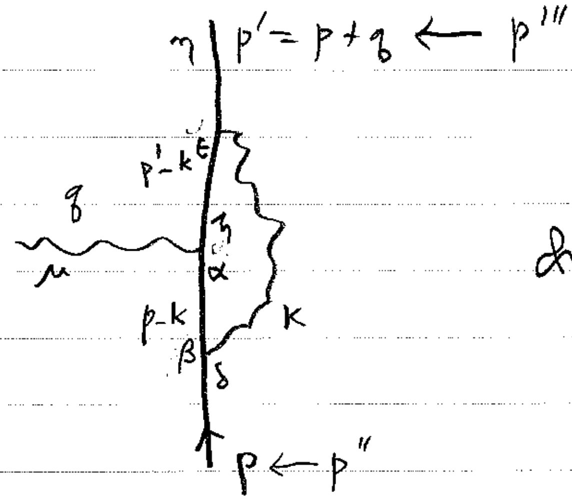

We leave out initial & final electron propagators.

$$\int (2\pi)^4 e\, \gamma^{\sigma}_{\eta\epsilon}\, \delta(p'''-k-p'+k) \quad \frac{-i}{(2\pi)^4} \frac{[-i(\not p'-\not k)+m]_{\epsilon\zeta}}{(p'-k)^2+m^2-i\epsilon}$$

$$(2\pi)^4 e\, \gamma^{\mu}_{\zeta\alpha}\, \delta(p-k-(p'-k)+q) \frac{-i}{(2\pi)^4} \frac{[-i(\not p-\not k)+m]_{\alpha\beta}}{(p-k)^2+m^2-i\epsilon} (2\pi)^4 e\, \gamma^{\rho}_{\beta\delta}\, \delta^4(p''-(p-k)+k)$$

$$\times \left(\frac{-i}{(2\pi)^4}\right) \frac{\eta_{\sigma\rho}}{k^2-i\epsilon} \quad d^4k\, d^4p''\, d^4p'''$$

$$\Gamma^{\mu}(p',p) = \frac{\text{diagram}}{(2\pi)^4 e}$$

$$= ie^2 \int \frac{d^4k}{(2\pi)^4}\, \gamma^{\sigma} \frac{[-i(\not p'-\not k)+m]}{(p'-k)^2+m^2-i\epsilon}\, \gamma^{\mu} \frac{[-i(\not p-\not k)+m]}{(p-k)^2+m^2-i\epsilon}\, \gamma_{\sigma} \frac{1}{k^2-i\epsilon}$$

Now we trick (11.3.2)

From (10.6):

$$\langle p'\sigma'|J^\mu(x)|p\sigma\rangle = e^{i(p-p')\cdot x}\langle p'\sigma'|J^\mu(0)|p\sigma\rangle$$

$$0 = \partial_\mu J^\mu(x) \quad \text{so}$$

$$0 = \langle p'\sigma'|\partial_\mu J^\mu(x)|p\sigma\rangle = i e^{i(p-p')\cdot x}(p-p')_\mu\langle p'\sigma'|J^\mu(0)|p\sigma\rangle$$

So

$$0 = (p-p')_\mu \langle p'\sigma'|J^\mu(0)|p\sigma\rangle,$$

$$\int d^3x\, \langle p'\sigma'|J^0(x)|p\sigma\rangle = \langle p'\sigma'|Q|p\sigma\rangle$$

$$= q\langle p'\sigma'|p\sigma\rangle$$

$$= \int d^3x\, e^{i(p-p')\cdot x}\langle p'\sigma'|J^0(0)|p\sigma\rangle$$

$$= (2\pi)^3\delta^3(p-p')\langle p'\sigma'|J^0(0)|p\sigma\rangle$$

So

$$\underbrace{\langle p\sigma'|J^0(0)|p\sigma\rangle}_{\text{same } p} = (2\pi)^{-3} q\,\delta_{\sigma'\sigma}$$

Spin $\frac{1}{2}$: Lorentz invariance implies

$$\langle p'\sigma' | J^\mu(0) | p\sigma \rangle = iq(2\pi)^{-3}\, \bar{u}(\vec{p}',\sigma')\, \Gamma^\mu(p',p)\, u(\vec{p},\sigma)$$

Γ^μ is a 4×4 matrix. We can expand it

in terms of 1 γ_ρ $[\gamma_\rho, \gamma_\sigma]$ $\gamma_5\gamma_\rho$ γ_5 etc.

