in which \( x \) stands for \( x_1, \ldots, x_k \) is a linear partial differential equation of order \( n = n_1 + \cdots + n_k \) in the \( k \) variables \( x_1, \ldots, x_k \). (A partial differential equation is a whole differential equation that has partial derivatives.)

Linear combinations of solutions of a linear homogeneous partial differential equation also are solutions of the equation. So if \( f_1 \) and \( f_2 \) are solutions of \( L f = 0 \), and \( a_1 \) and \( a_2 \) are constants, then \( f = a_1 f_1 + a_2 f_2 \) is a solution since \( L f = a_1 L f_1 + a_2 L f_2 = 0 \). Additivity of solutions is a property of all linear homogeneous differential equations, whether ordinary or partial.

The general solution \( f(x) = f(x_1, \ldots, x_k) \) of a linear homogeneous partial differential equation (6.15) is a sum \( f(x) = \sum_j a_j f_j(x) \) over a complete set of solutions \( f_j(x) \) of the equation with arbitrary coefficients \( a_j \).

A linear partial differential equation \( L f_i(x) = s(x) \) with a source term \( s(x) = s(x_1, \ldots, x_k) \) is an inhomogeneous linear partial differential equations because of the added source term.

Just as with ordinary differential equations, the difference \( f_{i_1} - f_{i_2} \) of two solutions of the inhomogeneous linear partial differential equation \( L f = s \) is a solution of the associated homogeneous equation \( L f = 0 \) (6.15)

\[
L \left[ f_{i_1}(x) - f_{i_2}(x) \right] = s(x) - s(x) = 0. \tag{6.16}
\]

So we can expand this difference in terms of the complete set of solutions \( f_j \) of the homogeneous linear partial differential equation \( L f = 0 \)

\[
f_{i_1}(x) - f_{i_2}(x) = \sum_j a_j f_j(x). \tag{6.17}
\]

Thus the general solution of the inhomogeneous linear partial differential equation \( L f = s \) is the sum of a particular solution \( f_{i_2} \) of \( L f = s \) and the general solution \( \sum_j a_j f_j \) of the associated homogeneous equation \( L f = 0 \)

\[
f_{i_1}(x) = f_{i_2}(x) + \sum_j a_j f_j(x). \tag{6.18}
\]

### 6.3 Notation for Derivatives

One often uses primes or dots to denote derivatives as in

\[
f' = \frac{df}{dx} \quad \text{or} \quad f'' = \frac{d^2f}{dx^2} \quad \text{and} \quad \dot{f} = \frac{df}{dt} \quad \text{or} \quad \ddot{f} = \frac{d^2f}{dt^2}.
\]

For higher or partial derivatives, one sometimes uses superscripts

\[
f^{(k)} = \frac{d^k f}{dx^k} \quad \text{and} \quad f^{(k,\ell)} = \frac{\partial^{k+\ell} f}{\partial x^k \partial y^\ell}. \tag{6.19}
\]