Energy of neutral inert particle

$$E = \sqrt{m^2c^4 + c^2p^2} \qquad E^2 = c^2p^2 + m^2c^4$$

$$E = \sqrt{m^2c^4 + \frac{c^2m^2v^2}{1-\frac{v^2}{c^2}}} \qquad p = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$= m\sqrt{1 + \frac{v^2}{1-v^2}} = m\sqrt{\frac{1-v^2+v^2}{1-v^2}}$$

$$E = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} \approx mc^2 + \frac{1}{2}mv^2$$

In grav. field

$$E = mc^2 + \frac{1}{2}mv^2 + mgz$$

ignore mc^2

$$E = \frac{1}{2}mv^2 + mgz$$

$$0 = \dot{E} = mv\dot{v} + mgv$$

$$= mv(\dot{v} + g) \qquad \dot{v} = -g$$

mech lots mud $z = 0$ $v = 0$

$E = 0$ where's energy?

In motion of molecules of mud.

Friction converts mechanical energy into heat.

Energy is conserved.

Heat is the kinetic energy of molecules or atoms.

B. Franklin showed electricity is a flow of charge, — of electrons.

People thought heat was a fluid too.

horse pulls rope ⇒ cannon is drilled. lots of heat generated.

drill bit does not get cold as barrel gets hot

also the amount of heat is proportional to the work done

heat (cal) = work × 0.24 cal/J

A cyclic process does as much work as it gets heat.

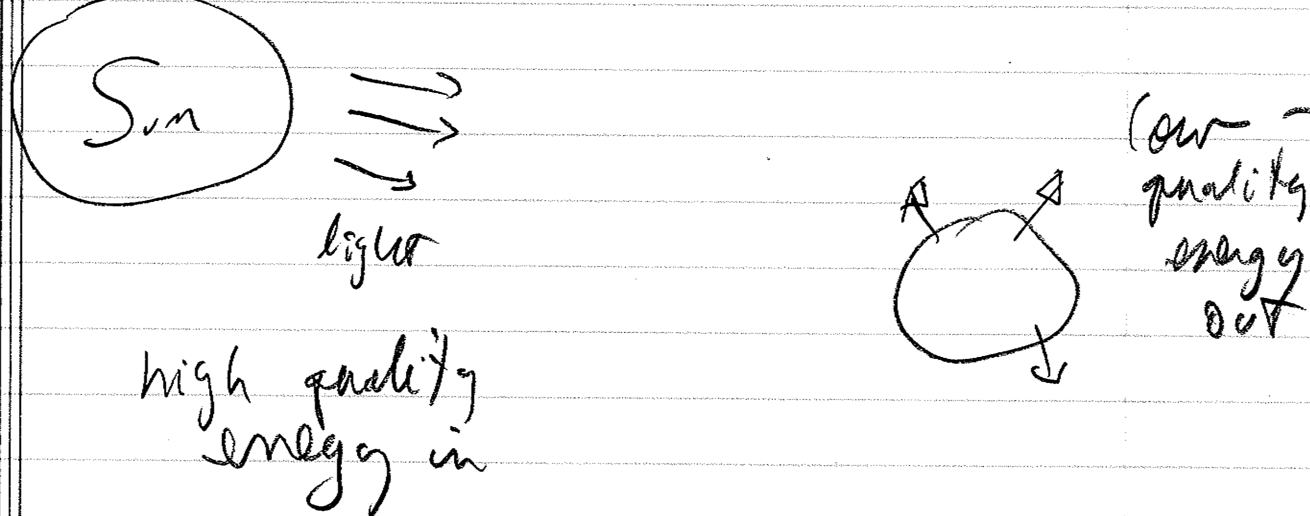
First law of thermodynamics is that energy is conserved.

Quality of energy is how useful it is. Can we make it do work? The available energy is the free energy

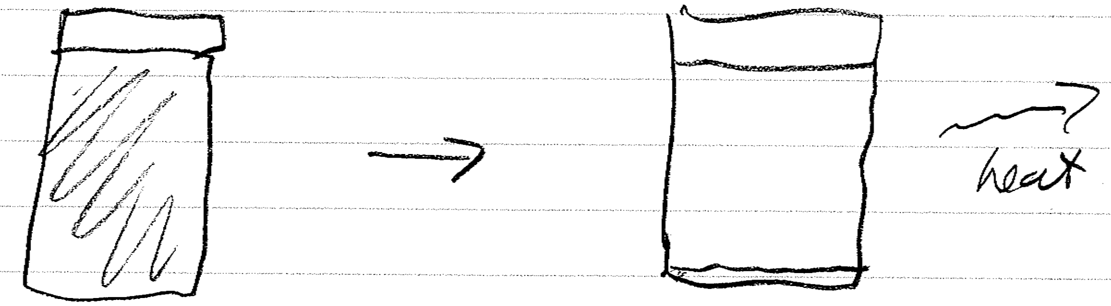
$$F = E - TS \qquad (1.4)$$

where T is the temperature and S is the entropy = disorder.

