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Condensates Break Chiral Symmetry

Kevin Cahill*

Center for Particle Physics
Department of Physics and Astronomy
University of New Mexico
Albuquerque, NM 87131-1156

Abstract

In the physical vacuum of QCD, the energy density of light-quark fields strongly coupled to slowly varying gluon fields can be negative. The states that drive this energy density lowest are condensates of pairs of quarks and antiquarks of nearly opposite momenta. These quark-antiquark condensates break chiral symmetry. They may also affect other features of hadronic physics, such as the range of the strong force and the confinement of color.

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*kevin@kevin.phys.unm.edu http://kevin.phys.unm.edu/~kevin/

1 The QCD Vacuum

The thesis of this note is that the energy density of strongly coupled light quarks can be negative and that this feature of QCD breaks chiral symmetry. The hamiltonian H_q of the u, d, and s quarks

$$H_q = \sum_{f=u,d,s} \int d^3x \, \bar{\psi}_f \left(\vec{\gamma} \cdot \vec{\nabla} - ig\gamma^0 A_{0a} \frac{\lambda_a}{2} - ig\vec{\gamma} \cdot \vec{A}_a \frac{\lambda_a}{2} + m_f \right) \psi_f \qquad (1)$$

can assume large negative mean values due to the term $-g \int d^3x \vec{J}_a \cdot \vec{A}_a$ when the gauge field $\vec{A}_a$ varies slowly with a modulus $|\vec{A}_a|$ that exceeds m_u/g by a sufficient margin [1]. For nearly constant gauge fields $\vec{A}_a$, the states that drive the energy lowest are condensates [2] of pairs of light quarks and antiquarks of opposite momenta; in such pairs the color charges cancel, but the color currents add. When $g|\vec{A}_a| \gg m_d$, the u and d quarks play very similar roles, and $s\bar{s}$ pairs become important when $g|\vec{A}_a| \gg m_s$.

If the gauge fields are not only slowly varying but also essentially abelian, in the sense that $gf_{abc}A^b_\mu A^c_\nu$ is small (*e.g.*, because $A^a_\mu(x) \simeq C^a(x)V_\mu(x)$), then the energy of the gauge fields is also small. If an essentially abelian gauge field, *e.g.*, $|\vec{A}_8|$, is relatively constant over a sphere of radius R beyond which it either remains constant or slowly drops to zero, then its energy density near the sphere can be of the order of $|\vec{A}_8|^2/R^2$ or less while that of the light-quark condensate can be as negative as $-|g\vec{A}_8|^4$. The physical vacuum of QCD is therefore a linear combination of states, each of which is approximately a coherent [3] state $|\vec{A}\rangle$ of a slowly varying, essentially abelian gauge field $\vec{A}_a(x)$ and an associated condensate of pairs of u and d quarks and $\bar{u}$ and $\bar{d}$ antiquarks of nearly opposite momenta:

$$|\Omega\rangle \simeq \int D\vec{A}_a \, f(\vec{A}_a) \prod_{S(A,u)} a^\dagger(\vec{p}_i, \sigma, u_i) a^{c\dagger}(\vec{q}_j, \tau, u_j) \prod_{S(A,d)} a^\dagger(\vec{p}_i, \sigma, d_i) a^{c\dagger}(\vec{q}_j, \tau, d_j) |\vec{A}\rangle. \qquad (2)$$

Here the sets $S(A,u)$ and $S(A,d)$ specify the momenta $\vec{p}, \vec{q}$, spins σ, τ, and colors i, j of the quarks and antiquarks, the operator $a^{c\dagger}(\vec{q}, \tau, d_j)$ creates a d antiquark of momentum $\vec{q}$, spin τ, and color j, the function $f(\vec{A}_a)$ is a weight function, and the s quarks have been suppressed.

