Superconductivity

Electrons form Cooper pairs which are bosons and which therefore can condense giving rise to superconductivity when the temperature drops enough.

Landau and Ginzburg let $\phi(x)$ stand for the Cooper pairs. Here $x \equiv \vec{x}$. One calls $\phi(x)$ and $M(x)$ <u>order parameters.</u>

Now $\phi(x)$ is a 2- charged complex field, so $\partial_i \phi \to D_i \phi \equiv \partial_i \phi - i2eA_i \phi$.

The free energy is

$$\tilde{f} = \frac{1}{4} F_{ij}^2 + |D_i \phi|^2 + a|\phi|^2 + \frac{b}{2}|\phi|^4 + \cdots$$

which is $U(1)$ invariant under $\phi \to e^{i2e\Lambda} \phi$ and $A_i \to A_i + \partial_i \Lambda$.

Take $b > 0$ and $a(T) = a_1(T - T_c)$, $a_1 > 0$. So for $T < T_c$, $a(T) < 0$, and $|\phi|^2$ gets a mean value when $T < T_c$

$$0 = a + b|\phi|^2 \implies |\phi| = \sqrt{\frac{-a}{b}} \equiv v.$$

Then for $T < T_c$ and $|\phi| = v$

$$\mathcal{F} = \frac{1}{4} F_{ij}^2 + |\partial_i \phi - i 2e A_i \phi|^2 + \ldots$$

$$= \frac{1}{2} \vec{B}^2 + (2ev)^2 A_i^2$$

so photon has mass $\frac{1}{2} M^2 = (2ev)^2$

$M^2 = 8(ev)^2$.

Meissner Effect

Below T_c the superconductor excludes $\vec{B}$. Why? Because

$$B = \nabla \times A$$

So if $\vec{A} = B_0 r \hat{\phi}$, then

$$B_z = B_0.$$

But then

$$F = \frac{1}{2} B_0^2 + (2ev)^2 B_0^2 r^2$$

which grows quadratically with r.
So the superconductor excludes magnetic fields.

London's penetration depth

Suppose $A = A_0 e^{-x/l}$.

Then $B \sim \frac{A_0}{l} e^{-x/l}$ and

$$F \sim \frac{1}{2} \frac{A_0^2}{l^2} e^{-2x/l} + (2ev)^2 A_0^2 e^{-2x/l}$$

$$\int F\,dx = A_0^2 \left(\frac{1}{2l^2} + (2ev)^2 \right) \frac{l}{2} \propto \frac{1}{2l} + (2ev)^2 l$$

So $0 = -\frac{1}{2l^2} + (2ev)^2$ or $2l^2 = \frac{1}{(2ev)^2}$

so

$$l \sim \frac{1}{\sqrt{8}\,ev} = \frac{1}{2\sqrt{2}\,ev}$$

$$= \frac{1}{2\sqrt{2}} \frac{1}{e} \sqrt{\frac{b}{-a}}.$$

To find the coherence length, we note that

$$\langle \phi(x)\phi(0) \rangle \sim \int d^3k \frac{e^{ikx}}{k^2+a} \sim \frac{e^{-\sqrt{-a}\,|x|}}{4\pi\,|x|}$$

So the coherence length is $l_\phi \sim \frac{1}{\sqrt{-a}}$.

The ratio of the two lengths is

$$\frac{l_c}{l_\phi} \sim \frac{1}{e}\sqrt{\frac{b}{-a}}\,\sqrt{-a} \sim \frac{\sqrt{b}}{e} = \frac{\sqrt{-a}}{ev}$$

$$= \frac{m_\phi}{m_A}.$$

Julian Antolin in class pointed out that the Cooper pairs can't easily radiate photons because the photons in a superconductor below T_c are MASSIVE. Thus, the Cooper pairs superconduct.