Since $p = (\epsilon_w - \epsilon_\ell)/(\epsilon_w + \epsilon_\ell) > 0$, the principal image charge pq at $(0, 0, -h)$ has the same sign as the charge q and so contributes a positive term proportional to pq^2 to the energy. So a lipid slab repels a nearby charge in water no matter what the sign of the charge.

A cell membrane is a phospholipid bilayer. The lipids avoid water and form a 4-nm-thick layer that lies between two 0.5-nm layers of phosphate groups which are electric dipoles. These electric dipoles cause the cell membrane to weakly *attract* ions that are within 0.5 nm of the membrane. □

Example 9.3 (Cylindrical Wave Guides) An electromagnetic wave traveling in the z-direction down a cylindrical wave guide looks like

$$\boldsymbol{E}\, e^{in\phi}\, e^{i(kz-\omega t)} \quad \text{and} \quad \boldsymbol{B}\, e^{in\phi}\, e^{i(kz-\omega t)} \tag{9.52}$$

in which $\boldsymbol{E}$ and $\boldsymbol{B}$ depend upon ρ

$$\boldsymbol{E} = E_\rho \hat{\boldsymbol{\rho}} + E_\phi \hat{\boldsymbol{\phi}} + E_z \hat{\boldsymbol{z}} \quad \text{and} \quad \boldsymbol{B} = B_\rho \hat{\boldsymbol{\rho}} + B_\phi \hat{\boldsymbol{\phi}} + B_z \hat{\boldsymbol{z}} \tag{9.53}$$

in cylindrical coordinates (section 6.4). If the wave guide is an evacuated, perfectly conducting cylinder of radius r, then on the surface of the wave guide the parallel components of $\boldsymbol{E}$ and the normal component of $\boldsymbol{B}$ must vanish which leads to the boundary conditions

$$E_z(r) = 0, \quad E_\phi(r) = 0, \quad \text{and} \quad B_\rho(r) = 0. \tag{9.54}$$

Since the E and B fields have subscripts, we will use commas to denote derivatives as in $\partial(\rho E_\phi)/\partial\rho \equiv (\rho E_\phi)_{,\rho}$ and so forth. In this notation, the vacuum forms $\boldsymbol{\nabla} \times \boldsymbol{E} = -\dot{\boldsymbol{B}}$ and $\boldsymbol{\nabla} \times \boldsymbol{B} = \dot{\boldsymbol{E}}/c^2$ of the Faraday and Maxwell-Ampère laws give us (exercise 9.14) the field equations

$$\begin{aligned} E_{z,\phi}/\rho - ikE_\phi &= i\omega B_\rho & inB_z/\rho - ikB_\phi &= -i\omega E_\rho/c^2 \\ ikE_\rho - E_{z,\rho} &= i\omega B_\phi & ikB_\rho - B_{z,\rho} &= -i\omega E_\phi/c^2 \\ \left[(\rho E_\phi)_{,\rho} - inE_\rho\right]/\rho &= i\omega B_z & \left[(\rho B_\phi)_{,\rho} - inB_\rho\right]/\rho &= -i\omega E_z/c^2. \end{aligned} \tag{9.55}$$

Solving them for the ρ and ϕ components of $\boldsymbol{E}$ and $\boldsymbol{B}$ in terms of their z components (exercise 9.15), we find

$$\begin{aligned} E_\rho &= \frac{-ikE_{z,\rho} + n\omega B_z/\rho}{k^2 - \omega^2/c^2} & E_\phi &= \frac{nkE_z/\rho + i\omega B_{z,\rho}}{k^2 - \omega^2/c^2} \\ B_\rho &= \frac{-ikB_{z,\rho} - n\omega E_z/c^2\rho}{k^2 - \omega^2/c^2} & B_\phi &= \frac{nkB_z/\rho - i\omega E_{z,\rho}/c^2}{k^2 - \omega^2/c^2}. \end{aligned} \tag{9.56}$$

The fields E_z and B_z obey the wave equations (11.91, exercise 6.6)

$$-\triangle E_z = -\ddot{E}_z/c^2 = \omega^2 E_z/c^2 \quad \text{and} \quad -\triangle B_z = -\ddot{B}_z/c^2 = \omega^2 B_z/c^2. \tag{9.57}$$