The 2×2 matrix $D^{(1/2,0)}$ that represents the Lorentz transformation (10.242)

$$L = e^{-i\theta_\ell J_\ell - i\lambda_\ell K_\ell} \tag{10.251}$$

is

$$D^{(1/2,0)}(\boldsymbol{\theta},\boldsymbol{\lambda}) = \exp\left(-i\boldsymbol{\theta}\cdot\boldsymbol{\sigma}/2 - \boldsymbol{\lambda}\cdot\boldsymbol{\sigma}/2\right). \tag{10.252}$$

And so the generic $D^{(1/2,0)}$ matrix is

$$D^{(1/2,0)}(\boldsymbol{\theta},\boldsymbol{\lambda}) = e^{-\boldsymbol{z}\cdot\boldsymbol{\sigma}/2} \tag{10.253}$$

with $\boldsymbol{\lambda} = \mathrm{Re}\boldsymbol{z}$ and $\boldsymbol{\theta} = \mathrm{Im}\boldsymbol{z}$. It is nonunitary and of unit determinant; it is a member of the group $SL(2,C)$ of complex unimodular 2×2 matrices. The (covering) group $SL(2,C)$ relates to the Lorentz group $SO(3,1)$ as $SU(2)$ relates to the rotation group $SO(3)$.

Example 10.31 (The Standard Left-Handed Boost) For a particle of mass $m > 0$, the "standard" boost that takes the 4-vector $k = (m,\mathbf{0})$ to $p = (p^0,\boldsymbol{p})$, where $p^0 = \sqrt{m^2+\boldsymbol{p}^2}$, is a boost in the $\hat{\boldsymbol{p}}$ direction

$$B(p) = R(\hat{\boldsymbol{p}})\, B_3(p^0)\, R^{-1}(\hat{\boldsymbol{p}}) = \exp\left(\alpha\, \hat{\boldsymbol{p}}\cdot\boldsymbol{B}\right) \tag{10.254}$$

in which $\cosh\alpha = p^0/m$ and $\sinh\alpha = |\boldsymbol{p}|/m$, as one may show by expanding the exponential (exercise 10.33).

For $\boldsymbol{\lambda} = \alpha\,\hat{\boldsymbol{p}}$, one may show (exercise 10.34) that the matrix $D^{(1/2,0)}(\mathbf{0},\boldsymbol{\lambda})$ is

$$\begin{aligned} D^{(1/2,0)}(\mathbf{0},\alpha\,\hat{\boldsymbol{p}}) &= e^{-\alpha\hat{\boldsymbol{p}}\cdot\boldsymbol{\sigma}/2} = I\,\cosh(\alpha/2) - \hat{\boldsymbol{p}}\cdot\boldsymbol{\sigma}\,\sinh(\alpha/2) \\ &= I\,\sqrt{(p^0+m)/(2m)} - \hat{\boldsymbol{p}}\cdot\boldsymbol{\sigma}\,\sqrt{(p^0-m)/(2m)} \\ &= \frac{p^0+m-\boldsymbol{p}\cdot\boldsymbol{\sigma}}{\sqrt{2m(p^0+m)}} \end{aligned} \tag{10.255}$$

in the third line of which the 2×2 identity matrix I is suppressed. □

Under $D^{(1/2,0)}$, the vector $(-I,\boldsymbol{\sigma})$ transforms like a 4-vector. For tiny $\boldsymbol{\theta}$ and $\boldsymbol{\lambda}$, one may show (exercise 10.36) that the vector $(-I,\boldsymbol{\sigma})$ transforms as

$$\begin{aligned} D^{\dagger(1/2,0)}(\boldsymbol{\theta},\boldsymbol{\lambda})(-I)D^{(1/2,0)}(\boldsymbol{\theta},\boldsymbol{\lambda}) &= -I + \boldsymbol{\lambda}\cdot\boldsymbol{\sigma} \\ D^{\dagger(1/2,0)}(\boldsymbol{\theta},\boldsymbol{\lambda})\,\boldsymbol{\sigma}\,D^{(1/2,0)}(\boldsymbol{\theta},\boldsymbol{\lambda}) &= \boldsymbol{\sigma} + (-I)\boldsymbol{\lambda} + \boldsymbol{\theta}\wedge\boldsymbol{\sigma} \end{aligned} \tag{10.256}$$