In what follows I shall compute the energy density of a such a light-quark condensate for the case of a constant gauge field $\vec{A}_8$. It will turn out that if $g|\vec{A}_8|$ is of the order of a GeV, then the mean value $\langle \frac{1}{2}(\bar{u}u + \bar{d}d) \rangle$ of the light-quark condensate is about $(260\,\mathrm{MeV})^3$ as required by soft-pion physics [4]. This condensate breaks chiral symmetry. It may also play a role in other features of hadronic physics, such as the short range of the strong force and the confinement of quarks.

2 A Particular Condensate

Let us consider the case of a constant gauge field $\vec{A}_8$ that points in the direction 8 of color space; the energy density and quark condensate associated with a slowly varying gauge field $\vec{A}_c(\vec{x})$ should be similar. If we call the quark colors red, green, and blue, then the condensate will be made of red and green u, d, and s quarks of momentum $\vec{p}$ and both spin indices σ; red and green u, d, and s antiquarks of momentum $-\vec{p}$ and both spin indices σ; blue u, d, and s quarks of momentum $-\vec{p}$ and both spin indices σ; and blue u, d, and s antiquarks of momentum $\vec{p}$ and both spin indices σ. The domains of integration $S(A, u)$ and $S(A, d)$ for the u and d quarks will be very similar when the gauge field $\vec{A}_8$ is intense, but the domain for the s quarks will be smaller. The component $|\Omega_A\rangle$ of the QCD vacuum associated with the gauge field $\vec{A}_8$ will then be

$$|\Omega_A\rangle = \prod_{S(A,u)} a^\dagger(\vec{p}_i, \sigma, u_i) a^{c\dagger}(-\vec{p}_i, \sigma, u_i) \prod_{S(A,d)} a^\dagger(\vec{p}_i, \sigma, d_i) a^{c\dagger}(-\vec{p}_i, \sigma, d_i)\, |\vec{A}_8\rangle \tag{3}$$

apart from the s quarks. These products over momentum, spin, and color are defined by box quantization in a volume V, and so the mean value of the hamiltonian H_q in the state $|\Omega_A\rangle$ is really an energy density.

The only quark operators that have non-zero mean values in the state $|\Omega_A\rangle$ are those that destroy and create the same kind of quark or antiquark. Thus if we normally order the quark hamiltonian (1), then the part of the magnetic term $H_{qm} = -g \int d^3x\, \vec{J}_a \cdot \vec{A}_a$ that involves the field $\psi_d(x)$ of the d

quark

$$\psi_{\ell d}(x) = \sum_\sigma \int \frac{d^3p}{(2\pi)^{3/2}} \left[u_\ell(\vec{p},\sigma,d_i) e^{ip\cdot x} a(\vec{p},\sigma,d_i) + v_\ell(\vec{p},\sigma,d_i) e^{-ip\cdot x} a^{c\dagger}(\vec{p},\sigma,d_i) \right] \tag{4}$$

has a mean value

$$\begin{aligned} E_{dm} &= \langle \Omega_A | H_{dm} | \Omega_A \rangle = \langle \Omega_A | \left(-g \int d^3x \, \vec{J}^d_a \cdot \vec{A}_a \right) | \Omega_A \rangle \\ &= \langle \Omega_A | \left(-ig \int d^3x \, \bar{\psi}_d \vec{\gamma} \cdot \vec{A}_a \frac{\lambda_a}{2} \psi_d \right) | \Omega_A \rangle \end{aligned} \tag{5}$$

given by

$$\begin{aligned} E_{dm} &= g\vec{A}_8 \cdot \sum_{\sigma,i} \frac{\lambda^8_{ii}}{2} \int_{S(A,d)} d^3p \, \left[u^\dagger(\vec{p}_i,\sigma,d_i)\gamma^0\vec{\gamma}\, u(\vec{p}_i,\sigma,d_i) \right. \\ &\qquad \left. - v^\dagger(-\vec{p}_i,\sigma,d_i)\gamma^0\vec{\gamma}\, v(-\vec{p}_i,\sigma,d_i) \right] . \end{aligned} \tag{6}$$

