$$[P_\mu, A(x)] = i\partial_\mu A(x) = i\frac{\partial A(x)}{\partial x^\mu}$$

$$\langle p'|\,[P_\mu, T(A(x_1)A(x_2))]\,|p\rangle$$

$$= (p'_\mu - p_\mu)\langle p'|\,T(A(x_1)A(x_2))\,|p\rangle$$

$$= \langle p'|\,\theta(x_1^0 - x_2^0)[P_\mu, A(x_1)A(x_2)]$$
$$+ \theta(x_2^0 - x_1^0)[P_\mu, A(x_2)A(x_1)]\,|p\rangle$$

$$= \langle p'|\,\theta(x_1^0 - x_2^0)\left[\left(i\frac{\partial}{\partial x_1^\mu}A(x_1)\right)A(x_2) + A(x_1)\left(i\frac{\partial}{\partial x_2^\mu}A(x_2)\right)\right]$$
$$+ \theta(x_2^0 - x_1^0)\left[\left(i\frac{\partial}{\partial x_2^\mu}A(x_2)\right)A(x_1) + A(x_2)\left(i\frac{\partial}{\partial x_1^\mu}A(x_1)\right)\right]|p\rangle$$

$$= \langle p'|\,\theta(x_1^0 - x_2^0)\left(i\frac{\partial}{\partial x_1^\mu} + i\frac{\partial}{\partial x_2^\mu}\right)A(x_1)A(x_2)$$
$$+ \theta(x_2^0 - x_1^0)\,i\left(\frac{\partial}{\partial x_1^\mu} + \frac{\partial}{\partial x_2^\mu}\right)A(x_2)A(x_1)\,|p\rangle$$

So since

$$\left(\frac{\partial}{\partial x_1^\mu} + \frac{\partial}{\partial x_2^\mu}\right)\theta(x_1^0 - x_2^0) = 0, \quad \text{we get}$$

$$(p' - p)_\mu \langle p'|\,T(A(x_1)A(x_2))\,|p\rangle$$
$$= i\left(\frac{\partial}{\partial x_1^\mu} + \frac{\partial}{\partial x_2^\mu}\right)\langle p'|\,T(A(x_1)A(x_2))\,|p\rangle$$

So

$$\langle p'|\,T(A(x_1)A(x_2))\,|p\rangle = e^{i(p-p')\cdot(c_1 x_1 + c_2 x_2)}F(x_1 - x_2)$$

where $c_1 + c_2 = 1$.

So the Fourier transform of such a time-ordered product is

$$\int d^4x_1 d^4x_2\, e^{-ik_1x_1 - ik_2x_2} \langle p'|T(A(x_1)A(x_2))|p\rangle \equiv G(k_1,k_2)$$

$$= \int d^4x_1 d^4x_2\, e^{-ik_1x_1 - ik_2x_2 + i(p-p')(c_1x_1+c_2x_2)} F(x_1-x_2)$$

Let $y = x_1 - x_2$, so $x_1 = y + x_2$

$$G(k_1,k_2) = \int d^4y\, d^4x_2\, e^{-ik_1(y+x_2) - ik_2x_2 + i(p-p')(x_2+c_1y)} F(y)$$

$$= (2\pi)^4 \delta^4(p-p'-k_1-k_2) \int d^4y\, e^{iy((p-p')c_1 - k_1)} F(y)$$

In general,

$$\int d^4x_1 d^4x_2 \cdots \langle p'|T(A(x_1)\cdots A(x_n))|p\rangle e^{-ik_ix_i}$$

$$\propto \delta^4(p-p'-k_1-k_2\cdots-k_n).$$

This expresses conservation of momentum

$$[Q, A_a(x)] = -q_a A(x)$$

$$(q_b - q_a)\langle p'_b| T(A_1(x_1)A_2(x_2))|p_a\rangle$$

$$= \langle p'_b|[Q, T(A_1(x_1)A_2(x_2))]|p_a\rangle$$

$$= -(q_1 + q_2)\langle p'_b|T(A_1(x_1)A_2(x_2))|p_a\rangle$$

So charge is conserved:

$$(q_b - q_a + q_1 + q_2)\langle p'_b|T(A_1(x_1)A_2(x_2))|p_a\rangle = 0$$