu) + (n_\nu + 1 - x_\nu/a) A^a_\mu(n) , \qquad (3)$$

and the field strength is given by the continuum formula

$$F^a_{\mu\nu}(x)=\partial_\nu A^a_\mu(x)-\partial_\mu A^a_\nu(x)+gf_{abc}A^b_\mu(x)A^c_\nu(x)\,. \tag{4}$$

The action is the sum over all plaquettes of the integrals over each plaquette of the square of the field strength

$$S=\sum_{p_{\mu\nu}}\frac{a^2}{2}\int \mathrm{d}x_\mu \mathrm{d}x_\nu F^c_{\mu\nu}(x)^2\,. \tag{5}$$

The mean-value in the vacuum of a euclidean-time-ordered operator $Q(A)$ may then be approximated by a normalized multiple integral over the $A^a_\mu(n)$

$$\langle\Omega|\mathcal{T}\,Q(A)\,|\Omega\rangle \approx \frac{\int \exp[-S(A)]\,Q(A)\,\prod_{\mu,a,n}\mathrm{d}A^a_\mu(n)}{\int \exp[-S(A)]\,\prod_{\mu,a,n}\mathrm{d}A^a_\mu(n)}\,, \tag{6}$$

which one may estimate by Creutz's Monte Carlo techniques [1].

The quantity normally used to study confinement in quarkless gauge theories is the Wilson loop, which is the mean-value in the vacuum of the path-and-time-ordered exponential

$$W(r,t)=\frac{1}{d}\langle\Omega|\mathcal{P}\mathcal{T}\exp\left(\mathrm{i}g\oint A^a_\mu T_a \mathrm{d}x_\mu\right)|\Omega\rangle\,, \tag{7}$$

where d is the dimension of the matrices T_a that represent the generators of the algebra of the gauge group and g is the coupling constant. Although Wilson loops vanish [18] in the exact theory, Creutz ratios [1] of Wilson loops defined as

$$\chi(r,t)\equiv-\log\left(\frac{W(r,t)\,W(r-a,t-a)}{W(r,t-a)\,W(r-a,t)}\right) \tag{8}$$

are finite and for large t provide an estimate of a^2 times the force between a quark and an antiquark separated by a distance r.

For a representation of a group with N generators satisfying the trace rule $\mathrm{Tr}(T_aT_b)=k\delta_{ab}$, the value of the Creutz ratio $\chi(r,t)$ in tree-level perturbation theory may be expressed in terms of the function

$$U(r,t)=(r/t)\arctan(r/t)+(t/r)\arctan(t/r)-\log(r^{-2}+t^{-2}) \tag{9}$$

as

$$\chi(r,t)=\frac{kNg^2}{\pi^2 d}[-U(r,t)-U(r-a,t-a)+U(r,t-a)+U(r-a,t)]\,. \tag{10}$$

Kirschner, Kripfganz, Ranft, and Schiller [19] have calculated the order-g^4th correction to this formula; some values of their formula are listed in table 1 for different values of the SU(2) inverse coupling $\beta=4/g^2$.

To measure Wilson loops and their Creutz ratios $\chi(r,t)$ by means of the noncompact method, I used a 10^4 periodic lattice, began with cold starts, in which all fields were initialized to zero, and allowed 1000 sweeps for thermalization. I measured Wilson loops every 10 sweeps, using all the different r-by-t loops that occur in a 10^4 lattice, including periodic translations and rotations by $\pi/2$. I made 100 measurements at $\beta=1$, fewer at $\beta=30$ and 400. Some of the resulting values for the Creutz ratios $\chi(r,r)$ are listed in table 1. These values are in approximate agreement both with perturbation theory and with the χ's obtained by Patrascioiu et al. [9], who found, for instance, at $\beta=1$, $\chi(2a,2a)=0.1$ and $\chi(3a,3a)=0.02$.