The spin sums [5]

$$\sum_\sigma u_\ell(\vec{p},\sigma,d_i) u^*_{\ell'}(\vec{p},\sigma,d_i) = \frac{1}{2p^0} \left[(p^\mu\gamma_\mu + im_d)\, \gamma^0 \right]_{\ell\ell'} \tag{7}$$

and

$$\sum_\sigma v_\ell(\vec{p},\sigma,d_i) v^*_{\ell'}(\vec{p},\sigma,d_i) = \frac{1}{2p^0} \left[(p^\mu\gamma_\mu - im_d)\, \gamma^0 \right]_{\ell\ell'} \tag{8}$$

imply the trace relations

$$\sum_\sigma u^\dagger(\vec{p}_i,\sigma,d_i)\gamma^0\vec{\gamma}\, u(\vec{p}_i,\sigma,d_i) = \sum_\sigma v^\dagger(\vec{p}_i,\sigma,d_i)\gamma^0\vec{\gamma}\, v(\vec{p}_i,\sigma,d_i) = -\frac{2\vec{p}}{p^0} \tag{9}$$

$$\sum_\sigma u^\dagger(\vec{p}_i,\sigma,d_i)\gamma^0\, u(\vec{p}_i,\sigma,d_i) = \sum_\sigma -v^\dagger(\vec{p}_i,\sigma,d_i)\gamma^0\, v(\vec{p}_i,\sigma,d_i) = -i\frac{2m_d}{p^0} \tag{10}$$

and

$$\sum_\sigma u^\dagger(\vec{p}_i,\sigma,d_i)\, u(\vec{p}_i,\sigma,d_i) = \sum_\sigma v^\dagger(\vec{p}_i,\sigma,d_i)\, v(\vec{p}_i,\sigma,d_i) = 2. \tag{11}$$

It follows from the trace relation (9) that the magnetic energy density of the d quarks and antiquarks of color i in the constant gauge field $\vec{A}_8$ is

$$E_{dm} = -2g\lambda^8_{ii} \int_{S(A,d)} \frac{d^3p}{p^0}\, \vec{A}_8 \cdot \vec{p}. \tag{12}$$

The trace relation (11) implies that in the state $|\Omega_A\rangle$ the mean value of the color charge density $J^{0d}_a = \psi^\dagger_d \frac{1}{2}\lambda_a \psi_d$ and therefore that of the second term of the hamiltonian (1) vanish. It follows from the Gell-Mann matrices λ_a and from the trace relation (9) that the color current $\vec{J}^d_a = -\psi^\dagger_d \gamma^0 \vec{\gamma} \frac{1}{2}\lambda_a \psi_d$ for $a \neq 8$ also vanishes.

The mean value of the hamiltonian H_q for d quarks and antiquarks of color i in the state $|\Omega_A\rangle$ is therefore

$$E_{di} = \int_{S(A,d,i)} d^3p \left(4p^0 - 2g\lambda^8_{ii} \frac{\vec{A}_8 \cdot \vec{p}}{p^0} \right) \tag{13}$$

where the domain of integration $S(\vec{A}_8, d, i)$ is the set of momenta $\vec{p}$ for which the integrand is negative

$$g\lambda^8_{ii} \vec{A}_8 \cdot \vec{p} > 2(\vec{p}^2 + m_d^2). \tag{14}$$

The set $S(\vec{A}_8, d, i)$ is empty unless $g\lambda^8_{ii}|\vec{A}_8| > 4m_d$, which requires the effective magnitude of the gauge field to be large compared to the mass m_d of the d quark.

Since the quark energy density E_{fi} depends upon the flavor f and the color i only through the dimensionless ratio

$$r = \frac{4m_f}{g\lambda^8_{ii}|\vec{A}_8|}, \tag{15}$$

we may write it as the integral

$$E_{fi} = -32\pi (g\lambda^8_{ii}|\vec{A}_8|)^4 \int_{1-\sqrt{1-r^2}}^{1+\sqrt{1-r^2}} dx\, \frac{x(2x - x^2 - r^2)^2}{\sqrt{x^2 + r^2}} \tag{16}$$

which has the value

$$E_{fi} = -32\pi (g\lambda^8_{ii}|\vec{A}_8|)^4$$

$$\left[\left(\frac{1}{5}(x^2+r^2)^2 - x^3 - \frac{1}{2}r^2x + \frac{4}{3}(x^2-2r^2)\right)\sqrt{x^2+r^2} + \frac{1}{2}r^4\operatorname{arcsinh}\left(\frac{x}{r}\right)\right]_{1-\sqrt{1-r^2}}^{1+\sqrt{1-r^2}}. \tag{17}$$

