Yukawa's Example

Consider the amplitude

$$G(q_1 \cdots q_4) = \int d^4x_1 \cdots d^4x_4 \, e^{-i q_i x_i} \langle 0 | T(\psi^\dagger(x_1)\psi(x_2)\psi^\dagger(x_3)\psi(x_4)) | 0 \rangle$$

Focus on

$$G(\quad) = \int d^4x_1 \cdots d^4x_4 \, e^{-i q_i x_i} \langle 0 | T(\psi^\dagger(x_1)\psi(x_2)) | p \rangle \langle p |$$

$$T(\psi^\dagger(x_3)\psi(x_4)) | 0 \rangle + O\,T$$

in which $|p\rangle$ is a π-meson of momentum $\vec{p}$. Then this amplitude will have a pole in

$$q = q_1 + q_2 = -q_3 - q_4 \quad \text{at} \quad q^2 = -m_\pi^2 .$$

$m_\pi \approx 140 \text{ MeV}/c^2$.

$$G(\quad) \propto \frac{\delta^4(q_1+q_2+q_3+q_4)}{(q_1+q_2)^2 + m_\pi^2 - i\epsilon}$$

The energy transfer $q_1^0 + q_2^0$ for $|\vec{q}_1| \ll m_\pi$ and $|\vec{q}_2| \ll m_\pi$ is of the order of $(\vec{q}_1^{\,2} - \vec{q}_2^{\,2})/2m_\pi$ and so

$$G(q_1 \cdots q_4) \sim \frac{\delta^4}{(\vec{q}_1 + \vec{q}_2)^2 + m_\pi^2}$$ but since

q_1 is outgoing and q_2 incoming

$$G \sim \frac{g}{(\overrightarrow{p_1 - p_2})^2 + m_\pi^2}$$

where the $\vec{p}$'s are the ordinary 3-momenta of the quarks.

Now $(\overrightarrow{p_1 - p_2})^2 + m_\pi^2$ does not vanish, but because m_π is the lightest hadron at ~ 140 MeV, G is affected by the pole even though it's in the unphysical region.

A local potential $e^{-m_\pi r}/4\pi r$ was suggested by Yukawa. In the Born approximation, it gives

$$\int d^3x_1 d^3x_2 d^3x_1' d^3x_2' \, e^{-ix_1 \cdot p_1 - i x_2 p_2 + i x_1' p_1' + i x_2' p_2'}$$

$$\times \frac{e^{-m_\pi |\overrightarrow{x_1 - x_2}|}}{4\pi |x_1 - x_2|} \delta^3(x_1 - x_1') \, \delta^3(x_2 - x_2')$$

$$= -(2\pi)^3 \delta^3(p_1 + p_2 - p_1' - p_2') \frac{1}{(\overrightarrow{p_1 - p_1'})^2 + m_\pi^2} .$$

due to the pion <u>particle</u>.