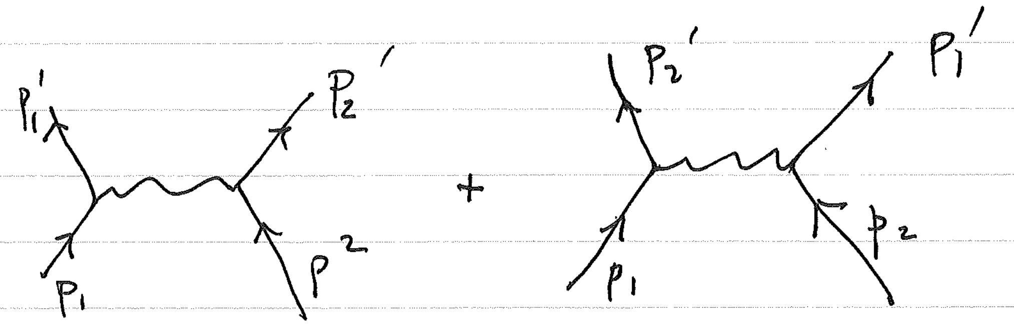
HW3

(1)



$$S = -\frac{g^2}{2}\int d^4x\, d^4y \langle 0|\, a(p_1')\ a(p_2')\ T\left(\psi^\dagger(x)\psi(x)\phi(x)\psi^\dagger(y)\psi(y)\phi(y)\right)$$
$$\times\, a^\dagger(p_1)\ a^\dagger(p_2)\ |0\rangle \sqrt{2E_p's}$$

Stop p_2 at y for a change

$$S = -g^3\int d^4x\, d^4y \langle 0| a_1' a_2'\ T\left(\psi^\dagger(x)\psi(x)\phi(x)\psi^\dagger(y)\frac{e^{-ip_2y}}{\sqrt{2E_{p_2}}}\ \phi(y)\right) a_1^\dagger\, |0\rangle\sqrt{2E_p's}$$

$$= -\frac{g^2\sqrt{2E's}}{\sqrt{2E_{p_1}2E_{p_2}}}\int d^4x\, d^4y \langle 0|a_1' a_2'\ T\left(\psi^\dagger(x)\phi(x)\psi^\dagger(y)\phi(y)\right)|0\rangle e^{-ip_2y - ip_1x}$$

$$= -g^2\int d^4x\, d^4y \langle 0|T(\phi(x)\phi(y))|0\rangle e^{-ip_2y-ip_1x}\left(e^{ip_1'x+ip_2'y} + e^{ip_2'x+ip_1'y}\right)$$

$$= -g^2\int d^4x\, d^4y\, \frac{d^4k}{(2\pi)^4}\, e^{-ik(x-y)}\frac{i}{k^2-m^2+i\epsilon}\, e^{-ip_2y-ip_1x}\left(e^{ip_1'x+ip_2'y} + e^{ip_2'x+ip_1'y}\right)$$

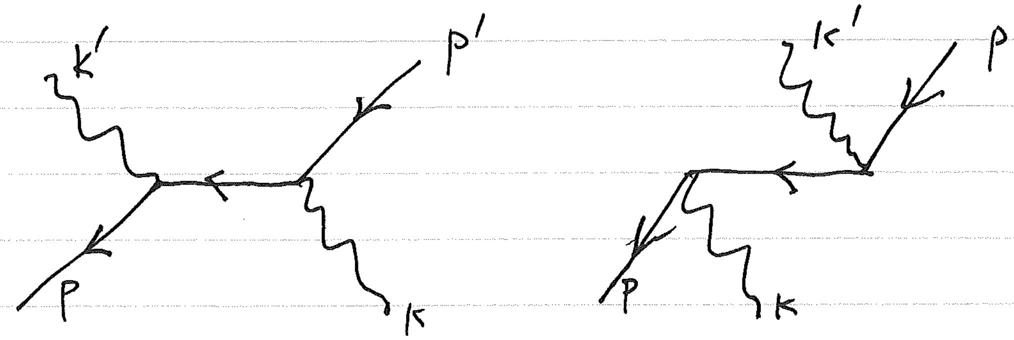
$$= -g^2\int \frac{d^4x\, d^4k\ i e^{-ip_1x-ikx}}{k^2-m^2+i\epsilon}\left(e^{ip_1'x}\delta(k+p_2'-p_2) + e^{ip_2'x}\delta(k+p_1'-p_2)\right)$$

$$S = -g^2 \int d^4x \left[\frac{i\, e^{-ip_1x - ix(p_2-p_2') + ip_1'x}}{(p_2-p_2')^2 - m^2 + i\epsilon} + \frac{i\, e^{-ip_1x - ix(p_2-p_1') + ip_2'x}}{(p_2-p_1')^2 - m^2 + i\epsilon} \right]$$

$$= -ig^2 (2\pi)^4 \delta^4(p_1+p_2-p_1'-p_2')$$

$$\times \left[\frac{1}{(p_2-p_2')^2 - m^2} + \frac{1}{(p_2-p_1')^2 - m^2} \right].$$

(2)



$$S = -\frac{g^2}{2} \sqrt{2E_p' s} \int d^4x\, d^4y \, \langle 0 | b' c' \, T(\psi^\dagger \psi \phi(x) \, \psi^\dagger \psi \phi(y)) \, b^\dagger c^\dagger | 0 \rangle$$

Stop $b^\dagger(p)$ at y

$$S = -g^2 \sqrt{2E_p' s} \int \langle 0 | b' c' \, T(\psi^\dagger(x) \psi(x) \phi(x) \psi(y) \phi(y)) \frac{e^{-ipy}}{\sqrt{2p^0}} c^\dagger | 0 \rangle d^4x\, d^4y$$

$$= \frac{-g^2 \sqrt{2E_p s}}{\sqrt{2p^0\, 2p'^0}} \int d^4x\, d^4y \, \langle 0 | c' \, T(\psi^\dagger(x) \phi(x) \psi(y) \phi(y)) c^\dagger | 0 \rangle e^{-ipy + ip'x}$$

$$= -g^2 \int d^4x\, d^4y \, \langle 0 | T(\psi^\dagger(x) \psi(y)) | 0 \rangle e^{-ipy + ip'x} \left(e^{-iky + ik'x} + e^{-ikx + ik'y} \right)$$

$$S = -g^2 \int d^4x\, d^4y\, \frac{d^4q}{(2\pi)^4} \frac{i e^{-iq(x-y) - ipy + ip'x}}{q^2 - m^2 + i\epsilon} \left(e^{-iky + ik'x} + e^{-ikx + ik'y} \right)$$

$$= -g^2 \int d^4x\, d^4q\, e^{-iqx + ip'x} \times \left[\frac{i e^{ik'x} \delta(q-p-k)}{q^2 - m^2 + i\epsilon} + \frac{i e^{-ikx} \delta(q-p+k')}{q^2 - m^2 + i\epsilon} \right]$$

$$= -g^2 \int d^4x\, e^{ixp'} \left[\frac{i e^{ik'x + ix(-p-k)}}{(p+k)^2 - m^2} + \frac{i e^{-ikx - ix(p-k')}}{(p-k')^2 - m^2} \right]$$

$$= -g^2 (2\pi)^4 \delta^{(4)}(p+k-p'-k') \left[\frac{i}{(p+k)^2 - m^2} + \frac{i}{(p-k')^2 - m^2} \right].$$