For small r the energy density E_{fi} is approximately

$$E_{fi} \simeq -32\pi(g\lambda^8_{ii}|\vec{A}_8|)^4\left(\frac{16}{15} - 4\left(\frac{4m_f}{g\lambda^8_{ii}|\vec{A}_8|}\right)^2\right). \tag{18}$$

Summing over the three colors, we get

$$E_f \simeq -64\pi(g|\vec{A}_8|)^4\left(\frac{16}{15} - 4\left(\frac{4m_f}{g|\vec{A}_8|}\right)^2\right) \tag{19}$$

which displays isospin symmetry when $4m_f \ll g|\vec{A}_8|$. If the gauge field is moderately strong $4m_s > g\lambda^8_{ii}|\vec{A}^8| \gg 4m_d$, then the energy density of the u–d condensate is

$$E_{ud} \simeq -128\pi(g|\vec{A}_8|)^4\left(\frac{16}{15} - 4\left(\frac{4m_{ud}}{g|\vec{A}_8|}\right)^2\right) \tag{20}$$

where $m^2_{ud} = (m^2_u + m^2_d)/2$. For stronger gauge fields, $g\lambda^8_{ii}|\vec{A}^8| >> 4m_s$, the energy density of the light-quark condensate is

$$E_{uds} \simeq -192\pi(g|\vec{A}_8|)^4\left(\frac{16}{15} - 4\left(\frac{4m_\ell}{g|\vec{A}_8|}\right)^2\right) \tag{21}$$

where $m^2_\ell = (m^2_u + m^2_d + m^2_s)/3$.

3 The Breakdown of Chiral Symmetry

The quark condensate occasioned by the constant gauge field $\vec{A}_8$ gives rise to a mean value of the space average of $\frac{1}{2}(\bar{u}u + \bar{d}d)$, which is an order parameter that traces the breakdown of chiral symmetry. It follows from the expansion (4) of the Dirac field and the trace relation (10) that this order parameter is

$$\begin{aligned}
\langle \tfrac{1}{2}(\bar{u}u + \bar{d}d)\rangle &= \langle \Omega_A | \int d^3x \tfrac{1}{2}(\bar{u}u + \bar{d}d) | \Omega_A \rangle \\
&= \tfrac{1}{2} \sum_{f=u}^{d} \sum_{i=1}^{3} \sum_{\sigma} \int_{S(A,f,i)} d^3p \left[u^\dagger(\vec{p}_i, \sigma, f_i) i\gamma^0 u(\vec{p}_i, \sigma, f_i) \right. \\
&\qquad \left. - v^\dagger(-\vec{p}_i, \sigma, f_i) i\gamma^0 v(-\vec{p}_i, \sigma, d_i) \right] \\
&= \sum_{f=u}^{d} \sum_{i=1}^{3} 2m_f \int_{S(A,f,i)} \frac{d^3p}{p^0}.
\end{aligned} \tag{22}$$

In terms of the ratio $r = 4m_f/(g\lambda^8_{ii}|\vec{A}_8|)$, it is

$$\begin{aligned}
\langle \tfrac{1}{2}(\bar{u}u + \bar{d}d)\rangle &= \sum_{f=u}^{d} \sum_{i=1}^{3} \frac{\pi}{4} m_f \left(g\lambda^8_{ii}|\vec{A}_8| \right)^2 \int_{1-\sqrt{1-r^2}}^{1+\sqrt{1-r^2}} dx \left(\frac{x^2}{\sqrt{x^2+r^2}} - \frac{x}{2}\sqrt{x^2+r^2} \right) \\
&= \sum_{f=u}^{d} \sum_{i=1}^{3} \frac{\pi}{4} m_f \left(g\lambda^8_{ii}|\vec{A}_8| \right)^2 \\
&\quad \left[\left(\frac{x}{2} - \frac{x^2+r^2}{6} \right) \sqrt{x^2+r^2} - \frac{r^2}{2} \text{arcsinh}\left(\frac{x}{r} \right) \right]_{1-\sqrt{1-r^2}}^{1+\sqrt{1-r^2}}.
\end{aligned} \tag{23}$$

