LATTICE GAUGE THEORY

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Deep connections between:

- quantum mechanics
- statistical mechanics
- path integral $\longleftrightarrow$ partition function
- $d \ \longleftrightarrow \ d+1$
- $3d$ QCD equivalent to $4d$ classical stat. mech.

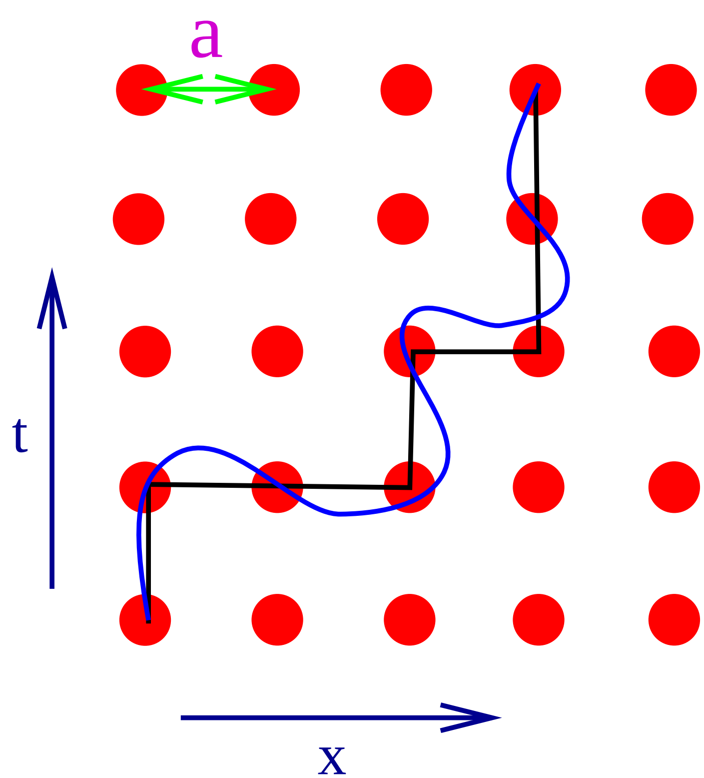
Computational project:

- Monte Carlo simulation
- Z_2 lattice gauge theory
- dramatic first order phase transition

Slides and sample program at: http://thy.phy.bnl.gov/~creutz/z2

Space-time Lattice

A mathematical trick
World lines $\longrightarrow$ discrete hops



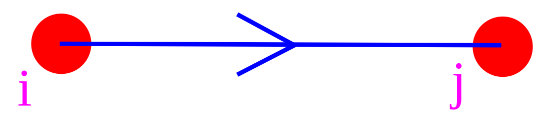
Lattice spacing a
$a \to 0$ for physics
$a = \text{cutoff} = \pi/\Lambda$

Wilson's formulation

Variables:

- Gauge fields generalize "phases"

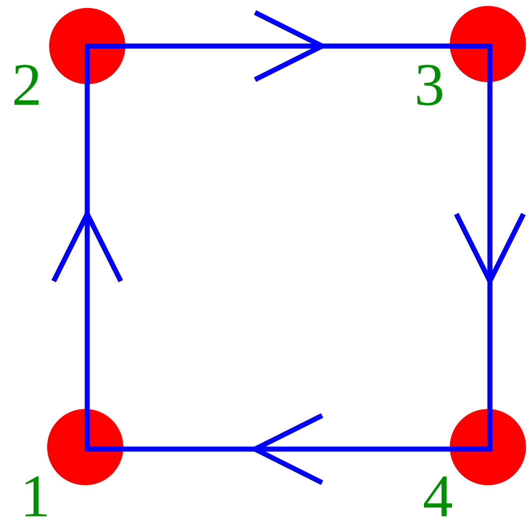
$$U_{i,j} \sim \exp(i \int_{x_i}^{x_j} A \cdot dx)$$



- On links connecting nearest neighbors
- U_{ij} = 3 by 3 unitary matrix $\in$ SU(3)
- 3 quarks in a proton

Dynamics:

- Sum over elementary squares, "plaquettes"



$$U_p = U_{1,2}U_{2,3}U_{3,4}U_{4,1}$$

- like a "curl"
- flux through corresponding plaquette.

$$S = \int d^4x \; (E^2 + B^2) \longrightarrow \sum_p \left(1 - \frac{1}{3}\mathrm{ReTr}U_p\right)$$

Quantum mechanics:

- via path integral
- sum over paths $\longrightarrow$ sum over phases

$$Z = \int (dU) e^{-\beta S}$$

- invariant group measure
- β defines the "bare" charge

$$\beta = \frac{6}{g_0^2}$$

- must renormalize as $a \to 0$
- "continuum limit"

Numerical Simulation

$$Z = \int dU e^{-\beta S}$$

10^4 lattice $\Rightarrow$

- $10^4 \times 4 \times 8 = 320,000$ dimensional integral
- 2 points/dimension $\Rightarrow$

$$2^{320,000} = 3.8 \times 10^{96,329} \qquad \text{terms}$$

- age of universe $\sim 10^{27}$ nanoseconds

Use statistical methods

- $Z \longleftrightarrow$ partition function
- $\frac{1}{\beta} \longleftrightarrow$ temperature