which is how the 4-vector $(t,\boldsymbol{x})$ transforms (10.241). Under a finite Lorentz transformation L the 4-vector $S^a \equiv (-I,\boldsymbol{\sigma})$ becomes

$$D^{\dagger(1/2,0)}(L)\, S^a\, D^{(1/2,0)}(L) = L^a{}_b S^b. \tag{10.257}$$

A massless field $\xi(x)$ that responds to a unitary Lorentz transformation $U(L)$ like

$$U(L)\,\xi(x)\,U^{-1}(L) = D^{(1/2,0)}(L^{-1})\,\xi(Lx) \qquad (10.258)$$

is called a **left-handed Weyl spinor**. We will see in example 10.32 why the action density for such spinors

$$\mathcal{L}_\ell(x) = i\,\xi^\dagger(x)\,(\partial_0 I - \boldsymbol{\nabla}\cdot\boldsymbol{\sigma})\,\xi(x) \qquad (10.259)$$

is Lorentz covariant, that is

$$U(L)\,\mathcal{L}_\ell(x)\,U^{-1}(L) = \mathcal{L}_\ell(Lx). \qquad (10.260)$$

Example 10.32 (Why $\mathcal{L}_\ell$ Is Lorentz Covariant) We first note that the derivatives ∂'_b in $\mathcal{L}_\ell(Lx)$ are with respect to $x' = Lx$. Since the inverse matrix L^{-1} takes x' back to $x = L^{-1}x'$ or in tensor notation $x^a = L^{-1a}{}_b\,x'^b$, the derivative ∂'_b is

$$\partial'_b = \frac{\partial}{\partial x'^b} = \frac{\partial x^a}{\partial x'^b}\frac{\partial}{\partial x^a} = L^{-1a}{}_b\,\frac{\partial}{\partial x^a} = \partial_a\,L^{-1a}{}_b. \qquad (10.261)$$

Now using the abbreviation $\partial_0 I - \boldsymbol{\nabla}\cdot\boldsymbol{\sigma} \equiv -\partial_a S^a$ and the transformation laws (10.257 & 10.258), we have

$$\begin{aligned} U(L)\,\mathcal{L}_\ell(x)\,U^{-1}(L) &= i\,\xi^\dagger(Lx)D^{(1/2,0)\dagger}(L^{-1})(-\partial_a S^a)D^{(1/2,0)}(L^{-1})\,\xi(Lx) \\ &= i\,\xi^\dagger(Lx)(-\partial_a L^{-1a}{}_b S^b)\,\xi(Lx) \\ &= i\,\xi^\dagger(Lx)(-\partial'_b S^b)\,\xi(Lx) = \mathcal{L}_\ell(Lx) \end{aligned} \qquad (10.262)$$

which shows that $\mathcal{L}_\ell$ is Lorentz covariant. □

Incidentally, the rule (10.261) ensures, among other things, that the divergence $\partial_a V^a$ is invariant

$$(\partial_a V^a)' = \partial'_a V'^a = \partial_b\,L^{-1b}{}_a\,L^a{}_c V^c = \partial_b\,\delta^b{}_c\,V^c = \partial_b\,V^b. \qquad (10.263)$$

Example 10.33 (Why ξ is Left Handed) The space-time integral S of the action density $\mathcal{L}_\ell$ is stationary when $\xi(x)$ satisfies the wave equation

$$(\partial_0 I - \boldsymbol{\nabla}\cdot\boldsymbol{\sigma})\,\xi(x) = 0 \qquad (10.264)$$

or in momentum space

$$(E + \boldsymbol{p}\cdot\boldsymbol{\sigma})\,\xi(p) = 0. \qquad (10.265)$$

Multiplying from the left by $(E - \boldsymbol{p}\cdot\boldsymbol{\sigma})$, we see that the energy of a particle created or annihilated by the field ξ is the same as its momentum $E = |\boldsymbol{p}|$ in accord with the absence of a mass term in the action density $\mathcal{L}_\ell$. And because the spin of the particle is represented by the matrix $\boldsymbol{J} = \boldsymbol{\sigma}/2$, the momentum-space relation (10.265) says that $\xi(p)$ is an eigenvector of $\hat{\boldsymbol{p}} \cdot \boldsymbol{J}$

$$\hat{\boldsymbol{p}} \cdot \boldsymbol{J}\, \xi(p) = -\frac{1}{2}\, \xi(p) \tag{10.266}$$

with eigenvalue $-1/2$. A particle whose spin is opposite to its momentum is said to have **negative helicity** or to be **left handed**. Nearly massless neutrinos are nearly left handed. □