If the static force between heavy quarks is independent of distance, corresponding to a linear confining potential, then the Creutz ratios $\chi(r,t)$ should be independent of r and t at least for large t. In his compact simulations [1,20], Creutz found at $\beta=3.28$, $\chi(3a,3a)=0.047$ and $\chi(2a,2a)=0.114$; at $\beta=2.80$, $\chi(3a,3a)=0.067$ and $\chi(2a,2a)=0.148$; at $\beta=2.49$, $\chi(3a,3a)=0.100$ and $\chi(2a,2a)=0.214$; at $\beta=2.29$, $\chi(3a,3a)=0.213$ and $\chi(2a,2a)=0.331$; and at $\beta=2.00$,

Table 1
Noncompact and perturbative Creutz ratios.

β	r/a, t/a	Noncompact	Perturbation
1	2,2	0.1254(14)	
	3,3	0.0233(39)	
30	2,2	0.00619(10)	0.00657
	3,3	0.00191(25)	0.00217
	4,4	0.00067(47)	0.00109
400	2,2	0.000511(12)	0.000496
	3,3	0.000175(28)	0.000163
	4,4	0.000071(44)	0.000082

$\chi(2a, 2a)=0.603$. These data show a clumping of the χ's in that as $\beta\to 2$, $\chi(3a, 3a)$ approaches $\chi(2a, 2a)$, an effect verified in other wilsonian simulations. In the present noncompact simulations, however, even at $\beta=1$ $\chi(3a, 3a)$ is nearly six times smaller than $\chi(2a, 2a)$. So there is no sign of confinement in these noncompact simulations.

These results and those of the earlier gauge-fixed noncompact simulations of SU(2) may present us with a clear conflict between the two methods. But the ratio of the energy scale of the continuum theory to that of Wilson's lattice theory is substantial, reaching $\Lambda_{\mathrm{MOM}}^{(\alpha=1)}/\Lambda_{\mathrm{W}}=57.5$ for SU(2) and 83.5 for SU(3) in the renormalization scheme based on momentum-space subtraction in Feynman gauge [21]. For SU(2) in the minimum-subtraction schemes [22], the ratios are $\Lambda_{\overline{\mathrm{MS}}}^{(\alpha=1)}/\Lambda_{\mathrm{W}}=19.89$ and $\Lambda_{\mathrm{MS}}^{(\alpha=1)}/\Lambda_{\mathrm{W}}=7.48$. Because the noncompact method imitates the continuum theory, it is reasonable to expect the scale of the noncompact method to be approximately in the range of those of the continuum theory, $\Lambda_{\mathrm{MS}}^{(\alpha=1)}\leqslant\Lambda_{\mathrm{NC}}\leqslant\Lambda_{\mathrm{MOM}}^{(\alpha=1)}$. Thus even if the two methods do represent substantially the same physics, the lattice spacing of the Wilson theory $a_{\mathrm{W}}(\beta)$ for SU(2) should be between about 7 and 58 times bigger than the lattice spacing $a_{\mathrm{NC}}(\beta)$ of the noncompact method at the same value of β. So to display with the noncompact method the clumping of the χ's seen with the compact method on a distance scale of a few $a_{\mathrm{W}}(2)$, one would have to run either at stronger coupling or on a much larger lattice.

It is necessary to run on a larger lattice because for $\beta<2$ the Wilson lattice spacing $a_{\mathrm{W}}(\beta)$, varies [1] as $\log(4/\beta)$. Thus even by running at $\beta=\frac{1}{4}$ one gains only by a factor 4.

We may, however, get some idea from presently available data as to whether the two methods are compatible by comparing them at a smaller value of the lattice spacing, for instance at $a_{\mathrm{W}}(\beta)=a_{\mathrm{NC}}(1)$. The noncompact lattice spacing $a_{\mathrm{NC}}(1)$ is likely to be from about 7 to 58 times smaller than the Wilson lattice spacing $a_{\mathrm{W}}(1)$ if the two methods represent the same physics apart from the energy scale Λ. Since the Wilson lattice spacing $a_{\mathrm{W}}(\beta)$ varies as $\log(4/\beta)$ for $\beta\leqslant 2$, it follows that $a_{\mathrm{W}}(2)$ should be twice as small as $a_{\mathrm{W}}(1)$. For $\beta\geqslant 2$, $a_{\mathrm{W}}(\beta)$ varies approximately perturbatively as

