(6.150) was *linear* in y. So we could set $P = \alpha(ry - s)$ and $Q = \alpha$. When P and Q are more complicated, integrating factors are harder to find or nonexistent.

Example 6.23 (Bodies Falling in Air) The downward speed v of a mass m in a gravitational field of constant acceleration g is described by the inhomogeneous first-order ODE $mv_t = mg - bv$ in which b represents air resistance. This equation is like (6.150) but with t instead of x as the independent variable, $r = b/m$, and $s = g$. Thus by (6.157), its solution is

$$v(t) = \frac{mg}{b} + \left(v(0) - \frac{mg}{b}\right) e^{-bt/m}. \tag{6.158}$$

The terminal speed mg/b is nearly 200 km/h for a falling man. A diving Peregrine falcon can exceed 320 km/h; so can a falling bullet. But mice can fall down mine shafts and run off unhurt, and insects and birds can fly.

If the falling bodies are microscopic, a statistical model is appropriate. The potential energy of a mass m at height h is $V = mgh$. The heights of particles at temperature T K follow Boltzmann's distribution (1.345)

$$P(h) = P(0) e^{-mgh/kT} \tag{6.159}$$

in which $k = 1.380\,6504 \times 10^{-23}$ J/K $= 8.617\,343 \times 10^{-5}$eV/K is his constant. The probability depends exponentially upon the mass m and drops by a factor of e with the **scale height** $S = kT/mg$, which can be a few kilometers for a small molecule. □

Example 6.24 (R-C Circuit) The **capacitance** C of a capacitor is the charge Q it holds (on each plate) divided by the applied voltage V, that is, $C = Q/V$. The current I through the capacitor is the time derivative of the charge $I = \dot{Q} = C\dot{V}$. The voltage across a **resistor** of R Ω (Ohms) through which a current I flows is $V = IR$ by Ohm's law. So if a time-dependent voltage $V(t)$ is applied to a capacitor in series with a resistor, then $V(t) = Q/C + IR$. The current I therefore obeys the first-order differential equation

$$\dot{I} + I/RC = \dot{V}/R \tag{6.160}$$

or (6.150) with $x \to t$, $y \to I$, $r \to 1/RC$, and $s \to \dot{V}/R$. Since r is a constant, the integrating factor $\alpha(x) \to \alpha(t)$ is

$$\alpha(t) = \alpha(t_0)\, e^{(t-t_0)/RC}. \tag{6.161}$$

Our general solution (6.157) of linear first-order ODEs gives us the expres-