For small r this condensate or order parameter is

$$\langle \tfrac{1}{2}(\bar{u}u + \bar{d}d)\rangle \simeq \sum_{f=u}^{d} \sum_{i=1}^{3} \frac{\pi}{4} m_f \left(g\lambda^8_{ii}|\vec{A}_8| \right)^2 \left[\frac{2}{3} + \frac{r^2}{2} \left(\ln\left(\frac{r}{4} \right) - \frac{1}{2} \right) \right]. \tag{24}$$

In the limit $r \to 0$ and summed over colors (and over u and d), it is

$$\langle \tfrac{1}{2}(\bar{u}u + \bar{d}d)\rangle \simeq \frac{\pi}{3}(m_u + m_d) \left(g|\vec{A}_8| \right)^2. \tag{25}$$

We may use this formula and Weinberg's relation (Eq.(19.4.46) of [4])

$$\langle \tfrac{1}{2}(\bar{u}u + \bar{d}d)\rangle \simeq \frac{m_\pi^2 F_\pi^2}{4(m_u + m_d)} \tag{26}$$

in which $F_\pi \simeq 184\,\mathrm{MeV}$ is the pion decay constant, to estimate the effective magnitude $\langle g|\vec{A}_8|\rangle$ of the gauge field in the physical vacuum of QCD as

$$\langle g|\vec{A}_8|\rangle \simeq \sqrt{\frac{3}{4\pi}}\,\frac{m_\pi F_\pi}{(m_u+m_d)}. \tag{27}$$

In the $\overline{\mathrm{MS}}$ scheme at a renormalization scale $\mu = 1\,\mathrm{GeV}$, the mass range $3\,\mathrm{MeV} < (m_u + m_d)/2 < 8\,\mathrm{MeV}$ of the Particle Data Group [6] implies that $770\,\mathrm{MeV} < \langle g|\vec{A}_8|\rangle < 2070\,\mathrm{MeV}$. Since $\mu = 1\,\mathrm{GeV}$ is somewhat high for hadronic physics, the low end of the range, $\langle g|\vec{A}_8|\rangle \simeq 800\,\mathrm{MeV}$, may be more reliable.

4 Exact Ground State

Because the hamiltonian (1) is quadratic in the quark fields, it is possible to find its exact ground state when the gluon fields are replaced by their mean values. For the case in which the gluons are represented by a coherent state $|A^3_c\rangle$ that is constant and points in the z-direction $\vec{A}_c = A^3_c$ with $A^0_c = 0$ and $c = 3$ or 8, the exact ground state $|\Omega^e_A\rangle$ may be defined in terms of the quantity

$$\alpha_i = \tfrac{1}{2} g A^3_c \lambda^c_{ii} \tag{28}$$

as a product over all momenta $\vec{p}$, colors i, and flavors f

$$|\Omega^e_A\rangle = \left(\prod_{\vec{p},i,f} C(\vec{p},i,f,A^3_c)\right)|A^3_c\rangle \tag{29}$$

