The other nonzero Γ's are

$$\Gamma^1_{22} = -r\,(1-kr^2) \qquad \Gamma^1_{33} = -r\,(1-kr^2)\sin^2\theta \tag{11.399}$$

$$\Gamma^2_{12} = \Gamma^3_{13} = \frac{1}{r} = \Gamma^2_{21} = \Gamma^3_{31} \tag{11.400}$$

$$\Gamma^2_{33} = -\sin\theta\,\cos\theta \qquad \Gamma^3_{23} = \cot\theta = \Gamma^3_{32}. \tag{11.401}$$

Our formulas (11.350 & 11.348) for the Ricci and curvature tensors give

$$R_{00} = R^n_{0n0} = [\partial_0 + \Gamma_0, \partial_n + \Gamma_n]^n{}_0. \tag{11.402}$$

Clearly the commutator of Γ_0 with itself vanishes, and one may use the formulas (11.397–11.401) for the other connections to check that

$$[\Gamma_0, \Gamma_n]^n{}_0 = \Gamma^n_{0\,k}\,\Gamma^k_{n\,0} - \Gamma^n_{n\,k}\,\Gamma^k_{0\,0} = 3\left(\frac{\dot a}{a}\right)^2 \tag{11.403}$$

and that

$$\partial_0\,\Gamma^n_{n\,0} = 3\,\partial_0\left(\frac{\dot a}{a}\right) = 3\,\frac{\ddot a}{a} - 3\left(\frac{\dot a}{a}\right)^2 \tag{11.404}$$

while $\partial_n \Gamma^n_{0\,0} = 0$. So the 00-component of the Ricci tensor is

$$R_{00} = 3\,\frac{\ddot a}{a}. \tag{11.405}$$

Similarly, one may show that the other non-zero components of Ricci's tensor are

$$R_{11} = -\frac{A}{1-kr^2} \qquad R_{22} = -r^2 A \quad \text{and} \quad R_{33} = -r^2 A\sin^2\theta \tag{11.406}$$

in which $A = a\ddot a + 2\dot a^2 + 2k$. The scalar curvature (11.351) is

$$R = g^{ab} R_{ba} = -\frac{6}{a^2}\left(a\ddot a + \dot a^2 + k\right). \tag{11.407}$$

In co-moving coordinates such as those of the Robertson-Walker metric (11.395) $u_i = (1,0,0,0)$, and so the energy-momentum tensor (11.370) is

$$T_{ij} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & p\,g_{11} & 0 & 0 \\ 0 & 0 & p\,g_{22} & 0 \\ 0 & 0 & 0 & p\,g_{33} \end{pmatrix}. \tag{11.408}$$

Its trace is

$$T = g^{ij}\,T_{ij} = -\rho + 3p. \tag{11.409}$$

Thus using our formula (11.395) for $g_{00} = -1$, (11.405) for R_{00}, (11.408)