One may add to this action density the **Majorana mass term**

$$\mathcal{L}_M(x) = -\tfrac{1}{2} m \left(\xi^\dagger(x)\, \sigma_2\, \xi^*(x) + \xi^\mathsf{T}(x)\, \sigma_2\, \xi(x) \right) \tag{10.267}$$

which is Lorentz covariant because the matrices σ_1 and σ_3 anti-commute with σ_2 which is antisymmetric (exercise 10.39). Since charge is conserved, only neutral fields like neutrinos can have Majorana mass terms.

The generators of the representation $D^{(0,1/2)}$ with $j = 0$ and $j' = 1/2$ are given by (10.247 & 10.249) with $\boldsymbol{J}^+ = 0$ and $\boldsymbol{J}^- = \boldsymbol{\sigma}/2$; they are

$$\boldsymbol{J} = \frac{1}{2}\boldsymbol{\sigma} \quad \text{and} \quad \boldsymbol{K} = i\frac{1}{2}\boldsymbol{\sigma}. \tag{10.268}$$

Thus 2×2 matrix $D^{(0,1/2)}(\boldsymbol{\theta}, \boldsymbol{\lambda})$ that represents the Lorentz transformation (10.242)

$$L = e^{-i\theta_\ell J_\ell - i\lambda_j K_j} \tag{10.269}$$

is

$$D^{(0,1/2)}(\boldsymbol{\theta}, \boldsymbol{\lambda}) = \exp\left(-i\boldsymbol{\theta} \cdot \boldsymbol{\sigma}/2 + \boldsymbol{\lambda} \cdot \boldsymbol{\sigma}/2\right) = D^{(1/2,0)}(\boldsymbol{\theta}, -\boldsymbol{\lambda}) \tag{10.270}$$

which differs from $D^{(1/2,0)}(\boldsymbol{\theta}, \boldsymbol{\lambda})$ only by the sign of $\boldsymbol{\lambda}$. The generic $D^{(0,1/2)}$ matrix is the complex unimodular 2×2 matrix

$$D^{(0,1/2)}(\boldsymbol{\theta}, \boldsymbol{\lambda}) = e^{\boldsymbol{z}^* \cdot \boldsymbol{\sigma}/2} \tag{10.271}$$

with $\boldsymbol{\lambda} = \mathrm{Re}\boldsymbol{z}$ and $\boldsymbol{\theta} = \mathrm{Im}\boldsymbol{z}$.

Example 10.34 (The Standard Right-Handed Boost) For a particle of mass $m > 0$, the "standard" boost (10.254) that transforms $k = (m, \boldsymbol{0})$ to $p = (p^0, \boldsymbol{p})$ is the 4×4 matrix $B(p) = \exp\left(\alpha\, \hat{\boldsymbol{p}} \cdot \boldsymbol{B}\right)$ in which $\cosh \alpha = p^0/m$ and $\sinh \alpha = |\boldsymbol{p}|/m$. This Lorentz transformation with $\boldsymbol{\theta} = \boldsymbol{0}$ and $\boldsymbol{\lambda} = \alpha\, \hat{\boldsymbol{p}}$

is represented by the matrix (exercise 10.35)

$$\begin{aligned} D^{(0,1/2)}(\mathbf{0}, \alpha\, \hat{\boldsymbol{p}}) = e^{\alpha \hat{\boldsymbol{p}}\cdot\boldsymbol{\sigma}/2} &= I \, \cosh(\alpha/2) + \hat{\boldsymbol{p}} \cdot \boldsymbol{\sigma} \, \sinh(\alpha/2) \\ &= I \, \sqrt{(p^0 + m)/(2m)} + \hat{\boldsymbol{p}} \cdot \boldsymbol{\sigma} \, \sqrt{(p^0 - m)/(2m)} \\ &= \frac{p^0 + m + \boldsymbol{p} \cdot \boldsymbol{\sigma}}{\sqrt{2m(p^0 + m)}} \end{aligned} \tag{10.272}$$

in the third line of which the 2×2 identity matrix I is suppressed. □

Under $D^{(0,1/2)}$, the vector $(I, \boldsymbol{\sigma})$ transforms as a 4-vector; for tiny $\boldsymbol{z}$