A system held at T can spontaneously drive a process if the free energy of the system falls.



Jar of water vapor.

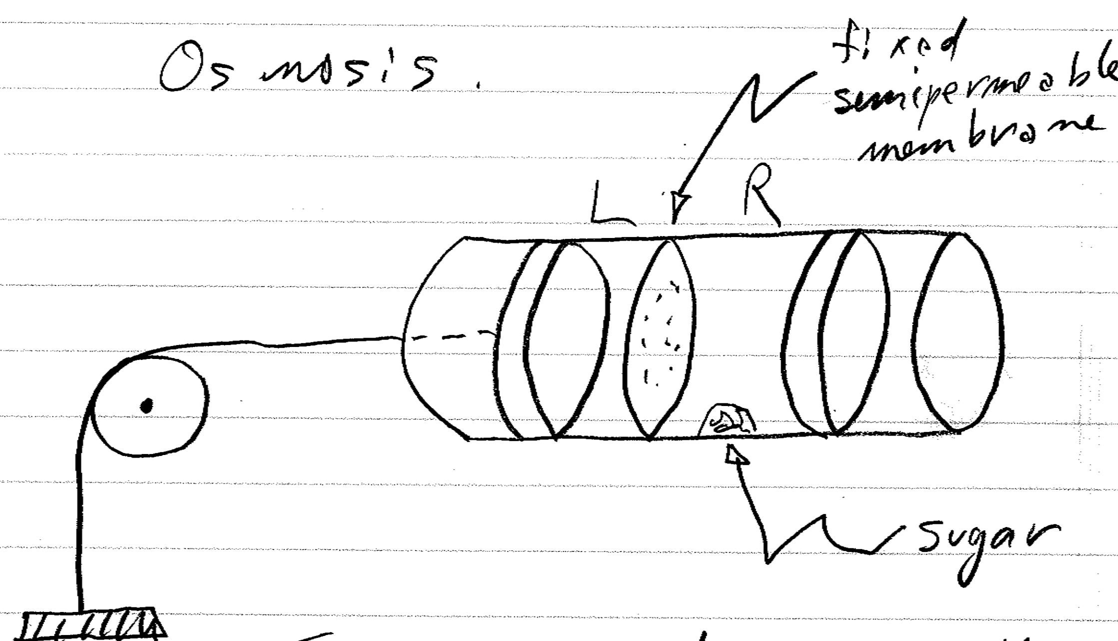


F, E & S decrease

So to make order, we must release waste heat.

Plants & animals consume order, not energy. They are free-energy transducers.

Osmosis.



Sugar can not cross the semipermeable membrane, but water can. There is much less sugar than water in the cylinder.

The sugar dissolves, and so, on the right side of the membrane, the sugar molecules bounce against the walls of the cylinder and the piston and the membrane along with the water molecules. So the pressure to the right of the membrane is higher. If the weight is not too heavy, this difference in pressure will raise it. This is osmosis.

If the weight is heavy, then it will pull the pistons to the left and leave only sugar on the right side of the membrane. This is reverse osmosis.

The maximum work done by the sugar is proportional to the number N of sugar molecules

$$W \approx N \times 4.1 \times 10^{-21}\,\mathrm{J} \times \gamma$$

where γ is a constant worked out in chapter 7. The number $4.1 \times 10^{-21}\,\mathrm{J}$ is

$$k\,T_r = 4.07 \times 10^{-21}\,\mathrm{J}$$

Since $F = E - TS$, the work done is the drop in the free energy F of the solution

$$W = T\Delta S.$$

So $\quad T\Delta S \approx N k_B T_L \times \gamma$

$$\Delta S = N k \gamma.$$

Here $\quad k = k_B = 1.38 \times 10^{-23}\ J/K.$

So

$$\frac{\Delta S}{k} = N\gamma$$

is proportional to the number of bits of information lost.