$$a_{\mathrm{W}}(\beta)=\Lambda_{\mathrm{W}}^{-1}(\beta/2n\gamma_0)^{\gamma_1/2\gamma_0^2}\exp(-\beta/4n\gamma_0) \qquad (11)$$

for SU(n), where

$$\gamma_0=(1/16\pi^2)(11n/3)\,,$$
$$\gamma_1=(1/16\pi^2)^2(34n^2/3)\,. \qquad (12)$$

So by $\beta=2.5$, the Wilson lattice spacing $a_{\mathrm{W}}(\beta)$ will have dropped by another factor of 3.5, for a total shrinkage by a factor of 7. Thus if the ratio of the scale factors $\Lambda_{\mathrm{NC}}/\Lambda_{\mathrm{W}}=7$, then $\chi_{\mathrm{NC}}(r, t)$ at $\beta=1$ should resemble $\chi_{\mathrm{W}}(r, t)$ at $\beta=2.5$. But Creutz's values for χ_{W} at $\beta=2.5$, cited earlier, exceed my values for χ_{NC} at $\beta=1$, quoted in table 1, by about 1.7 for the 2×2 loop and by about 4.3 for the 3×3 one. So the ratio of the scale factors seems larger than 7.

By $\beta=3.28$, the Wilson lattice spacing is about 51 times smaller than at $\beta=1$. Thus if the ratio of the scale factors $\Lambda_{\mathrm{NC}}/\Lambda_{\mathrm{W}}=51$, then $\chi_{\mathrm{NC}}(r, t)$ at $\beta=1$ should resemble $\chi_{\mathrm{W}}(r, t)$ at $\beta=3.28$. For the 2×2 ratios, we have approximate agreement: $\chi_{\mathrm{NC}}(2a, 2a)=0.125$ and $\chi_{\mathrm{W}}(2a, 2a)=0.11$. But for the 3×3 ratios, we have a conflict: $\chi_{\mathrm{NC}}(3a, 3a)=0.023$ and $\chi_{\mathrm{W}}(3a, 3a)=0.048$. The compact method assigns to the static force between heavy quarks greater strength at $r\approx 3a_{\mathrm{NC}}(1)\approx 3a_{\mathrm{W}}(3.28)$ than does the noncompact method. This discrepancy suggests that the two methods may describe different physics.

I have also run noncompact simulations of SU(2) on a 5^4 lattice to see whether the Creutz ratios of the noncompact method exhibit asymptotic freedom, according to the criterion introduced by Creutz [23]. My very preliminary results suggest that the χ_{NC}'s seem consistent with asymptotic freedom at $\beta=400$, which is very weak coupling, $g_0=0.1$, but not at $\beta=30$ or $\beta=1$.

In conclusion, both the present noncompact simulations of SU(2), which were done without gauge fixing, and earlier gauge-fixed noncompact simulations [9,10] show no sign of quark confinement on a 10^4 lattice. This absence of confinement offers some support for Gribov's [14] view that confinement occurs only in QCD with light quarks and not in pure or heavy-quark QCD. Yet because the lattice spacing of Wilson's method is probably much bigger than that of the noncompact method at the same value of the coupling, it is possible that noncompact simulations on much larger lattices might exhibit some sort of confinement. But the present simulations suggest that even when such differences of scale are taken into account, the static $q\bar{q}$ force of the noncompact method falls off much faster with distance than does that of Wilson's formalism.

I am grateful to S. Adler, G. Bodwin, C. Chandler, S. Coleman, J. Collins, M. Creutz, N. Duric, V.N. Gribov, P. Horowitz, E. Kovacs, S. Prasad, V. Rebhahn, D. Sinclair, J. Smit, and A. White for useful conversations, to F. Pipkin for hospitality during my sabbatical year at Harvard, and to A. White for hospitality at Argonne during the summer of 1989. This research was done on a Ridge 32C computer.

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