in which the operator $C(\vec{p},i,f,A^3_c)$ is

$$\begin{aligned} C(\vec{p},i,f,A^3_c) = \frac{1}{2p^0 s}\Big\{&\big(p_0^2-\alpha_i p_3-p^0 s\big)\, a^\dagger_{if}(\vec{p},\tfrac{1}{2})a^{c\dagger}_{if}(-\vec{p},\tfrac{1}{2})a^\dagger_{if}(\vec{p},-\tfrac{1}{2})a^{c\dagger}_{if}(-\vec{p},-\tfrac{1}{2}) \\ &+ \frac{\alpha_i p_3}{p^0+m_f}\Big[(p_1-ip_2)\, a^\dagger_{if}(\vec{p},\tfrac{1}{2})a^{c\dagger}_{if}(-\vec{p},\tfrac{1}{2}) - (p_1+ip_2)\, a^\dagger_{if}(\vec{p},-\tfrac{1}{2})a^{c\dagger}_{if}(-\vec{p},-\tfrac{1}{2})\Big] \\ &+ \frac{\alpha_i\,(p_0^2+p^0 m_f-p_3^2)}{p^0+m_f}\Big[a^\dagger_{if}(\vec{p},\tfrac{1}{2})a^{c\dagger}_{if}(-\vec{p},-\tfrac{1}{2}) + a^\dagger_{if}(\vec{p},-\tfrac{1}{2})a^{c\dagger}_{if}(-\vec{p},\tfrac{1}{2})\Big] \\ &+ \big(p_0^2-\alpha_i p_3+p^0 s\big)\Big\} \end{aligned} \tag{30}$$

where $a^{c\dagger}_{if}(-\vec{p}, \frac{1}{2}) = a^{c\dagger}(-\vec{p}, \frac{1}{2}, i, f)$ creates antiquarks of color i and flavor f and s is the square root

$$s = \sqrt{p_0^2 - 2\alpha_i p_3 + \alpha_i^2} \geq 0. \tag{31}$$

The state $|\Omega_A^e\rangle$ is an eigenstate of the hamiltonian (1) with eigenvalue

$$E(A_c^3) = \sum_{i,f} \int d^3p \left(2p^0 - 2\frac{\alpha_i p_3}{p^0} - 2s \right) \tag{32}$$

in which the integrand $E(\vec{p}, i, f, A_c^3) = 2p^0 - 2\alpha_i p_3/p^0 - 2s$ is negative definite. When $\alpha_i p_3 \gg p_0^2$, which is a generous version of the inequality (14), this integrand is approximately

$$E(\vec{p}, i, f, A_c^3) \simeq 4p^0 - 4\frac{\alpha_i p_3}{p^0}, \tag{33}$$

which is that of the integral (13). But when $p_0^2 \gg \alpha_i p_3$ and $p_0^2 \gg \alpha_i^2$, it is

$$E(\vec{p}, i, f, A_c^3) \simeq -\alpha_i^2 \, \frac{m_f^2 + p_1^2 + p_2^2}{(p^0)^3}. \tag{34}$$

In terms of the cutoff Λ, the quadratically divergent energy density (32) is

$$E(A_c^3) \sim -\left(A_c^3\right)^2 \Lambda^2. \tag{35}$$

It exerts a force, $\sim 2A_c^3\Lambda^2$, on the mean value of the gauge field A_c^3 driving it to larger values.

5 Conclusions and Speculations

We have seen that a vacuum component $|\Omega_A\rangle$ consisting of a coherent state of a slowly varying gauge field $\vec{A}_c$ and an associated quark-antiquark condensate (3) possesses a negative energy density ρ_Ω of the order of $-(g|\vec{A}_c|)^4$. The $q\bar{q}$ condensate in the component $|\Omega_A\rangle$ breaks chiral symmetry with an order parameter $\langle\frac{1}{2}(\bar{u}u + \bar{d}d)\rangle$ that agrees with soft-pion physics if the effective strength $g|\vec{A}_c|$ of the gauge field is of the order of 800 MeV.

The real vacuum is an integral over all such components $|\Omega_A\rangle$. In the temporal gauge, the gauge field A_a^0 is absent, and the vacuum is an integral (2) over all gauge transformations $\omega(\vec{x})$ of the image $|\Omega_A^\omega\rangle$ of a state like the component $|\Omega_A\rangle$ under the gauge transformation $\omega(\vec{x})$. This integral removes any breaking of rotational invariance associated with the uniform field $\vec{A}_8$. The mean value of any gauge-invariant operator, for instance the quark hamiltonian H_q, is a double integral

$$\langle\Omega|H_q|\Omega\rangle = \int D\vec{A}_a \int D\vec{A}'_a \, \langle\Omega_A|H_q|\Omega_{A'}\rangle \tag{36}$$

in which most of the off-diagonal terms are very small.

