PROBLEMS

1.1 *Dorm-room dynamics*

a. An air conditioner cools down your room, removing thermal energy. Yet it *consumes* electrical energy. Is there a contradiction with the First Law?

b. Could you design a high-tech device that sits in your window, continuously converting the unwanted thermal energy in your room to electricity, which you then sell to the power company? Explain.

1.2 *Thompson's experiment*

Long ago, people did not use SI units.

a. Benjamin Thompson actually said that his cannon-boring apparatus could bring 25.5 pounds of cold water to the boiling point in 2.5 hours. Supposing that "cold" water is at 20 °C, find the power input into the system by his horses, in watts. [*Hint:* A kilogram of water weighs 2.2 pounds. That is, Earth's gravity pulls it with a force of $1\ \mathrm{kg} \times g = 2.2$ pound.]

b. James Joule actually found that 1 pound of water increases in temperature by one degree Fahrenheit (or 0.56 °C) after he input 770 foot pounds of work. How close was he to the modern value of the mechanical equivalent of heat?

1.3 *Metabolism*

Metabolism is a generic term for all of the chemical reactions that break down and "burn" food, thereby releasing energy. Here are some data for metabolism and gas exchange in humans.

food	kcal/g	liters O_2/g	liters CO_2/g
carbohydrate	4.1	0.81	0.81
fat	9.3	1.96	1.39
protein	4.0	0.94	0.75
alcohol	7.1	1.46	0.97

The table gives the energy released, the oxygen consumed, and the carbon dioxide released upon metabolizing the given food, per gram of food.

a. Calculate the energy yield per liter of oxygen consumed for each food type and note that it is roughly constant. Thus, we can determine a person's metabolic rate simply by measuring her rate of oxygen consumption. In contrast, the CO_2/O_2 ratios are different for the different food groups; this circumstance allows us to estimate what is actually being used as the energy source, by comparing oxygen intake to carbon dioxide output.

b. An average adult at rest uses about 16 liters of O_2 per hour. The corresponding heat release is called the "basal metabolic rate" (BMR). Find it, in kcal/hour and in kcal/day.

c. What power output does this correspond to in watts?

d. Typically, the CO_2 output rate might be 13.4 liters per hour. What, if anything, can you say about the type of food materials being consumed?

e. During exercise, the metabolic rate increases. Someone performing hard labor for 10 hours a day might need about 3500 kcal of food per day. Suppose the person does mechanical work at a steady rate of 50 W over 10 hours. We can define the body's efficiency as the ratio of mechanical work done to excess energy intake (beyond the BMR calculated in (b)). Find this efficiency.

1.4 *Earth's temperature*

The Sun emits energy at a rate of about $3.9 \cdot 10^{26}$ W. At Earth, this sunshine gives an incident energy flux I_e of about $1.4\,\mathrm{kW\,m^{-2}}$. In this problem, you'll investigate whether any other planets in our solar system could support the sort of water-based life we find on Earth.

Consider a planet orbiting at distance d from the Sun (and let d_e be Earth's distance). The Sun's energy flux at distance d is $I = I_e(d_e/d)^2$, because energy flux decreases as the inverse square of distance. Call the planet's radius R, and suppose that it absorbs a fraction α of the incident sunlight, reflecting the rest back into space. The planet intercepts a disk of sunlight of area πR^2, so it absorbs a total power of $\pi R^2 \alpha I$. Earth's radius is about 6400 km.

The Sun has been shining for a long time, but Earth's temperature is roughly stable: The planet is in a steady state. For this to happen, *the absorbed solar energy must get reradiated back to space as fast as it arrives* (see Figure 1.2). Because the rate at which a body radiates heat depends on its temperature, we can find the expected mean temperature of the planet, using the formula

$$\text{radiated heat flux} = \alpha\sigma T^4.$$

In this formula, σ denotes the number $5.7 \cdot 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$ (the "Stefan–Boltzmann constant"). The formula gives the rate of energy loss per unit area of the radiating body (here, the Earth). You needn't understand the derivation of this formula but make sure you do understand how the units work.

a. Using this formula, work out the average temperature at the Earth's surface and compare your answer to the actual value of 289 K.

b. Using the formula, work out how far from the Sun a planet the size of Earth may be, as a multiple of d_e, and still have a mean temperature greater than freezing.

c. Using the formula, work out how close to the Sun a planet the size of Earth may be, as a multiple of d_e, and still have a mean temperature below boiling.

d. *Optional:* If you know the planets' orbital radii, which ones are then candidates for water-based life, using this rather oversimplified criterion?

