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TOWARD A NEUTRINO MASS MATRIX

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One may identify the general properties of the neutrino mass matrix by generating many random mass matrices and testing them against the results of the neutrino experiments.

There are three light, active neutrinos whose fields $\nu_e$, $\nu_\mu$, and $\nu_\tau$ are left handed; there also probably are three right-handed neutrinos whose fields $\nu_{re}$, $\nu_{r\mu}$, and $\nu_{r\tau}$ do not participate in the electroweak interactions and are said to be sterile. Fields that mix must transform in the same way. So it makes sense to combine the three left-handed (two-component) neutrinos $\nu_e$, $\nu_\mu$, and $\nu_\tau$ with the left-handed fields $s_e$, $s_\mu$, and $s_\tau$ that are the charge conjugates of the right-handed neutrino fields $\nu_{re}$, $\nu_{r\mu}$, $\nu_{r\tau}$, i.e., $s_e = -i\sigma_2 \nu_{re}^* = (\nu_{re2}, -\nu_{re1})$, etc. The six left-handed neutrino fields form a six-vector, $N = (\nu_e, \nu_\mu, \nu_\tau, s_e, s_\mu, s_\tau)$. The only mass term available for two left-handed neutrino fields $N_i$ and $N_j$ is $M_{ij} (N_i N_j - \frac{1}{2} N_i^2 - \frac{1}{2} N_j^2) + \text{h.c.}$ The six-by-six complex mass matrix

\[ M = \begin{pmatrix} F & D^* \\ D & E \end{pmatrix} \]

is symmetric; here $\top$ means transpose. In most models the quantities $M_{ij}$ are the mean values of Higgs fields in the vacuum. Because the complex mass matrix $M$ is symmetric, it may be factored $M = U M U^\top$ by a matrix $U$ that is unitary, and a matrix $M$ that is diagonal and positive. The elements $m_i$ of $M$ are the masses of the neutrinos. The vector $N_m$ of six mass eigenfields is $N_m = U^\top N$. The mass eigenfields are the normal modes of the action. They are Majorana neutrinos.

The six-by-six complex mass matrix $M$ involves 42 real parameters. One may explore this large space and test various properties of the mass matrix $M$ by generating many random mass matrices that share those properties, and by computing the extent to which they fit the results of the neutrino experiments. For instance, one may define an angle $x_\nu$ by

\[ \sin^2 x_\nu = \frac{\text{Tr}(E^\dagger F + F^\dagger E)}{\text{Tr}(2D^\dagger D + E^\dagger E + F^\dagger F)} \]

and test whether $x_\nu$ can be very small. This angle characterizes where the six neutrinos lie on a continuum that extends from three purely Dirac neutrinos, $x_\nu = 0$, to six purely Majorana neutrinos, $x_\nu = \frac{\pi}{2}$. When the angle $x_\nu$ is small, as in a theory in which $B - L$ is nearly conserved, the six masses $m_i$ coalesce into three pairs of nearly degenerate masses, and the six neutrinos form three nearly Dirac neutrinos, a condition that has been called pseudo-Dirac. This $B - L$ or small-$x_\nu$ property explains the large mixing angles and tiny mass differences seen
experimentally and allows the masses of the neutrinos to lie in the eV range with \( \sum_i m_i \lesssim 8 \text{ eV} \) which is a cosmological bound.\(^4\) To test this property, I used the software package LAPACK\(^5\) to factorize 10,000 random mass matrices \( \mathcal{M} \) every parameter of each of which was a complex number \( z = x + iy \) with \( x \) and \( y \) chosen randomly and uniformly from the interval \([-1\text{eV},1\text{eV}]\). The solar neutrinos were taken to have an energy of 1 MeV, and the probability \( P(\nu_i \rightarrow \nu_f) = |A(\nu_i \rightarrow \nu_f)|^2 \) with \( A(\nu_i \rightarrow \nu_f) = \sum_j U^*_{fj} U_{ij} \exp(-im_d^2 L/(2E)) \) was averaged over one revolution of the Earth about the Sun. The atmospheric neutrinos were averaged over the atmosphere and over energies in the range of 1–30 GeV weighted by the flux of atmospheric muon neutrinos as a function of energy and local zenith angle given by the Bartol group.\(^6\) The resulting scatter plots fit the gross features of the solar and atmospheric experiments quite well with \( \sin \theta^d = 0.003 \) when two additional properties are added.\(^2\) The first is a constraint on inter-generational mixing which I imposed by suppressing the singly off-diagonal matrix elements of \( D, E, \) and \( F \) by 0.2 and the doubly off-diagonal matrix elements by 0.04. The second is a weak mass hierarchy which I implemented by scaling the \( i,j \)-th elements of the sub-matrices \( E, F, \) and \( D \) of the mass matrix \( \mathcal{M} \), Eq. (1), by the factors \( f(i) \ast f(j) \) where \( f = (0.2,1,2) \). These three properties also lead to general agreement with the results of the LSND and KARMEN2 experiments. If the physical mass matrix \( \mathcal{M} \) shares these properties, and if MiniBooNE can achieve a sensitivity of 0.001 for \( \bar{\nu}_\mu \rightarrow \bar{\nu}_e \) and a precision of 0.01 for \( \nu_\mu \rightarrow \nu_\mu \), then it has a good chance of seeing the appearance of \( \bar{\nu}_e \) and the disappearance of \( \bar{\nu}_\mu \).\(^2\)