Of course, the actual energy density of the vacuum is small and non-negative. But by using normal ordering, we have been ignoring the zero-point energies of the fields. Zero-point energies augment ρ_Ω by a positive or negative energy density that is quartically divergent unless the number of Fermi fields is equal to the number of Bose fields and is quadratically divergent unless the supertrace $\sum(-1)^{2j} m_j^2$ of the squared masses of all particles vanishes. The large negative energy density ρ_Ω may make it possible to cancel a large positive energy density due to the breaking of supersymmetry.

The condensates described in this paper may explain why massless gluons give rise to a short-range force. To see this let us specialize to the component $|\Omega_A\rangle$ of the *QCD* vacuum that has a constant gauge field A_8^1 pointing in direction 1 of space and direction 8 of color space. In this component the mean value of the hamiltonian of the gauge fields contains a mass term for the fields A_b^m for $m \neq 1$

$$\sum_{a,b,c,m} \tfrac{1}{2} f_{a8b} f_{a8c} \langle A_8^1\rangle^2 A_b^m A_c^m = \sum_{b,c,m} \tfrac{1}{2} M_{bc}^2 A_b^m A_c^m \tag{37}$$

in which the mass matrix is

$$M_{bc}^2 = \langle g A_8^1\rangle^2 \sum_a f_{a8b} f_{a8c} = \langle g A_8^1\rangle^2 \left(T_8^2\right)_{bc} \tag{38}$$

where the 8×8 matrix T_8 is a generator of the group *SU(3)* in the adjoint representation. Since the matrix T_8 is hermitian, the eigenvalues of the mass matrix M_{bc}^2 are all non-negative.

The QCD vacuum (2) is an integral over slowly varying gauge fields and their correlated condensates. In this vacuum $|\Omega\rangle$, every space component A_c^i of the gauge field acquires a non-zero mean value. Thus gluons are massive in the vacuum of QCD, and according to the estimate (27), the mass of the gluon is in the range of hundreds of MeV.

The condensates of this paper may also explain quark confinement [7, 8]. Because a color-electric field moves quarks in one direction and antiquarks in the opposite direction, the quark condensate of the QCD vacuum in this model is not stable in the presence of color-electric fields. Thus volumes of space that are traversed by color-electric fields have less quark-antiquark condensate [9] and so a higher energy density than that of the physical vacuum. Hence the surface of a hadron is exposed to a pressure that is equal to the difference between the energy density ρ_Ω of the physical vacuum outside the hadron and the energy density ρ_h inside the hadron which, due to the color-electric fields, is somewhat higher than ρ_Ω. This pressure p

$$p \simeq \rho_h - \rho_\Omega \tag{39}$$

confines quarks because it squeezes their color-electric fields. In this view quarks are confined not because of the energy of their color-electric fields but because their color-electric fields are excluded by the physical vacuum. With the estimate (27) of $\langle g|\vec{A}_8|\rangle$, a drop of only 1% in the hadronic quark-antiquark condensate would produce a confining pressure of the order of $p \simeq (1\text{GeV})^4$.

The standard picture of quark confinement rests upon computer simulations of lattice gauge theories. In these theories the gauge fields are replaced by the elements of a compact gauge group. This compactification generates unwanted effects known as artifacts. Most [10] if not all [11] of the string tension measured in compact lattice gauge theory is due to such artifacts. In non-compact lattice simulations [12], there is no sign of confinement unless the action contains auxiliary fields, which introduce other kinds of artifacts. Thus in all lattice simulations whether compact or non-compact, confinement has appeared only when accompanied by significant lattice artifacts. The artifacts that drive confinement on the lattice may represent the long-distance effects of a true, but unknown, microscopic lattice action. But it is also possible that these artifacts are merely artifacts, have nothing to do with confinement, and do not survive in the continuum limit [11].

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