Black holes are not really black. Stephen Hawking (1942–) has shown that the intense gravitational field of a black hole of mass M radiates at temperature

$$T = \frac{\hbar\, c^3}{8\pi\, k\, G\, M} \tag{11.392}$$

in which $k = 8.617343 \times 10^{-5}\,\text{eV K}^{-1}$ is Boltzmann's constant, and $\hbar$ is Planck's constant $h = 6.6260693 \times 10^{-34}\,\text{J s}$ divided by 2π, $\hbar = h/(2\pi)$.

The black hole is entirely converted into radiation after a time

$$t = \frac{5120\,\pi\, G^2}{\hbar\, c^4}\, M^3 \tag{11.393}$$

proportional to the cube of its mass.

11.48 Cosmology

Astrophysical observations tell us that on the largest observable scales, space is **flat** or very nearly flat; that the visible universe contains at least 10^{90} particles; and that the cosmic microwave background radiation is isotropic to one part in 10^5 apart from a Doppler shift due the motion of the Earth. These and other observations suggest that potential energy expanded our universe by $\exp(60) = 10^{26}$ during an **era of inflation** that could have been as brief as 10^{-35} s. The potential energy that powered inflation became the radiation of the **Big Bang**. During and after inflation, the (negative) gravitational potential energy kept the total energy constant.

During the first three minutes, some of that radiation became hydrogen, helium, neutrinos, and **dark matter**. But for 50,000 years after the Big Bang, most of the energy of the visible universe was radiation. Because the momentum of a particle but not its mass falls with the expansion of the universe, this **era of radiation** gradually gave way to an **era of matter**. This transition happened when the temperature kT of the universe fell to 0.81 eV.

The era of matter lasted for 10.3 billion years. After 380,000 years, the universe had cooled to $kT = 0.26$ eV, and less than 1% of the atoms were ionized. Photons no longer scattered off a plasma of electrons and ions. The universe became **transparent**. The photons that last scattered just before this **initial transparency** became the **cosmic microwave background radiation** or **CMBR** that now surrounds us, red-shifted to 2.7255 ± 0.0006 K. Between 10 and 17 million years after the Big Bang, the temperature of the known universe fell from 373 to 273 K. If and where very early, very heavy stars had produced carbon, nitrogen, and oxygen, biochemistry would have started.

The era of matter has been followed by the current **era of dark energy** during which the energy of the visible universe is mostly a potential energy called **dark energy** (something like a **cosmological constant**). Dark energy has been accelerating the expansion of the universe for the past 3.5 billion years and may continue to do so forever.

It is now 13.817 ± 0.048 billion years after the Big Bang, and the dark-energy density is $\rho_{de} = 5.827 \times 10^{-30}\, c^2\, \text{g}\, \text{cm}^{-3}$ or 68.5 percent ($\pm\, 1.8\%$) of the **critical energy density** $\rho_c = 3H_0^2/8\pi G = 1.87837\, h^2 \times 10^{-29}\, c^2\, \text{g}\, \text{cm}^{-3}$ needed to make the universe flat. Here $H_0 = 100\, h\, \text{km}\, \text{s}^{-1}\, \text{Mpc}^{-1}$ is the **Hubble constant**, one parsec is 3.262 light-years, the Hubble time is $1/H_0 = 9.778\, h^{-1} \times 10^9$ years, and $h = 0.673 \pm 0.012$ is not to be confused with Planck's constant.

Matter makes up $31.5 \pm 1.8\,\%$ of the critical density, and baryons only $4.9 \pm 0.06\,\%$ of it. Baryons are $15\,\%$ of the total matter in the visible universe. The other $85\,\%$ does not interact with light and is called **dark matter**.

Einstein's equations (11.376) are second-order, non-linear partial differential equations for 10 unknown functions $g_{ij}(x)$ in terms of the energy-momentum tensor $T_{ij}(x)$ throughout the universe, which of course we don't know. The problem is not quite hopeless, however. The ability to choose arbitrary coordinates, the appeal to symmetry, and the choice of a reasonable form for T_{ij} all help.

Hubble showed us that the universe is expanding. The cosmic microwave background radiation looks the same in all spatial directions (apart from a Doppler shift due to the motion of the Earth relative to the local supercluster of galaxies). Observations of clusters of galaxies reveal a universe that is homogeneous on suitably large scales of distance. So it is plausible that the universe is **homogeneous** and **isotropic** in space, but not in time. One may show (Carroll, 2003) that for a universe of such symmtery, the line element in **comoving coordinates** is

$$ds^2 = -dt^2 + a^2 \left[\frac{dr^2}{1 - k\, r^2} + r^2 \left(d\theta^2 + \sin^2\theta\, d\phi^2 \right) \right]. \tag{11.394}$$

Whitney's embedding theorem tells us that any smooth four-dimensional manifold can be embedded in a flat space of eight dimensions with a suitable **signature**. We need only four or five dimensions to embed the space-time described by the line element (11.394). If the universe is closed, then the