We now make the gauge transformation

$$A'_b(x) = A_b(x) + \partial_b \Lambda(x) \quad \text{and} \quad \psi'(x) = e^{iq\Lambda(x)}\psi(x) \tag{16.169}$$

and replace the fields $A_b(x)$ and $\psi(x)$ everywhere in the numerator and (separately) in the denominator in the ratio (16.168) of path integrals by their gauge transforms (16.169) $A'_\mu(x)$ and $\psi'(x)$. This change of variables changes nothing; it's like replacing $\int_{-\infty}^{\infty} f(x)\,dx$ by $\int_{-\infty}^{\infty} f(y)\,dy$, and so

$$\langle\Omega|\mathcal{T}\left[\mathcal{O}_1 \dots \mathcal{O}_n\right]|\Omega\rangle = \langle\Omega|\mathcal{T}\left[\mathcal{O}_1 \dots \mathcal{O}_n\right]|\Omega\rangle' \tag{16.170}$$

in which the prime refers to the gauge transformation (16.169).

We've seen that the action S is gauge invariant. So is the measure $DA\,D\psi$, and we now restrict ourselves to operators $\mathcal{O}_1 \dots \mathcal{O}_n$ that are *gauge invariant*. So in the right-hand side of equation (16.170), the replacement of the fields by their gauge transforms affects only the term $\delta[\boldsymbol{\nabla} \cdot \boldsymbol{A}]$ that enforces the Coulomb-gauge condition

$$\langle\Omega|\mathcal{T}\left[\mathcal{O}_1 \dots \mathcal{O}_n\right]|\Omega\rangle = \frac{\displaystyle\int \mathcal{O}_1 \dots \mathcal{O}_n\, e^{iS}\, \delta[\boldsymbol{\nabla} \cdot \boldsymbol{A} + \triangle\Lambda]\, DA\, D\psi}{\displaystyle\int e^{iS}\, \delta[\boldsymbol{\nabla} \cdot \boldsymbol{A} + \triangle\Lambda]\, DA\, D\psi}. \tag{16.171}$$

We now have two choices. If we integrate over all gauge functions $\Lambda(x)$ in both the numerator and the denominator of this ratio (16.171), then apart from over-all constants that cancel, the mean value in the vacuum of the time-ordered product is the ratio

$$\langle\Omega|\mathcal{T}\left[\mathcal{O}_1 \dots \mathcal{O}_n\right]|\Omega\rangle = \frac{\displaystyle\int \mathcal{O}_1 \dots \mathcal{O}_n\, e^{iS}\, DA\, D\psi}{\displaystyle\int e^{iS}\, DA\, D\psi} \tag{16.172}$$

in which we integrate over all matter fields, gauge fields, and gauges. That is, **we do not fix the gauge**.

The analogous formula for the euclidean time-ordered product is

$$\langle\Omega|\mathcal{T}_e\left[\mathcal{O}_1 \dots \mathcal{O}_n\right]|\Omega\rangle = \frac{\displaystyle\int \mathcal{O}_1 \dots \mathcal{O}_n\, e^{-S_e}\, DA\, D\psi}{\displaystyle\int e^{-S_e}\, DA\, D\psi} \tag{16.173}$$

in which the euclidean action S_e is the space-time integral of the energy density. This formula is quite general; it holds in nonabelian gauge theories and is important in lattice gauge theory.

Our second choice is to multiply the numerator and the denominator