$$\begin{aligned} D^{\dagger(0,1/2)}(\boldsymbol{\theta}, \boldsymbol{\lambda}) \, I \, D^{(0,1/2)}(\boldsymbol{\theta}, \boldsymbol{\lambda}) &= I + \boldsymbol{\lambda} \cdot \boldsymbol{\sigma} \\ D^{\dagger(0,1/2)}(\boldsymbol{\theta}, \boldsymbol{\lambda}) \, \boldsymbol{\sigma} \, D^{(0,1/2)}(\boldsymbol{\theta}, \boldsymbol{\lambda}) &= \boldsymbol{\sigma} + I \boldsymbol{\lambda} + \boldsymbol{\theta} \wedge \boldsymbol{\sigma} \end{aligned} \tag{10.273}$$

as in (10.241).

A massless field $\zeta(x)$ that responds to a unitary Lorentz transformation $U(L)$ as

$$U(L) \, \zeta(x) \, U^{-1}(L) = D^{(0,1/2)}(L^{-1}) \, \zeta(Lx) \tag{10.274}$$

is called a **right-handed Weyl spinor**. One may show (exercise 10.38) that the action density

$$\mathcal{L}_r(x) = i \, \zeta^\dagger(x) \, (\partial_0 I + \boldsymbol{\nabla} \cdot \boldsymbol{\sigma}) \, \zeta(x) \tag{10.275}$$

is Lorentz covariant

$$U(L) \, \mathcal{L}_r(x) \, U^{-1}(L) = \mathcal{L}_r(Lx). \tag{10.276}$$

Example 10.35 (Why ζ Is Right Handed) An argument like that of example (10.33) shows that the field $\zeta(x)$ satisfies the wave equation

$$(\partial_0 I + \boldsymbol{\nabla} \cdot \boldsymbol{\sigma}) \, \zeta(x) = 0 \tag{10.277}$$

or in momentum space

$$(E - \boldsymbol{p} \cdot \boldsymbol{\sigma}) \, \zeta(p) = 0. \tag{10.278}$$

Thus, $E = |\boldsymbol{p}|$, and $\zeta(p)$ is an eigenvector of $\hat{\boldsymbol{p}} \cdot \boldsymbol{J}$

$$\hat{\boldsymbol{p}} \cdot \boldsymbol{J} \, \zeta(p) = \frac{1}{2} \, \zeta(p) \tag{10.279}$$

with eigenvalue 1/2. A particle whose spin is parallel to its momentum is said to have **positive helicity** or to be **right handed**. Nearly massless antineutrinos are nearly right handed. □

The Majorana mass term

$$\mathcal{L}_M(x) = -\tfrac{1}{2}m\left(\zeta^\dagger(x)\,\sigma_2\,\zeta^*(x) + \zeta^{\mathsf{T}}(x)\,\sigma_2\,\zeta(x)\right) \tag{10.280}$$

like (10.267) is Lorentz covariant.

10.33 The Dirac Representation of the Lorentz Group

Dirac's representation of $SO(3,1)$ is the direct sum $D^{(1/2,0)} \oplus D^{(0,1/2)}$ of $D^{(1/2,0)}$ and $D^{(0,1/2)}$. Its generators are the 4×4 matrices

$$\boldsymbol{J} = \frac{1}{2}\begin{pmatrix} \boldsymbol{\sigma} & 0 \\ 0 & \boldsymbol{\sigma} \end{pmatrix} \quad \text{and} \quad \boldsymbol{K} = \frac{i}{2}\begin{pmatrix} -\boldsymbol{\sigma} & 0 \\ 0 & \boldsymbol{\sigma} \end{pmatrix}. \tag{10.281}$$

Dirac's representation uses the **Clifford algebra** of the gamma matrices γ^a which satisfy the anticommutation relation

$$\{\gamma^a, \gamma^b\} \equiv \gamma^a\,\gamma^b + \gamma^b\,\gamma^a = 2\eta^{ab} I \tag{10.282}$$

in which η is the 4×4 diagonal matrix (10.223) with $\eta^{00} = -1$ and $\eta^{jj} = 1$ for $j = 1$, 2, and 3, and I is the 4×4 identity matrix.