The ideas of pages 4-6 will be developed in the later chapters. Don't worry if some of this is mysterious now.

UNITS

1. Length has dimension $\mathcal{L}$ measured in meters m in SI units.

2. Mass has $\mathcal{M}$ — kilograms, kg.

3. Time has $\mathcal{T}$ — seconds, s.

4. Speed has $\mathcal{L}/\mathcal{T}$ — m/s $=$ m s^{-1}.

5. Acceleration $\mathcal{L}\mathcal{T}^{-2}$ — m s^{-2}.

6. Force has $\mathcal{M}\mathcal{L}\mathcal{T}^{-2}$ — kg m s^{-2}, newtons N.

7. Energy = Force $\times$ Length has dimensions of $\mathcal{M}\mathcal{L}^2\mathcal{T}^{-2}$ = kg m^2 s^{-2} = joules = J.

8. Electric charge has $\mathcal{Q}$ — coulombs = coul.

9. Current has $\mathcal{Q}\mathcal{T}^{-1}$ — coul s^{-1} = amperes = A.

10. Temperature is more subtle. Ice melts at 273 K, $T_r = 295$ K.

giga $= 10^9$, mega $= 10^6$, kilo $= 10^3$, centi $= 10^{-2}$

milli $= 10^{-3}$, micro $= 10^{-6}$, nano $= 10^{-9}$, pico $= 10^{-12}$.

These are written $G\ M\ k\ c\ m\ \mu\ n$ & p.

The Earth is about 4.6 Gy old.

A piconewton $\mathrm{pN} = 10^{-12}\ \mathrm{N}$. Forces in cells are of the order of pN.

Angles are dimensionless, pure numbers.

Dimensional quantities can be thought of as the product of a number and a unit.

Example: 2b. A pound of water is $\frac{1}{2.2}$ kg of water. But a pound of force is $\frac{g\,\mathrm{kg}}{2.2} = \frac{9.8}{2.2}\ \mathrm{m\,s^{-2}\,kg} = \frac{9.8}{2.2}\ \mathrm{N}$. So

$$\frac{1}{2.2}\ \mathrm{kg}\ {}^\circ\mathrm{F} = \frac{1}{2.2}\ \mathrm{kg} \times \frac{10^3\ \mathrm{g}}{\mathrm{kg}} \times {}^\circ\mathrm{F} \times \frac{0.56\ {}^\circ\mathrm{C}}{1\ {}^\circ\mathrm{F}}$$

$\boxed{\mathrm{cal} = \mathrm{g}\,{}^\circ\mathrm{C}}$

$$= \frac{10^3 \times 0.56}{2.2}\ \mathrm{cal} = 770\ \text{foot pounds}$$

$$= 770 \times \text{foot} \times \frac{12\ \mathrm{in}}{1\ \text{foot}} \times \frac{2.54\ \mathrm{cm}}{1\ \mathrm{in}} \times \frac{1\ \mathrm{m}}{10^2\ \mathrm{cm}} \times \frac{9.8\ \mathrm{N}}{2.2}$$

or

$560 \text{ cal} = 770 \times 12 \times 2.54 \times 10^{-2} \times 9.8 \text{ J}$

So

$$1 \text{J} = \frac{560}{770 \times 12 \times 2.54 \times 10^{-2} \times 9.8} \text{ cal}$$

$$= 0.2435 \text{ cal.}$$

Google says

$$1 \text{J} = 0.2390057736 \text{ cal.}$$

So Joule got very close.

$$\frac{.2435 - .239}{.239} = 0.019 \text{ or } 2\%.$$

Keeping track of dimensions can help students find errors.

Mathematics deals with pure numbers. So the x in exp(x) must be a number, a dimensionless quantity. Similarly, in sin(x), log(x), etc., the x must be a pure number.

$$[F] = \left[\frac{q_1 q_2}{4\pi\epsilon_0 r^2}\right] = \frac{m\,kg}{s^2}$$

$$= \frac{coul^2}{[\epsilon_0]\, m^2}$$

So

$$\frac{1}{[\epsilon_0]} = \frac{m^3}{coul^2}\frac{kg}{s^2} \quad \text{and}$$

$$[\epsilon_0] = \frac{coul^2\, s^2}{m^3\, kg} = \frac{Q^2 \mathcal{T}^2}{\mathcal{L}^3 \mathcal{M}}.$$

C has dimensions of farad.