1.5 *Franklin's estimate*

The estimate of Avogadro's number in Section 1.5.1 came out too small partly because we used the molar mass of water, not of oil. We can look up the molar mass and mass density of some sort of oil available in the eighteenth century in the *Handbook of chemistry and physics* (Lide, 2001). The *Handbook* tells us that the principal component of olive oil is oleic acid and gives the molar mass of oleic acid (also known as 9-octadecenoic acid or $CH_3(CH_2)_7CH{=}CH(CH_2)_7COOH$) as $282\,\mathrm{g\,mole^{-1}}$. We'll see in Chapter 2 that oils and other fats are triglycerides, made up of three fatty acid chains, so we estimate the molar mass of olive oil as a bit more than three times the value for oleic acid. The *Handbook* also gives the density of olive oil as $0.9\,\mathrm{g\,cm^{-3}}$.

Make an improved estimate of N_{mole} from these facts and Franklin's original observation.

1.6 *Atomic sizes, again*

In 1858, J. Waterston found a clever way to estimate molecular sizes from macroscopic properties of a liquid, by comparing its surface tension and heat of vaporization.

The surface tension of water, Σ, is the work per unit area needed to create more free surface. To define it, imagine breaking a brick in half. The two pieces have two new surfaces. Let Σ be the work needed to create these new surfaces, divided by their total area. The analogous quantity for liquid water is the surface tension.

The **heat of vaporization** of water, Q_{vap}, is the energy per unit volume we must add to liquid water (just below its boiling point) to convert it completely to steam (just above its boiling point). That is, the heat of vaporization is the energy needed to separate every molecule from every other one.

Picture a liquid as a cubic array with N molecules per centimeter in each of three directions. Each molecule has weak attractive forces to its six nearest neighbors. Suppose it takes energy ϵ to break one of these bonds. Then the complete vaporization of $1\,\mathrm{cm}^3$ of liquid requires that we break all the bonds. The corresponding energy cost is $Q_{\mathrm{vap}} \times (1\,\mathrm{cm}^3)$.

Next consider a molecule on the *surface* of the fluid. It has only five bonds—the nearest neighbor on the top is missing (suppose this is a fluid–vacuum interface). Draw a picture to help you visualize this situation. Thus, to create more surface area requires that we break some bonds. The energy needed to do that, divided by the new area created, is Σ.

a. For water, $Q_{\mathrm{vap}} = 2.3 \cdot 10^9\,\mathrm{J\,m^{-3}}$ and $\Sigma = 0.072\,\mathrm{J\,m^{-2}}$. Estimate N.

b. Assuming the molecules are closely packed, estimate the approximate molecule diameter.

c. What estimate for Avogadro's number do you get?

1.7 *Tour de France*

A bicycle rider in the Tour de France eats a lot. If his total daily food intake were burned, it would liberate about 8000 kcal of heat. Over the three or four weeks of the race, his weight change is negligible, less than 1%. Thus, his energy input and output must balance.

Let's first look at the mechanical work done by the racer. A bicycle is incredibly efficient. The energy lost to internal friction, even including the tires, is negligible. The expenditure against air drag is, however, significant, amounting to 10 MJ per day. Each day, the rider races for 6 hours.

a. Compare the 8000 kcal input to the 10 MJ of work done. Something's missing! Could the missing energy be accounted for by the altitude change in a hard day's racing?

Regardless of how you answered (a), next suppose that on one particular day of racing there's no net altitude change, so that we must look elsewhere to see where the missing energy went. We have so far neglected another part of the energy equation: the rider gives off *heat.* Some of this is radiated. Some goes to warm up the air he breathes in. But by far the greatest share goes somewhere else.

The rider *drinks a lot of water.* He doesn't need this water for his metabolism—he is actually creating water when he burns food. Instead, nearly all that liquid water leaves his body as water *vapor*. The thermal energy needed to vaporize water appeared in Problem 1.6.

b. How much water would the rider have to drink for the energy budget to balance? Is this reasonable?

Next let's go back to the 10 MJ of mechanical work done by the rider each day.

c. The wind drag for a situation like this is a backward force of magnitude $f = Bv^2$, were B is some constant. We measure B in a wind-tunnel to be $1.5\ \mathrm{kg\ m^{-1}}$. If we simplify by supposing a day's racing to be at constant speed, what is that speed? Is your answer reasonable?