Remarkably, the generators of the Lorentz group

$$J^{ij} = \epsilon_{ijk} J_k \quad \text{and} \quad J^{0j} = K_j \tag{10.283}$$

may be represented as commutators of gamma matrices

$$J^{ab} = -\frac{i}{4}\,[\gamma^a, \gamma^b]. \tag{10.284}$$

They transform the gamma matrices as a 4-vector

$$[J^{ab}, \gamma^c] = -i\gamma^a\,\eta^{bc} + i\gamma^b\,\eta^{ac} \tag{10.285}$$

(exercise 10.40) and satisfy the commutation relations

$$i[J^{ab}, J^{cd}] = \eta^{bc}\,J^{ad} - \eta^{ac}\,J^{bd} - \eta^{da}\,J^{cb} + \eta^{db}\,J^{ca} \tag{10.286}$$

of the Lorentz group (Weinberg, 1995, p. 213–217) (exercise 10.41).

The gamma matrices γ^a are not unique; if S is any 4×4 matrix with an inverse, then the matrices $\gamma'^a \equiv S\gamma^a S^{-1}$ also satisfy the definition (10.282). The choice

$$\gamma^0 = -i\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \text{and} \quad \boldsymbol{\gamma} = -i\begin{pmatrix} 0 & \boldsymbol{\sigma} \\ -\,\boldsymbol{\sigma} & 0 \end{pmatrix} \tag{10.287}$$

makes $\boldsymbol{J}$ and $\boldsymbol{K}$ block diagonal (10.281) and lets us assemble a left-handed spinor ξ and a right-handed spinor ζ neatly into a 4-component spinor

$$\psi = \begin{pmatrix} \xi \\ \zeta \end{pmatrix}. \tag{10.288}$$

Dirac's action density for a 4-spinor is

$$\mathcal{L} = -\overline{\psi}\,(\gamma^a \partial_a + m)\,\psi \equiv -\overline{\psi}\,(\not{\partial} + m)\,\psi \tag{10.289}$$

in which

$$\overline{\psi} \equiv i\psi^\dagger \gamma^0 = \psi^\dagger \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} \zeta^\dagger & \xi^\dagger \end{pmatrix}. \tag{10.290}$$

The kinetic part is the sum of the left-handed $\mathcal{L}_\ell$ and right-handed $\mathcal{L}_r$ action densities (10.259 & 10.275)

$$-\overline{\psi}\,\gamma^a \partial_a \psi = i\xi^\dagger\,(\partial_0 I - \boldsymbol{\nabla}\cdot\boldsymbol{\sigma})\,\xi + i\,\zeta^\dagger\,(\partial_0 I + \boldsymbol{\nabla}\cdot\boldsymbol{\sigma})\,\zeta. \tag{10.291}$$

If ξ is a left-handed spinor transforming as (10.258), then the spinor

$$\zeta = \sigma_2\,\xi^* \equiv \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} \xi_1^\dagger \\ \xi_2^\dagger \end{pmatrix} \tag{10.292}$$

transforms as a right-handed spinor (10.274), that is (exercise 10.42)

$$e^{\boldsymbol{z}^*\cdot\boldsymbol{\sigma}/2}\sigma_2\,\xi^* = \sigma_2\left(e^{-\boldsymbol{z}\cdot\boldsymbol{\sigma}/2}\xi\right)^*. \tag{10.293}$$

Similarly, if ζ is right handed, then $\xi = \sigma_2\,\zeta^*$ is left handed.

The simplest 4-spinor is the Majorana spinor

$$\psi_M = \begin{pmatrix} \xi \\ \sigma_2 \xi^* \end{pmatrix} = \begin{pmatrix} \sigma_2 \zeta^* \\ \zeta \end{pmatrix} = -i\gamma^2 \psi_M^* \tag{10.294}$$

whose particles are the same as its antiparticles.

If two Majorana spinors $\psi_M^{(1)}$ and $\psi_M^{(2)}$ have the same mass, then one may combine them into a Dirac spinor

$$\psi_D = \frac{1}{\sqrt{2}}\left(\psi_M^{(1)} + i\psi_M^{(2)}\right) = \frac{1}{\sqrt{2}} \begin{pmatrix} \xi^{(1)} + i\xi^{(2)} \\ \zeta^{(1)} + i\zeta^{(2)} \end{pmatrix} = \begin{pmatrix} \xi_D \\ \zeta_D \end{pmatrix}. \tag{10.295}$$