$E = \frac{1}{2}\frac{q^2}{C}$ so

$$[C] = \frac{Q^2}{\mathcal{E}} = \frac{Q^2}{\mathcal{M}\mathcal{L}^2\mathcal{T}^{-2}} = \frac{Q^2\mathcal{T}^2}{\mathcal{M}\mathcal{L}^2}$$

so

$$1\ F = \text{coul}^2\ s^2\ kg^{-1}\ m^{-2}.$$

$$\frac{[C]}{\mathcal{L}} = [\epsilon_0] = \frac{F}{m} = F\,m^{-1}.$$

Dimensional Analysis

ζ (zeta) is the viscous-friction coefficient

$$[\zeta] = \frac{[\text{force}]}{[\text{speed}]} = \frac{\mathcal{M}\mathcal{L}\mathcal{T}^{-2}}{\mathcal{L}\mathcal{T}^{-1}} = \frac{\mathcal{M}}{\mathcal{T}} = \mathcal{M}\mathcal{T}^{-1}$$

diffusion constant D has dimension

$$[D] = \mathcal{L}^2/\mathcal{T} = \mathcal{L}^2\mathcal{T}^{-1} \quad \text{so}$$

$$[\zeta D] = \mathcal{M}\mathcal{L}^2\mathcal{T}^{-2} = [\text{Energy}].$$

$$\eta D = E_{\text{thermal}}$$

Thus Einstein estimated the kinetic energy of a room-temperature molecule.

Notation:

N often will denote a number

V volume m^3

q charge coul

$$\left[\frac{dN}{dt}\right] = T^{-1} = s^{-1}$$

$$[Q] = \left[\frac{dV}{dt}\right] = m^3 s^{-1}$$

$$I = \left[\frac{dq}{dt}\right] = \text{coul } s^{-1}.$$

$$[c] = \left[\frac{N}{V}\right] = m^{-3}$$ concentration or number density

$[\rho_m] = kg\, m^{-3}$ mass density

$[\rho_q] =$ coul m^{-3} charge density

Molecules are Tiny

$N_{mole} = 6.0 \times 10^{23}$ molecules per mole

So 12g of C^{12} is $6\ 10^{23}$ carbon atoms.

1773 Franklin noted that a teaspoon of oil $\approx 5\, cm^3$ can cover half an acre of water or $2000\, m^2$.

$$L \times 2000\, m^2 = 5\, cm^3 = 5\ 10^{-6}\ m^3$$

$$L = \frac{5}{2000}\ 10^{-6}\ m = 2.5\ 10^{-9}\ m$$

$= 2.5$ nm.

plausible for a molecule of oil (Olive oil).

$N_{mole} = 6 \ 10^{23}$ is a pure number.

How big is a molecule of water H_2O? A mole is 18 g. or 18 cm^3. So

$$\frac{18 \ cm^3}{V_{H_2O}} = 6 \ 10^{23}$$

$$V_{H_2O} = 3 \ 10^{-23} \ cm^3$$

$$= 30 \ 10^{-24} \ cm^3$$

$$= 30 \ \mathring{A}^3$$

$$1 \mathring{A} = 10^{-8} \ cm = 10^{-10} \ m = 0.1 \ nm$$

$$V_{H_2O} = 0.03 \ nm^3$$

$$L_{H_2O} \simeq 3 \mathring{A}$$

molar mass is mass of mole

isomer is a chemically distinct

arrangement of atoms of a molecule

If the molecule can easily flip

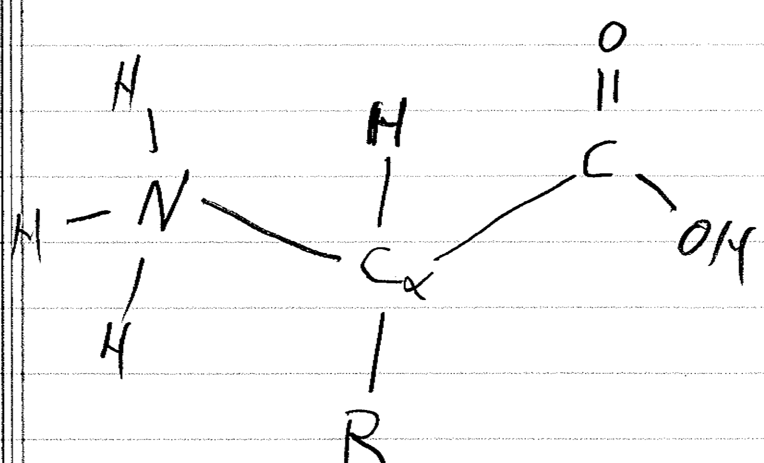
between isomers, it is "labile."