Another test of the properties of the neutrino mass matrix \( \mathcal{M} \) is provided by the passage of high-energy neutrinos through the Earth.\(^7\) In momentum space the Dirac equation for the 24-vector \( (N, i\sigma_2 N^*) \) of neutrino fields is

\[
0 = \begin{pmatrix}
E + \vec{p} \cdot \vec{V} + V & -M^\dagger \\
-M & E - \vec{p} \cdot \vec{V}
\end{pmatrix}
\begin{pmatrix}
N \\
N^*
\end{pmatrix}
\]

in which the hermitian matrix \( V \) represents the interaction of the neutrinos with the electrons and quarks of the Earth. The matrix \( V \) is diagonal with elements \( V = \left( G_F/\sqrt{2} \right) (2N_e - N_n, -N_n, -N_n, 0, 0, 0, 0) \) in which \( N_e \) and \( N_n \) are the number densities of electrons and neutrons. By neglecting neutrinos of positive helicity, antineutrinos of negative helicity, and the terms \( V^2 \) and \( [V,M] \), one finds for the square of the mass operator the six-by-six hermitian matrix \( M(p)^2 = M\dagger M \pm 2|\vec{p}| V = U^\dagger M_d(p)^2 U \) in which \( U \) is another unitary matrix and the plus sign is for neutrinos and the minus sign for anti-neutrinos. I used the PREM model\(^8\) to divide the Earth into 82 shells each with its approximate values of the densities \( N_e \) and \( N_n \). My code computes the amplitude for a neutrino to propagate through the Earth, which is the path-ordered exponential

\[
A = \mathcal{P} \left\{ \exp \left[ i \int dx \frac{M(p)^2}{(2|\vec{p}|)} \right] \right\},
\]

as a product of up to 82 factors of the form \( U^\dagger \exp \left[ i \Delta x \frac{M_d(p)^2}{(2|\vec{p}|)} \right] U \). For 16 random mass matrices \( \mathcal{M} \) with small \( x_{ij} \), little inter-generational mixing, and a weak
mass hierarchy, the probabilities \( P_{\text{SK}}(\nu_\mu \rightarrow \nu_\mu) \) that a muon neutrino of energy 100 GeV would traverse the Earth at a given azimuthal angle \( \theta \) and enter the Super Kamiokande detector are plotted in Fig. 1. Because high-energy interactions with matter suppress active-sterile neutrino oscillations, the probability \( P_{\text{SK}}(\nu_\mu \rightarrow \nu_\mu) \) remains close to unity in 13 of the cases. But in three cases it drops quite far below unity. Further work is required to determine what additional properties, if any, would cause such random mass matrices to display the oscillations and neutral-current events observed by the Super-Kamiokande collaboration.\(^9\)

Figure 1: The probability that a 100-GeV muon neutrino would traverse the Earth are plotted against the cosine of the azimuthal angle \( \theta \) for 16 random mass matrices \( M \) with \( \sin \varphi = 0.003 \), little inter-generational mixing, and a weak mass hierarchy.

References