Use $\bar{u}(p',\sigma')\,(i\not{p}' + m) = 0$
and $(i\not{p} + m)\, u(p\sigma) = 0$

to eliminate all terms but

$$\bar{u}(p'\sigma')\,\Gamma^\mu(p',p)\,u(p,\sigma)$$

$$= \bar{u}(p'\sigma')\Big[\gamma^\mu F(k^2) - \frac{i}{2m}(p+p')^\mu G(k^2)$$

$$+ \frac{(p-p')^\mu}{2m} H(k^2)\Big]\, u(p,\sigma). \qquad (10.6.10)$$

where $k = p' - p$ and
where $F, G, \& H$ are real since J^μ is
hermitian (which implies $\beta\, \Gamma^{\mu\dagger}(p'p)\,\beta = -\Gamma^\mu(p\,p')$).

The F & G terms satisfy $\bar{u}\,(p'-p)_\mu \Gamma^\mu u = 0$
but H doesn't. So $H \equiv 0$.

$$\bar{u}\Big\{(p'-p)_\mu \gamma^\mu = -i\big[\,(i\not{p}'+m) - (i\not{p}+m)\,\big]\Big\}u = 0$$

$$(\not{p}'-\not{p})(\not{p}'+\not{p}) = p'^2 - p^2 = -m^2 + m^2 = 0$$

As $p' \to p$

$$\langle p\sigma' | J^\mu(0) | p\sigma\rangle = iq(2\pi)^{-3}\,\bar{u}(p\sigma')\left[\gamma^\mu F(0) - \frac{i}{m} p^\mu G(0)\right] u(p\sigma)$$

$$\{\gamma^\mu, i\not{p} + m\} = 2m\gamma^\mu + 2ip^\mu \quad \text{so}$$

$$\bar{u}\gamma^\mu u = -i\frac{p^\mu}{m}\,\bar{u}(p,\sigma')u(p,\sigma)$$

$$\bar{u}(p\sigma')u(p\sigma) = \delta_{\sigma'\sigma}\frac{m}{p^0}$$

So

$$\langle p\sigma' | J^\mu(0) | p\sigma\rangle = \frac{q}{(2\pi)^3}\left(\frac{p^\mu}{p^0}\right)\delta_{\sigma'\sigma}\left[F(0) + G(0)\right]$$

Since

$$\langle p\sigma' | J^0(0) | p\sigma\rangle = \frac{q}{(2\pi)^3}\,\delta_{\sigma'\sigma}$$

we have

$$F(0) + G(0) = 1.$$

As shown on pp 456 & 457 the magnetic moment μ is

$$\mu = \frac{q\,F(0)}{2m}.$$

$$G(0) = \frac{-e^2 m^2}{4\pi^2} \int_0^1 dx \int_0^x dy \; \frac{x(1-x)}{m^2 x^2}$$

$$= -\frac{e^2}{4\pi^2} \int_0^1 dx \; (1-x) = -\frac{e^2}{4\pi^2}\left(1 - \frac{1}{2}\right)$$

$$= -\frac{e^2}{8\pi^2}$$

So

$$\mu = \frac{q\, F(0)}{2m} = \frac{q\,(1 - G(0))}{2m}$$

$$= \frac{e}{2m}\left(1 + \frac{e^2}{8\pi^2}\right)$$

$$= \frac{e}{2m}\left(1 + \frac{\alpha}{2\pi}\right) = \frac{e}{2m}\left(1 + \frac{1}{2\pi 137}\right)$$

$$= \frac{e}{2m}(1.001161) .$$ Schwinger 1948