5.27 Show that the Yukawa Green's function (5.151) reproduces the Yukawa potential (5.141) when $n = 3$. Use $K_{1/2}(x) = \sqrt{\pi/2x}\, e^{-x}$ (9.99).

5.28 Derive the two explicit formulas (5.188) and (5.189) for the square-root of a complex number.

5.29 What is $(-i)^i$? What is the most general value of this expression?

5.30 Use the indefinite integral (5.223) to derive the principal-part formula (5.224).

5.31 The Bessel function $J_n(x)$ is given by the integral

$$J_n(x) = \frac{1}{2\pi i} \oint_C e^{(x/2)(z-1/z)} \, \frac{dz}{z^{n+1}} \tag{5.352}$$

along a counter-clockwise contour about the origin. Find the generating function for these Bessel functions, that is, the function $G(x, z)$ whose Laurent series has the $J_n(x)$'s as coefficients

$$G(x, z) = \sum_{n=-\infty}^{\infty} J_n(x)\, z^n. \tag{5.353}$$

5.32 Show that the Heaviside function $\theta(y) = (y + |y|)/(2|y|)$ is given by the integral

$$\theta(y) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} e^{iyx} \, \frac{dx}{x - i\epsilon} \tag{5.354}$$

in which ϵ is an infinitesimal positive number.

5.33 Show that the integral of $\exp(ik)/k$ along the contour from $k = L$ to $k = L + iH$ and then to $k = -L + iH$ and then down to $k = -L$ vanishes in the double limit $L \to \infty$ and $H \to \infty$.

5.34 Use a ghost contour and a cut to evaluate the integral

$$I = \int_{-1}^{1} \frac{dx}{(x^2+1)\sqrt{1-x^2}} \tag{5.355}$$

by imitating example 5.30. Be careful when picking up the poles at $z = \pm i$. If necessary, use the explicit square-root formulas (5.188) and (5.189).

5.35 Redo the previous exercise (5.34) by defining the square roots so that the cuts run from $-\infty$ to -1 and from 1 to ∞. Take advantage of the evenness of the integrand and integrate on a contour that is slightly above the whole real axis. Then add a ghost contour around the upper half plane.

5.36 Show that if u is even and v is odd, then the Hilbert transforms (5.265) imply (5.267).