Consider for some field $A(x)$

$$\langle 0| T\{A(x_1)\, A(x_2)\}|0\rangle$$

$$= \theta(x_1^0 - x_2^0)\ \langle 0| A(x_1)\, A(x_2)|0\rangle$$
$$+\ \theta(x_2^0 - x_1^0)\ \langle 0| A(x_2)\, A(x_1)|0\rangle$$

Focus on one of these terms and insert a complete set of states

$$\langle 0| A(x_1) \sum |n\rangle\langle n|\, A(x_2)|0\rangle .$$

Pick one state $|\vec{p}\rangle$ and zoom in on it

$$\int \langle 0| A(x_1)\, |p\rangle\langle p|\, A(x_2)|0\rangle\, d^3p\ \theta(x_1^0 - x_2^0)$$

Recall that

$$A(x_1) = e^{-iP\cdot x}\, A(0)\, e^{+iP\cdot x}$$

so that

$$\int d^3p\ \ \langle 0|A(x_1)|p\rangle\langle p|\, A(x_2)|0\rangle = \int d^3p\, |\langle 0|A(0)|p\rangle|^2 e^{-ip\cdot(x_2 - x_1)}$$

So

$$\langle 0|T\{A(x_1)\, A(x_2)\}|0\rangle = \theta(x_1^0 - x_2^0)\int d^3p\ |\langle 0|A(0)|p\rangle|^2 e^{-ip(x_2 - x_1)} + O.T.$$

where O.T. means "other terms."

Now $$\theta(t) = -\frac{1}{2\pi i}\int_{-\infty}^{\infty} d\omega\, \frac{e^{-i\omega t}}{\omega + i\epsilon}$$

So

$$\langle 0|T\{A(x_1)A(x_2)\}|0\rangle = -\frac{1}{2\pi i}\int_{-\infty}^{\infty} d\omega \frac{e^{-i\omega(x_1^0-x_2^0)}}{\omega+i\epsilon}$$

$$\times \int d^3p \; |\langle 0|A(0)|p\rangle|^2 e^{-ip(x_2-x_1)} \qquad + \text{O.T.}$$

We now take this into momentum space

$$G(q_1,q_2) = \int d^4x_1 d^4x_2 \, e^{-iq_1x_1 - iq_2x_2} \langle 0|T\{A(x_1)A(x_2)\}|0\rangle$$

$$= -\frac{1}{2\pi i}\int_{-\infty}^{\infty} d\omega \frac{e^{-i\omega(x_1^0-x_2^0)}}{\omega+i\epsilon} \int d^3p |\langle 0|A(0)|p\rangle|^2 e^{-ip(x_2-x_1)-iq_1x_1-iq_2x_2} d^4x_1 d^4x_2 + \text{O.T.}$$

$$= -\frac{1}{2\pi i}\int_{-\infty}^{\infty} d\omega \frac{e^{-ix_1^0(\omega+p^0-q_1^0)+ix_2^0(\omega+p^0+q_2^0)} dx_1^0 dx_2^0}{\omega+i\epsilon}\int d^3p |\langle 0|A(0)|p\rangle|^2 (2\pi)^6 \delta^3(-\vec{p}+\vec{q}_1)\,\delta^3(\vec{p}+\vec{q}_2)$$

$$= -\frac{1}{2\pi i}\int_{-\infty}^{\infty} d\omega \frac{1}{\omega+i\epsilon}\int d^3p |\langle 0|A(0)|p\rangle|^2 (2\pi)^6 \delta^3(-\vec{p}+\vec{q}_1)\,\delta^3(\vec{p}+\vec{q}_2)$$
$$(2\pi)^2 \delta(\omega+p^0-q_1^0)\,\delta(\omega+p^0+q_2^0)$$

$$= -\frac{1}{2\pi i}\frac{1}{-\sqrt{\vec{q}_1^{\,2}+m^2}+i\epsilon+q_1^0} |\langle 0|A(0)|\vec{q}_1\rangle|^2 (2\pi)^8 \delta^3(\vec{q}_2+\vec{q}_1)\,\delta(q_2^0+q_1^0)$$

$$= i(2\pi)^7 \delta^4(q_1+q_2) |\langle 0|A(0)|\vec{q}_2\rangle|^2 \frac{1}{-\sqrt{\vec{q}_1^{\,2}+m^2}+q_1^0+i\epsilon}$$

Set $q = q_1 = -q_2$. Then

$$G(q_1, q_2) = \frac{i(2\pi)^7 \delta^4(q_1+q_2) \, |\langle 0|A(0)|+q\rangle|^2}{-\sqrt{\vec{q}^{\,2}+m^2} + q^0 + i\epsilon}$$

$$= \frac{i(2\pi)^7 \delta^4(q_1+q_2) \, |\langle 0|A(0)|\vec{q}\rangle|^2}{\left(\sqrt{\vec{q}^{\,2}+m^2} - i\epsilon\right)^2 - q^{0\,2}} \left(-q^0 - \sqrt{\vec{q}^{\,2}+m^2} + i\epsilon\right).$$

Near the pole G looks like

$$G(q_1, q_2)\Big|_{q^2 \simeq -m^2} = i(2\pi)^7 \frac{\delta^4(q_1+q_2) \, |\langle 0|A(0)|\vec{q}\rangle|^2}{q^2 + m^2 - i\epsilon} \left(-2\sqrt{\vec{q}^{\,2}+m^2}\right)$$

So the Fourier transform has a pole at $q^2 = q_1^2 = (-q_2)^2 = -m^2$

$$G(q_1, q_2) = \int d^4x_1 d^4x_2 \, e^{-iq_1 \cdot x_1 - iq_2 x_2} \langle 0|T(A(x_1)A(x_2))|0\rangle$$

$$= i(2\pi)^7 \delta^4(q_1+q_2) \frac{|\langle 0|A(0)|\vec{q}\rangle|^2}{q^2+m^2-i\epsilon} \left(-2\sqrt{q^2+m^2}\right)$$

(near $q^2 = -m^2$)

if there exists a one-particle state $|q\rangle$ such that $\langle 0|A(0)|\vec{q}\rangle \neq 0$.