Actin Fibers in HELA Cells

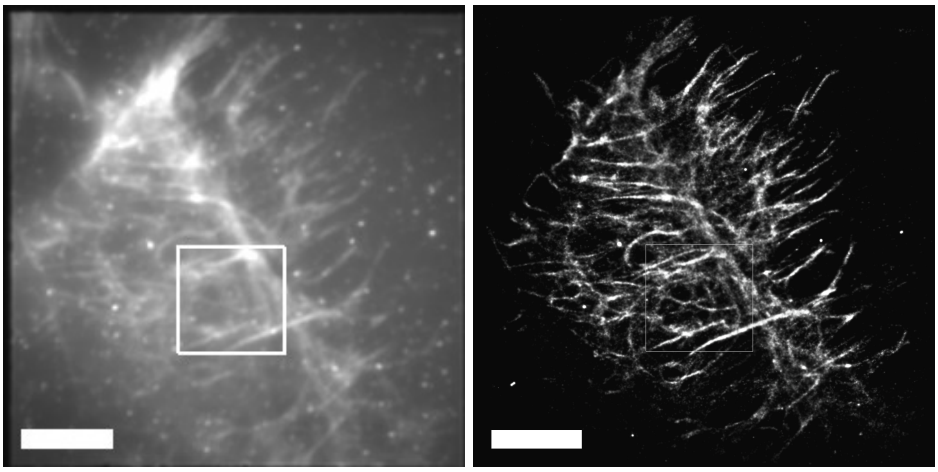

Figure 13.2 Conventional (left, fuzzy) and dSTORM (right, sharp) images of actin fibers in HELA cells. The actin is labeled with Alexa Fluor 647 Phalloidin. The white rectangles are 5 microns in length. Images courtesy of Fang Huang and Keith Lidke.

Example 13.7 (Single-Molecule Super-Resolution Microscopy) If the wavelength of visible light were a nanometer, microscopes would yield much sharper images. Each photon from a (single-molecule) fluorophore entering the lens of a microscope would follow ray optics and be focused within a tiny circle of about a nanometer on a detector. Instead, a photon arrives not at $\boldsymbol{x} = (x_1, x_2)$ but at $\boldsymbol{y_i} = (y_{1i}, y_{2i})$ with gaussian probability

$$P(\boldsymbol{y_i}) = \frac{1}{2\pi\sigma^2}\, e^{-(\boldsymbol{y_i}-\boldsymbol{x})^2/2\sigma^2} \tag{13.76}$$

where $\sigma \approx 150$ nm is about a quarter of a wavelength. What to do?

In the **centroid** method, one collects $N \approx 500$ points $\boldsymbol{y_i}$ and finds the point $\boldsymbol{x}$ that maximizes the joint probability of the N image points

$$P = \prod_{i=1}^{N} P(\boldsymbol{y_i}) = d^N \prod_{i=1}^{N} e^{-(\boldsymbol{y_i}-\boldsymbol{x})^2/(2\sigma^2)} = d^N \exp\left[-\sum_{i=1}^{N} (\boldsymbol{y_i}-\boldsymbol{x})^2/(2\sigma^2)\right] \tag{13.77}$$

where $d = 1/2\pi\sigma^2$ by solving for $k = 1$ and 2 the equations

$$\frac{\partial P}{\partial x_k} = 0 = P\,\frac{\partial}{\partial x_k}\left[-\sum_{i=1}^{N} (\boldsymbol{y_i}-\boldsymbol{x})^2/(2\sigma^2)\right] = \frac{P}{\sigma^2}\sum_{i=1}^{N} (y_{ik} - x_k)\,. \tag{13.78}$$

This **maximum-likelihood** estimate of the image point $\boldsymbol{x}$ is the average of