Find "typical" equilibrium configurations C

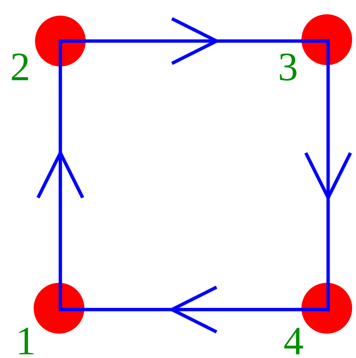
$$P(C) \sim e^{-\beta S(C)}$$

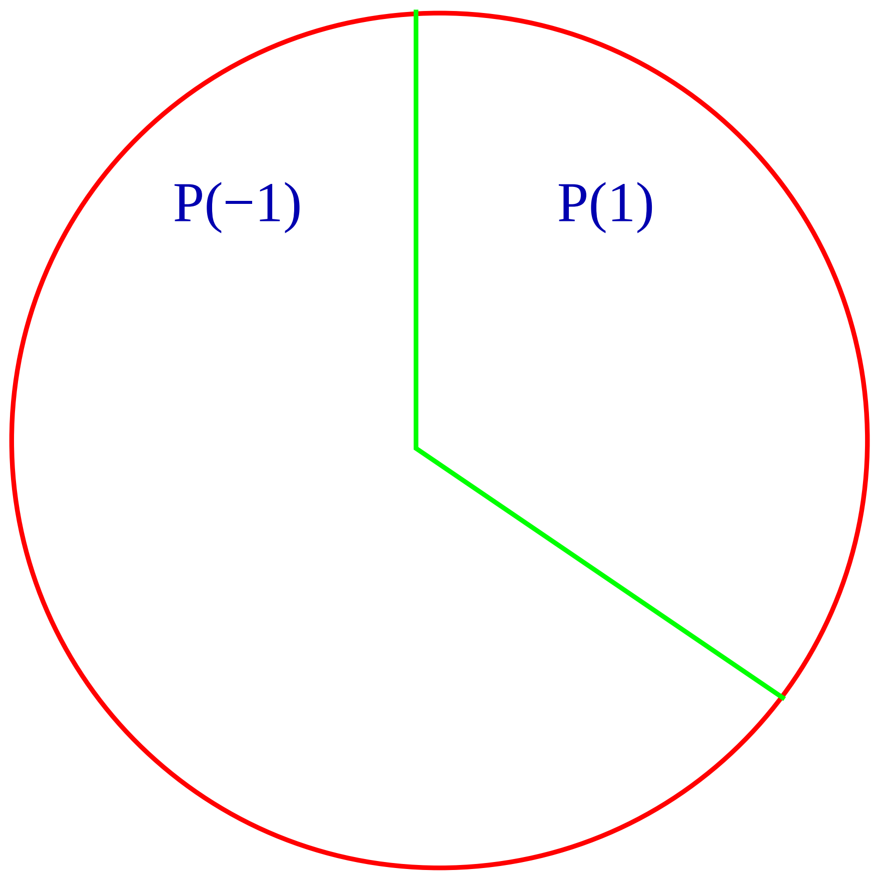
Use a Markov process

$$C \to C' \to \ldots$$

Z_2 example: (L. Jacobs, C. Rebbi, MC)

$$U = \pm 1 \qquad S = -\sum_p UUUU \qquad P(1) = \frac{e^{-\beta S(1)}}{e^{-\beta S(1)} + e^{-\beta S(-1)}}$$





Some Experiments

- M.C, L. Jacobs, C. Rebbi, Phys. Rev. Lett. 42, 1390-1393 (1979).
- M. C., L. Jacobs, C. Rebbi, Phys. Rev. D20, 1915-1922 (1979).
- Quarks, Gluons, and Lattices, M. Creutz (Cambridge, 1983).

Thermal cycle:
- start ordered, $\beta \sim 1.0$
- update while gradually reducing $\beta \sim 0.$
- reheat back to $\beta \sim 1.0$
- strong hysteresis

Coexisting phases:
- set $\beta = \beta_t = \frac{1}{2} \log(1 + \sqrt{2})$
- start ordered
- update 100 times
- start random
- update 100 times
- distinct final states

Mixed starts:

- order lattice
- randomize the first third
- run for several $\beta \sim \beta_t$
- the appropriate phase takes over
- determines β_t numerically

Vary the dimension:

- $d = 2$ no transition
- $d = 3$ second order, Ising
- $d = 4$ first order
- $d = 5$ first order?

Programming fun:

- let d be a parameter
- store U's as bits in words
- update in parallel with logical operations

Add spins to the sites:

- $S = \beta_G \sum U_p + \beta_S \sum S_i U_{ij} S_j$
- limiting cases: Z_2 gauge, Ising
- M.C. Phys. Rev. D21, 1006-1012 (1980)

Wilson loops:

- multiply U's around a closed loop
- expectation falls with size
- small beta: $\sim \exp(-\mathrm{area})$
- large beta: $\sim \exp(-\mathrm{perimeter})$

Clock models:

- $Z_= \{\pm 1\} \ \rightarrow \ Z_N = \{e^{2\pi i n/N} | n = 0, 1, ..., N-1\}$
- $S = \sum \mathrm{Re} U_p$
- two transitions for $N >= 5$
- $N \rightarrow \infty$ gives $U(1)$

$U(1) = \{e^{i\theta}\}$

- one transition
- electrodynamics
- confining vs. free charges

Non-Abelian theories

- U's matrices
- $SU(3)$: 3 by 3 unitary, $|U| = 1$

Add fermions ...

Go to lattice gauge conferences