HW 1

1) First $d=2$

$$\left(\int_{-\infty}^{\infty} e^{-x^2}\,dx\right)^2 = \int_0^{\infty} e^{-r^2} 2\pi r\,dr = \pi \int_0^{\infty} e^{-u}\,du = \pi$$

So $$\int_{-\infty}^{\infty} e^{-x^2}\,dx = \sqrt{\pi}.$$

Now

$$\int_{-\infty}^{\infty} e^{-x^2}\,d^dx = \left(\int_{-\infty}^{\infty} e^{-x^2}\,dx\right)^d = \pi^{d/2}$$

$$= \int_0^{\infty} \Omega_d\, x^{d-1} e^{-x^2}\,dx$$

Let $t = x^2$, $dt = 2x\,dx$, $dx = dt/2x$,

$x^{d-1} = t^{(d-1)/2}$, so

$$\pi^{d/2} = \Omega_d \int_0^{\infty} t^{\frac{d-1}{2}} e^{-t} \frac{dt}{2t^{1/2}} = \Omega_d \int_0^{\infty} t^{\frac{d}{2}-1} e^{-t} \frac{dt}{2}$$

$$= \Omega_d\, \Gamma\left(\tfrac{d}{2}\right)/2$$

So $$\Omega_d = \frac{2\pi^{d/2}}{\Gamma\left(\frac{d}{2}\right)}.$$

HW 1

2) $\mathcal{L}_F \equiv -\frac{1}{4}(Z_3-1)\, F^{\mu\nu}F_{\mu\nu}$

To lowest order in $\mathcal{L}_F$

$$\langle q'| T e^{i\int \mathcal{L}_F d^4x} |q\rangle = -\frac{i}{4}(Z_3-1)\langle q'|\int F^{\mu\nu}F_{\mu\nu}\, d^4x\, |q\rangle .$$

The trick is to integrate by parts:

$$\int (\partial^\mu A^\nu - \partial^\nu A^\mu)(\partial_\mu A_\nu - \partial_\nu A_\mu)\, d^4x$$

$$= \int d^4x\, -A^\nu \partial^\mu(\partial_\mu A_\nu - \partial_\nu A_\mu) + A^\mu \partial^\nu(\partial_\mu A_\nu - \partial_\nu A_\mu)$$

$$= \int d^4x\, A^\nu(-2\Box)A_\nu + A^\nu(\partial^\mu \partial_\nu A_\mu) + A^\mu \partial^\nu \partial_\mu A_\nu$$

$$= \int d^4x\, A_\mu(-2\partial_\rho \partial^\rho \eta^{\mu\nu})A_\nu + 2A_\mu \partial^\mu \partial^\nu A_\nu$$

$$= \int d^4x\, A_\mu\, 2(-\eta^{\mu\nu}\partial_\rho \partial^\rho + \partial^\mu \partial^\nu)A_\nu$$

$$\langle q'| T e^{i\int \mathcal{L}_F d^4x} |q\rangle = \langle 0|a(q',s')(-i)(Z_3-1)$$

$$\times \int \frac{d^4x\, d^3p\, d^3k}{(2\pi)^3\sqrt{2p^0 2k^0}}\, e^{-ipx}\, \epsilon^{\mu *}(p,t)\, a^\dagger(p,t)\, (-\eta_{\mu\nu}\partial_\rho\partial^\rho + \partial_\mu\partial_\nu)$$

$$\times\, e^{ikx}\, \epsilon^\nu(k,u)\, a(k,u)\, a^\dagger(q,s)|0\rangle$$

$$= -i(Z_3-1)\int \frac{d^4x\, e^{-iq'x+iqx}}{(2\pi)^3\sqrt{2q'^0 2q^0}}\, \epsilon^{\mu *}(q',s')(-\eta_{\mu\nu}\partial_\rho\partial^\rho + \partial_\mu\partial_\nu)\, e^{iqx}\, \epsilon^\nu(q,s)$$

$$= \frac{-i(Z_3-1)}{(2\pi)^3 2\sqrt{q^0 q'^0}} \int d^4x \, e^{ix(q-q')} \epsilon^{\mu *}(q', s') (\eta_{\mu\nu} q^2 - q_\mu q_\nu) \epsilon^\nu(q, s)$$

$$= -i\pi \frac{(Z_3-1)}{q^0} \delta^4(q-q') \epsilon^{\mu *}(q, s') (\eta_{\mu\nu} q^2 - q_\mu q_\nu) \epsilon^\nu(q, s)$$

in which we identify

$$\Pi^{*\mu\nu}_{2F}(q) = -(Z_3-1)(q^2\eta^{\mu\nu} - q^\mu q^\nu).$$