

PROBLEMS

3.1 *White-collar crime*

- a. You are a city inspector. You go undercover to a bakery and buy 30 loaves of bread marked 500 g. Back at the lab you weigh them and find their masses to be 493, 503, 486, 489, 501, 498, 507, 504, 493, 487, 495, 498, 494, 490, 494, 490, 497, 503, 498, 495, 503, 496, 492, 492, 495, 498, 490, 490, 497, and 482 g. You go back to the bakery and issue a warning. Why?
- b. Later you return to the bakery (this time, they know you). They sell you 30 more loaves of bread. You take them home, weigh them, and find their masses to be 504, 503, 503, 503, 501, 500, 500, 501, 505, 501, 501, 500, 508, 503, 503, 500, 503, 501, 500, 502, 502, 501, 503, 501, 501, 502, 503, 501, 502, and 500 g. You're satisfied, because all the loaves weigh at least 500 g. But your boss reads your report and tells you to go back and close the shop down. What did she notice that you missed?

3.2 *Relative concentration versus altitude*

Earth's atmosphere has roughly four molecules of nitrogen for every oxygen molecule at sea level; more precisely, the ratio is 78:21. Assuming a constant temperature at all altitudes (not really very accurate), what is the ratio at an altitude of 10 km? Explain why your result is qualitatively reasonable. [*Hint:* This problem concerns the number density of oxygen molecules as a function of height. The density is related in a simple way to the *probability* that a given oxygen molecule will be found at a particular height. You know how to calculate such probabilities.]

[*Remark:* Your result is also applicable to the sorting of macromolecules by sedimentation to equilibrium (see Problem 5.2).]

3.3 *Stop the dance*

A suspension of virus particles is flash-frozen and chilled to a temperature of nearly absolute zero. When the suspension is gently thawed, it is found to be still virulent. What conclusion do we draw about the nature of hereditary information?

3.4 *Photons*

Section 3.3.3 reviewed Muller's and Timoféeff's empirical results that the rate of induced mutations is proportional to the radiation exposure. Not only X-rays can induce mutations; even ultraviolet light will work (that's why you wear sunblock). To get a feeling for what is so shocking about these results, notice that they imply that there's no "safe," or threshold, dose level. The amount of damage (probability of damaging a gene) is directly proportional to the total radiation exposure. Extrapolating to the smallest possible dose, we must conclude that even a single photon of UV light has the ability to cause permanent genetic damage to a skin cell and its progeny. (Photons are the packets of light mentioned in Section 1.5.3.)

- a. Somebody tells you that a single ultraviolet photon carries an energy equivalent of about 10 electron volts (eV, see Appendix B). You propose a damage mechanism: A photon delivers that energy into a volume the size of the cell nucleus and heats it up; then the increased thermal motion knocks the chromosomes apart in some

- way. Is this a reasonable proposal? Why or why not? [Hint: Use Equation 1.2, and the definition of calorie found just below it, to calculate the temperature change.]
- b. Turning the result around, suppose that that photon's energy is delivered to a small volume L^3 and heats it up. We might suspect that if it heats up the region to boiling, this change could disrupt any genetic message contained in that volume. How small must L be for this amount of energy to heat that volume up to boiling (from 30°C to 100°C)? What could we conclude about the size of a gene if this proposal were correct?

3.5 T_2 Effusion

Figure 3.6 shows how to check the Boltzmann distribution of molecular speeds experimentally. Interpreting the data, however, requires some analysis.

Figure 3.17 shows a box full of gas with a tiny pinhole of area A , which slowly allows gas molecules to escape into a region of vacuum. You can assume that the gas molecules have a nearly equilibrium distribution inside the box; the disturbance caused by the pinhole is small. The gas molecules have a known mass m . The number density of gas in the box is c . The emerging gas molecules pass through a velocity selector, which admits only those with speed in a particular range, from u to $u + du$. A detector measures the total number of molecules arriving per unit time. It is located a distance d from the pinhole, on a line perpendicular to the hole, and its sensitive region is of area A_* .

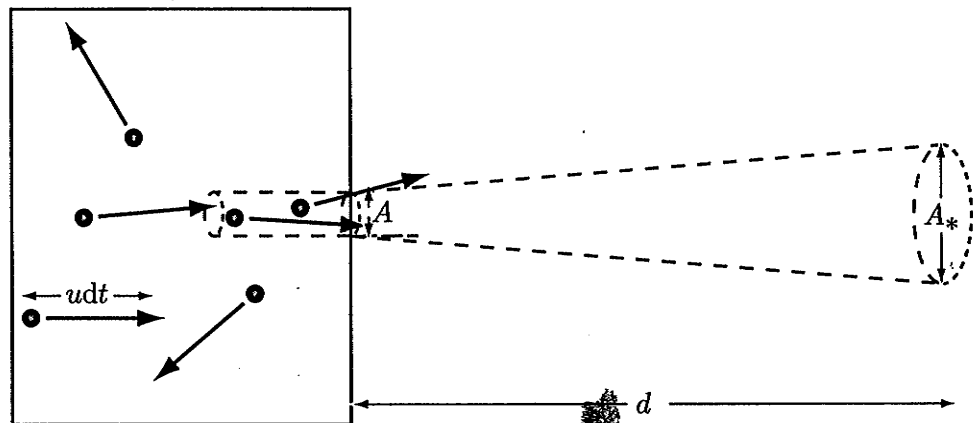


Figure 3.17: (Schematic.) Gas escaping from a pinhole of area A in the wall of a box. The number density of gas molecules is c inside the box and zero outside. A detector counts the rate at which molecules land on a sensitive region of area A_* . The six arrows in the box depict schematically six molecules, all with one particular speed $u = |\mathbf{v}|$. Of these, only two will emerge from the box in time dt , and of those two, only one will arrive at the detector a distance d away.

- a. The detector catches only those molecules emitted in certain directions. If we imagine a sphere of radius d centered on the pinhole, then the detector covers only a fraction α of the full sphere. Find α .

Thus, the fraction of all gas molecules whose \mathbf{v} makes them candidates for detection is $P(\mathbf{v})d^3\mathbf{v}$, where \mathbf{v} points perpendicular to the pinhole and has magnitude u and

$d^3v = 4\pi\alpha u^2 du$. Of these, the molecules that actually emerge from the box in time dt will be those initially located within a cylinder of cross-sectional area A and length udt (see the dashed cylinder in the figure).

b. Find the total number of gas molecules per unit time arriving at the detector.

c. Some authors report their results in terms of the transit time $\tau = d/u$ instead of u . Rephrase your answer to (b) in terms of τ and $d\tau$, not u and du .

[Note: In practice, the selected velocity range du depends on the width of the slots in Figure 3.6, and on the value of u selected. For thin slots, du is roughly a constant times u . Thus, the solid curve drawn in Figure 3.7 consists of your answer to (b), multiplied by another factor of u , and normalized; the experimental points reflect the detector response, similarly normalized.]