13.10 Redo the polling example (13.14–13.16) for the case of a slightly better poll in which 16 people were asked and 13 said they'd vote for Nancy Pelosi. What's the probability that she'll win the election? (You may use Maple or some other program to do the tedious integral.)

13.11 For the case in which N and $N - n$ are big, derive (13.52 & 13.53) from (13.43 & 13.51).

13.12 For the case in which N, $N - n$, and n are big, derive (13.54 & 13.55) from (13.43 & 13.51).

13.13 Without using the fact that the Poisson distribution is a limiting form of the binomial distribution, show from its definition (13.58) and its mean (13.60) that its variance is equal to its mean, as in (13.62).

13.14 Show that Gauss's approximation (13.74) to the binomial distribution is a normalized probability distribution with mean $\langle x \rangle = \mu = pN$ and variance $V[x] = pqN$.

13.15 Derive the approximations (13.88 & 13.89) for binomial probabilities for large N.

13.16 Compute the central moments (13.27) of the gaussian (13.75).

13.17 Derive formula (13.84) for the probability that a gaussian random variable falls within an interval.

13.18 Show that the expression (13.91) for $P(y|600)$ is negligible on the interval $(0, 1)$ except for y near $3/5$.

13.19 Determine the constant A of the homogeneous solution $\langle \boldsymbol{v}(t) \rangle_{gh}$ and derive expression (13.141) for the general solution $\langle \boldsymbol{v}(t) \rangle$ to (13.139).

13.20 Derive equation (13.142) for the variance of the position $\boldsymbol{r}$ about its mean $\langle \boldsymbol{r}(t) \rangle$. You may assume that $\langle \boldsymbol{r}(0) \rangle = \langle \boldsymbol{v}(0) \rangle = 0$ and that $\langle (\boldsymbol{v} - \langle \boldsymbol{v}(t) \rangle)^2 \rangle = 3kT/m$.

13.21 Derive equation (13.172) for the ensemble average $\langle \boldsymbol{r}^2(t) \rangle$ for the case in which $\langle \boldsymbol{r}^2(0) \rangle = 0$ and $d\langle \boldsymbol{r}^2(0) \rangle / dt = 0$.

13.22 Use (13.183) to derive the lower moments (13.185 & 13.186) of the distributions of Gauss and Poisson.

13.23 Find the third and fourth moments μ_3 and μ_4 for the distributions of Poisson (13.178) and Gauss (13.175).

13.24 Derive formula (13.190) for the first five cumulants of an arbitrary probability distribution.

13.25 Show that like the characteristic function, the moment-generating function $M(t)$ for an average of several independent random variables factorizes $M(t) = M_1(t/N)\, M_2(t/N) \cdots M_N(t/N)$.

13.26 Derive formula (13.197) for the moments of the log-normal probability distribution (13.196).

13.27 Why doesn't the log-normal probability distribution (13.196) have a sensible power-series about $x = 0$? What are its derivatives there?

13.28 Compute the mean and variance of the exponential distribution (13.198).

13.29 Show that the chi-square distribution $P_{3,G}(v, \sigma)$ with variance $\sigma^2 = kT/m$ is the Maxwell-Boltzmann distribution (13.100).

13.30 Compute the inverse Fourier transform (13.174) of the characteristic function (13.203) of the symmetric Lévy distribution for $\nu = 1$ and 2.

13.31 Show that the integral that defines $P^{(2)}(y)$ gives formula (13.239) with two Heaviside functions. Hint: keep x_1 and x_2 in the interval $(0, 1)$.

13.32 Derive the normal distribution (13.224) in the variable (13.223) from the central limit theorem (13.221) for the case in which all the means and variances are the same.

13.33 Show that Fisher's matrix (13.257) is symmetric $F_{k\ell} = F_{\ell k}$ and non-negative (1.38), and that when it is positive (1.39), it has an inverse.

13.34 Derive the integral equations (13.259 & 13.260) from the normalization condition $\int P(\boldsymbol{x}; \boldsymbol{\theta})\, d^N x = 1$.

13.35 Derive the Cramér-Rao lower bound (13.275) on the variance $V[t_k]$ from the inequality (13.270).

13.36 Show that the variance $V[u_{\sigma^2}^{(N)}]$ of Bessel's estimator (13.254) is given by (13.281).

13.37 Compute the fourth central moment (13.27) of Gauss's probability distribution $P_G(x; \mu, \sigma^2)$.

13.38 Show that when the real $N \times M$ matrix G has rank M, the matrices $P = G\,G^+$ and $P_\perp = 1 - P$ are projection operators that are mutually orthogonal $P(I - P) = (I - P)P = 0$.

13.39 Show that Kolmogorov's distance D_N is bounded as in (13.314).

13.40 Show that Kolmogorov's distance D_N is the greater of D_N^+ and D_N^-.

13.41 Derive the formulas (13.317) for D_N^+ and D_N^-.

13.42 Compute the exact limiting value D_∞ of the Kolmogorov distance between $P_G(x, 0, 1)$ and $P_S(x, 3, 1)$. Use the cumulative probabilities (13.321 & 13.194) to find the value of x that maximizes their difference. Using Maple or some other program, you should find $x = 0.6276952185$ and then $D_\infty = 0.0868552356$.

13.43 Show that when the data do come from the theoretical probability distribution (assumed to be continuous) but are in bins of width w, then the limiting value D_∞ of the Kolmogorov distance is given by (13.324).