HW 6

The interactions of some Dirac fermions $\psi_a(x)$ with some gauge fields $A_\mu^b(x)$ is similar to that of QED but with more indices:

$$e\bar{\psi}\gamma^\mu\psi A_\mu \to e\bar{\psi}_a\gamma^\mu t^b_{aa'}\psi_{a'}A^b_\mu .$$

Here the hermitian matrices t^b generate the Lie algebra

$$[t^a, t^b] = i f_{abc} t^c$$

of the gauge group.

With this brief background, compute the amplitude $i\mathcal{M}$ for $f_a + f_b \to f_c + f_d$ to lowest order in e. For simplicity, assume that the group is $SU(2)$, in which case

$$t^a = \frac{1}{2}\sigma^a$$

and the interaction is

$$\mathcal{H}(x) = \sum_{\alpha,\beta=1}^{2}\sum_{a=1}^{3} e\,\bar{\psi}_\alpha\gamma^\mu\frac{1}{2}\sigma^a_{\alpha\beta}\psi_\beta A^a_\mu .$$

The gauge-boson propagator is

$$\frac{-ig_{\mu\nu}\delta_{ab}}{q^2+i\epsilon}.$$