A chiral molecule looks

different in a mirror.

D & L sugars Pasteur 1857.

Most amino acids are chiral



The α-carbon has four different

bonds, unless R = H (glycine).

An isolated molecule has a well-defined energy when it's in its state of lowest energy.

H_2 = H — H in its ground state

is bound by ~ 4.75 eV. It takes

4.75 eV to separate the two H's.

$1 eV = 1.6 \times 10^{-19} coul \times 1 volt = 1.6 \times 10^{-19} J$

$2H_2 + O_2 \rightarrow 2H_2O$ + heat + light

is an exothermic reaction

$2H_2O$ + energy $\rightarrow 2H_2 + O_2$

can be done by electrolysis. Thus, hydrogen is unlikely to be an important fuel — unless fusion reactors become cheap.

Light comes in photons $E = h\nu$

h = Planck's constant.

$kT_r \cong \frac{1}{40}\,eV \approx 4.1\ pN\ nm$

Ideal Gas Law

$$pV = NkT = nRT$$

\# molecules

\# moles

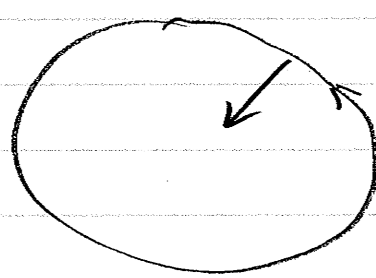
T is in K $\qquad 0°C = 273\ K.$

Cells use energy to make order.

Formulas

p = mass × speed $\qquad \vec{p} = m\vec{v}$

for speeds $<< c = 3 \times 10^8\ m\,s^{-1}$.



$$a = r\omega^2 = \frac{v^2}{r}$$

$$\vec{F} = \frac{d\vec{p}}{dt}$$

torque = $r \times F$

work = $\vec{F} \cdot \vec{L}$

$$p = \frac{F}{A}$$

$$E_k = \frac{1}{2} m v^2 \qquad v \ll c$$

spring $\quad f = kx \qquad E = \frac{1}{2} k x^2$

$$E_p = mgh$$

$$E_q = qV$$

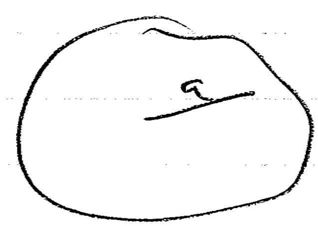
$$E = -\frac{dV}{dx}$$

$$\vec{F} = q\vec{E} \qquad \text{if } \vec{B} \text{ or } \vec{v} = 0$$

$$V(\vec{r}) = \frac{q}{4\pi\epsilon\underbrace{|\vec{r}|}_{\text{length of } \vec{r}}}$$

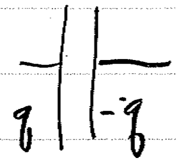
electrostatic energy $= \frac{1}{2} \sum_{ij} \frac{q_i q_j}{4\pi\epsilon|r_{ij}|}$

$$= \frac{q^2}{8\pi\epsilon a}$$



q

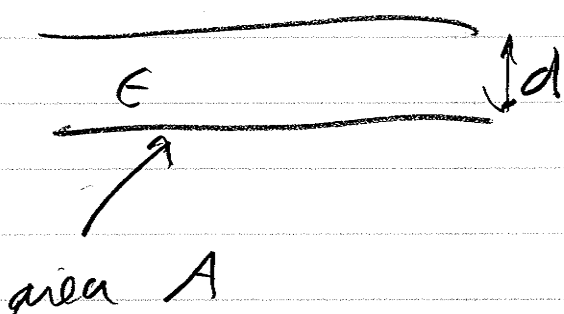
Ohm $V = IR$ $P = I^2 R$ Watts



$\Delta V = \frac{q}{C}$

$E_c = \frac{1}{2} \frac{q^2}{C}$

$C = \frac{A\epsilon}{d}$



$J = 0.24 \underbrace{g}_{\text{gram of water}} {}^\circ C$