The **Dirac mass term**

$$-m\,\overline{\psi}_D \psi_D = -m\left(\zeta_D^\dagger \xi_D + \xi_D^\dagger \zeta_D\right) \tag{10.296}$$

conserves charge. For a Majorana field, it reduces to

$$\begin{aligned} -\tfrac{1}{2}m\overline{\psi}_M\psi_M = & -\tfrac{1}{2}m\left(\zeta^\dagger\xi + \xi^\dagger\zeta\right) = -\tfrac{1}{2}m\left(\xi^\dagger\sigma_2\xi^* + \xi^{\mathsf{T}}\sigma_2\xi\right) \\ = & -\tfrac{1}{2}m\left(\zeta^\dagger\sigma_2\zeta^* + \zeta^{\mathsf{T}}\sigma_2\zeta\right) \end{aligned} \tag{10.297}$$

a Majorana mass term (10.267 or 10.280).

10.34 The Poincaré Group

The elements of the Poincaré group are products of Lorentz transformations and translations in space and time. The Lie algebra of the Poincaré group therefore includes the generators $\boldsymbol{J}$ and $\boldsymbol{K}$ of the Lorentz group as well as the hamiltonian H and the momentum operator $\boldsymbol{P}$ which respectively generate translations in time and space.

Suppose $T(y)$ is a translation that takes a 4-vector x to $x+y$ and $T(z)$ is a translation that takes a 4-vector x to $x+z$. Then $T(z)T(y)$ and $T(y)T(z)$ both take x to $x+y+z$. So if a translation $T(y) = T(t,\boldsymbol{y})$ is represented by a unitary operator $U(t,\boldsymbol{y}) = \exp(iHt - i\boldsymbol{P}\cdot\boldsymbol{y})$, then the hamiltonian H and the momentum operator $\boldsymbol{P}$ commute with each other

$$[H, P^j] = 0 \quad \text{and} \quad [P^i, P^j] = 0. \tag{10.298}$$

We can figure out the commutation relations of H and $\boldsymbol{P}$ with the angular-momentum $\boldsymbol{J}$ and boost $\boldsymbol{K}$ operators by realizing that $P^a = (H, \boldsymbol{P})$ is a 4-vector. Let

$$U(\boldsymbol{\theta},\boldsymbol{\lambda}) = e^{-i\boldsymbol{\theta}\cdot\boldsymbol{J} - i\boldsymbol{\lambda}\cdot\boldsymbol{K}} \tag{10.299}$$

be the (infinite-dimensional) unitary operator that represents (in Hilbert space) the infinitesimal Lorentz transformation

$$L = I + \boldsymbol{\theta}\cdot\boldsymbol{R} + \boldsymbol{\lambda}\cdot\boldsymbol{B} \tag{10.300}$$

where $\boldsymbol{R}$ and $\boldsymbol{B}$ are the six 4×4 matrices (10.232 & 10.233). Then because P is a 4-vector under Lorentz transformations, we have

$$U^{-1}(\boldsymbol{\theta},\boldsymbol{\lambda})PU(\boldsymbol{\theta},\boldsymbol{\lambda}) = e^{+i\boldsymbol{\theta}\cdot\boldsymbol{J} + i\boldsymbol{\lambda}\cdot\boldsymbol{K}}Pe^{-i\boldsymbol{\theta}\cdot\boldsymbol{J} - i\boldsymbol{\lambda}\cdot\boldsymbol{K}} = (I + \boldsymbol{\theta}\cdot\boldsymbol{R} + \boldsymbol{\lambda}\cdot\boldsymbol{B})\,P \tag{10.301}$$

or using (10.273)

$$\begin{aligned} (I + i\boldsymbol{\theta}\cdot\boldsymbol{J} + i\boldsymbol{\lambda}\cdot\boldsymbol{K})\,H\,(I - i\boldsymbol{\theta}\cdot\boldsymbol{J} - i\boldsymbol{\lambda}\cdot\boldsymbol{K}) &= H + \boldsymbol{\lambda}\cdot\boldsymbol{P} \\ (I + i\boldsymbol{\theta}\cdot\boldsymbol{J} + i\boldsymbol{\lambda}\cdot\boldsymbol{K})\,\boldsymbol{P}\,(I - i\boldsymbol{\theta}\cdot\boldsymbol{J} - i\boldsymbol{\lambda}\cdot\boldsymbol{K}) &= \boldsymbol{P} + H\boldsymbol{\lambda} + \boldsymbol{\theta}\wedge\boldsymbol{P}. \end{aligned} \tag{10.302}$$