The product of the two eigenvalues is the constant $\mu_+\mu_- = \det \mathcal{M} = -m^2$ so as μ_- goes down, μ_+ must go up. Minkowski, Yanagida, and Gell-Mann, Ramond, and Slansky invented this "**seesaw**" mechanism as an explanation of why neutrinos have such small masses, less than 1 eV$/c^2$. If $mc^2 =$ 10 MeV, and $\mu_- c^2 \approx 0.01$ eV, which is a plausible light-neutrino mass, then the rest energy of the huge mass would be $Mc^2 = 10^7$ GeV. This huge mass would point at new physics, beyond the standard model. Yet the small masses of the neutrinos may be related to the weakness of their interactions. □

If we return to the orthogonal transformation (1.304) and multiply column ℓ of the matrix O and row ℓ of the matrix O^{T} by $\sqrt{|R^{(d)}_\ell|}$, then we arrive at the **congruency transformation** of Sylvester's theorem

$$R = C\,\hat{R}^{(d)} C^{\mathsf{T}} \tag{1.306}$$

in which the diagonal entries $\hat{R}^{(d)}_\ell$ are either ± 1 or 0 because the matrices $C_{k\ell} = \sqrt{|R^{(d)}_\ell|}\,O_{k\ell}$ and C^{T} have absorbed the factors $|R^{(d)}_\ell|$.

Example 1.40 (Equivalence Principle) If G is a real, symmetric 4×4 matrix then there's a real 4×4 matrix $D = C^{\mathsf{T}-1}$ such that

$$G_d = D^{\mathsf{T}} G D = \begin{pmatrix} g_1 & 0 & 0 & 0 \\ 0 & g_2 & 0 & 0 \\ 0 & 0 & g_3 & 0 \\ 0 & 0 & 0 & g_4 \end{pmatrix} \tag{1.307}$$

in which the diagonal entries g_i are ± 1 or 0. Thus there's a real 4×4 matrix D that casts the real nonsingular symmetric metric g_{ik} of spacetime at any given point into the diagonal metric $\eta_{j\ell}$ of flat spacetime by the congruence

$$g_d = D^{\mathsf{T}} g D = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \eta. \tag{1.308}$$

Usually one needs different Ds at different points. Since one can implement the congruence by changing coordinates, it follows that in any gravitational field, one may choose free-fall coordinates in which all physical laws take the same form as in special relativity without acceleration or gravitation at least over suitably small volumes of space-time (section 11.39). □