5.5 The function $f(z) = 1/z$ is analytic in the region $|z| > 0$. Compute the integral of $f(z)$ counter-clockwise along the unit circle $z = e^{i\theta}$ for $0 \le \theta \le 2\pi$. The contour lies entirely within the domain of analyticity of the function $f(z)$. Did you get zero? Why? If not, why not?

5.6 Let $P(z)$ be the polynomial

$$P(z) = (z - a_1)(z - a_2)(z - a_3) \tag{5.342}$$

with roots $a_1$, $a_2$, and $a_3$. Let $R$ be the maximum of the three moduli $|a_k|$. (a) If the three roots are all different, evaluate the integral

$$I = \oint_C \frac{dz}{P(z)} \tag{5.343}$$

along the counter-clockwise contour $z = 2Re^{i\theta}$ for $0 \le \theta \le 2\pi$. (b) Same exercise, but for $a_1 = a_2 \ne a_3$.

5.7 Compute the integral of the function $f(z) = e^{az}/(z^2 - 3z + 2)$ along the counterclockwise contour $\mathcal{C}_\square$ that follows the perimeter of a square of side 6 centered at the origin. That is, find

$$I = \oint_{\mathcal{C}_\square} \frac{e^{az}}{z^2 - 3z + 2}\, dz. \tag{5.344}$$

5.8 Use Cauchy's integral formula (5.36) and Rodrigues's expression (5.37) for Legendre's polynomial $P_n(x)$ to derive Schlaefli's formula (5.38).

5.9 Use Schlaefli's formula (5.38) for the Legendre polynomials and Cauchy's integral formula (5.32) to compute the value of $P_n(-1)$.

5.10 Evaluate the counter-clockwise integral around the unit circle $|z| = 1$

$$\oint \left(3\sinh^2 2z - 4\cosh^3 z\right) \frac{dz}{z}. \tag{5.345}$$

5.11 Evaluate the counter-clockwise integral around the circle $|z| = 2$

$$\oint \frac{z^3}{z^4 - 1}\, dz. \tag{5.346}$$

5.12 Evaluate the contour integral of the function $f(z) = \sin wz/(z - 5)^3$ along the curve $z = 6 + 4(\cos t + i \sin t)$ for $0 \le t \le 2\pi$.

5.13 Evaluate the contour integral of the function $f(z) = \sin wz/(z - 5)^3$ along the curve $z = -6 + 4(\cos t + i \sin t)$ for $0 \le t \le 2\pi$.

5.14 Is the function $f(x, y) = x^2 + iy^2$ analytic?

5.15 Is the function $f(x, y) = x^3 - 3xy^2 + 3ix^2y - iy^3$ analytic? Is the function $x^3 - 3xy^2$ harmonic? Does it have a minimum or a maximum? If so, what are they?