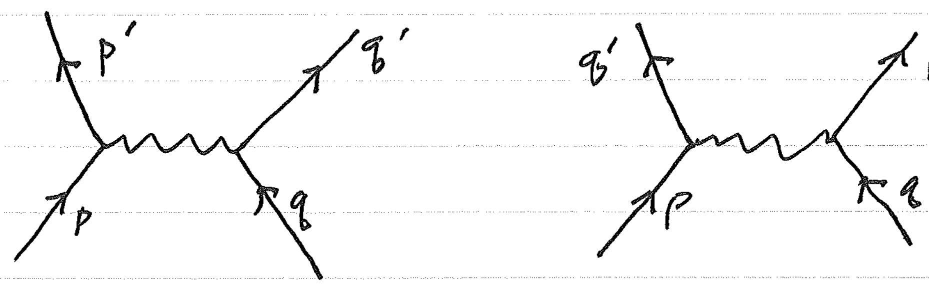
$e^- e^- \rightarrow e^- e^-$



The Feynman rules give

$$iM = \frac{\bar{u}(p',s')(-ie\gamma^\mu)u(p,s)(-i\eta_{\mu\nu})\ \bar{u}(q',t')(-ie\gamma^\nu)u(q,t)}{(p-p')^2}$$

$$\pm \frac{\bar{u}(q',t')(-ie\gamma^\mu)u(p,s)(-i\eta_{\mu\nu})\ \bar{u}(p',s')(-ie\gamma^\nu)u(q,t)}{(p-q')^2}$$

Which sign is right? Fermi statistics suggest a minus sign. Let's check carefully:

$$S = \sqrt{2E's}\ \langle 0|a(q',t')a(p',s')\ T\left[\exp\left(-i\int e\bar{\Psi}\gamma^\mu\Psi A_\mu\, d^4x\right)\right]a^\dagger(p,s)\,a^\dagger(q,t)|0\rangle$$

$d^4x_1 d^4x_2$

Stop $a^\dagger(q,t)$ at x_2 get 2:

$$= -e^2\sqrt{2E's}\ \langle 0|a(q',t')a(p',s')\int T\left(\bar{\Psi}^-(x_1)\gamma^\mu\Psi^+(x_1)A_\mu(x_1)\bar{\Psi}^-(x_2)\gamma^\nu\Psi^+(x_2)A_\nu(x_2)\right)$$

$$a^\dagger(p,s)\,a^\dagger(q,t)\ |0\rangle d^4x_1 d^4x_2$$

$$= -e^2\frac{\sqrt{2E's}}{\sqrt{2E_q 2E_p}}\langle 0|a(q',t')a(p',s')\int e^{-iqx_2-ipx_1}\,T\left(\bar{\Psi}^-(x_1)\gamma^\mu_{\ \wedge}{}^{u(p,s)} A_\mu(x_1)\bar{\Psi}^-(x_2)\gamma^\nu A_\nu(x_1)|0\rangle\right.$$

$$\times\ u(q,t)\ d^4x_1\, d^4x_2$$

There is a relative minus sign because if $\bar{\psi}^{-}(x_1)$ makes $a^{+}(q',t')$, then it must cross $a(p',s')$:

$$S = -e^2 \int e^{-iqx_2 - ipx_1} \langle 0|T(A_\mu(x_1) A_\nu(x_2))|0\rangle$$

$$\times \Big[\bar{u}(p',s')\gamma^\mu u(p,s)\, \bar{u}(q',t')\gamma^\nu u(q,t)\, e^{ip'x_1 + iq'x_2}$$

$$- \bar{u}(q',t')\gamma^\mu u(p,s)\, \bar{u}(p',s')\gamma^\nu u(q,t)\, e^{iq'x_1 + ip'x_2} \Big] d^4x_1 d^4x_2$$

$$= -e^2 \int e^{-iqx_2 - ipx_1} \frac{e^{-ik(x_1-x_2)}(-ig^{\mu\nu})}{k^2} \frac{d^4k}{(2\pi)^4}$$

$$\times \Big[\bar{u}(p's')\gamma^\mu u(ps)\bar{u}(q't')\gamma^\nu u(qt) e^{ip'x_1 + iq'x_2}$$

$$- \bar{u}(q't')\gamma^\mu u(ps)\, \bar{u}(p's')\gamma^\nu u(qt) e^{iq'x_1 + ip'x_2} \Big] d^4x_1 d^4x_2$$

$$= ie^2 \iint \Big[\delta^4(k - q + q')\, \bar{u}(p's')\gamma^\mu u(ps)\, \bar{u}(q't')\gamma_\mu u(qt) e^{ip'x_1}$$

$$- \delta^4(k - q + p')\, \bar{u}(q't')\gamma^\mu u(ps)\, u(p's')\gamma_\mu u(q,t) e^{iq'x_1} \Big]$$

$$\times \frac{e^{-ipx_1 - ikx_1}}{k^2} d^4k\, d^4x_1$$

$$= ie^2 \int \Big[\frac{\bar{u}(p's')\gamma^\mu u(p,s)\, \bar{u}'\gamma_\mu u}{(q-q')^2} e^{ip'x_1 - ipx_1 - i(q-q')x_1}$$

$$- \frac{\bar{u}(q't')\gamma^\mu u(ps)\, u'\gamma_\mu u}{(q-p')^2} e^{iq'x_1 - ipx_1 - i(q-p')x_1} \Big] d^4x_1$$

$$S = i e^2 (2\pi)^4 \delta^4(p'+q'-p-q) \times \left[\frac{\bar{u}(p's')\gamma^\mu u(ps)\, \bar{u}(q't')\gamma_\mu u(qt)}{(q-q')^2} - \frac{\bar{u}(q't')\gamma^\mu u(ps)\, \bar{u}(p's')\gamma_\mu u(qt)}{(q-p')^2} \right]$$

$$= i \mathcal{M} (2\pi)^4 \delta^4(p'+q'-p-q) \quad \text{with}$$

$$i\mathcal{M} = i e^2 \left[\frac{\bar{u}(p's')\gamma^\mu u(ps)\, \bar{u}(q't')\gamma_\mu u(qt)}{(p-p')^2} - \frac{\bar{u}(q't')\gamma^\mu u(ps)\, \bar{u}(p's')\gamma_\mu u(qt)}{(p-q')^2} \right].$$

Now $e\,\mu^- \to e\,\mu^-$ is simpler; it has only one diagram:

$$S = i (2\pi)^4 \delta^4(p'+q'-p-q) \frac{\bar{u}(p',s')\gamma^\mu u(p,s)\, \bar{u}(q',t')\gamma_\mu u(q,t)}{(p-p')^2}$$

is the amplitude for