

Recall in PS notation

$$|\vec{p}^{\prime}\rangle = \sqrt{2\varepsilon_p} a_p^\dagger |0\rangle = \sqrt{2p^0} a_p^\dagger |0\rangle$$

and

$$\langle \hat{p}' | \vec{q}' \rangle = 2p^0 (2\pi)^3 \delta^{(3)}(\vec{p} - \vec{q}).$$

The S-matrix for the scattering of $|p_1 p_2\rangle$ to $|p'_1 p'_2\rangle$ is

$$\begin{aligned} S &= \langle p'_1 p'_2 | U(\infty, -\infty) | p_1 p_2 \rangle \\ &= \langle p'_1 p'_2 | T \left[\exp(-i \int_{-\infty}^{\infty} V(t) dt) \right] | p_1 p_2 \rangle \end{aligned}$$

where if $H = H_0 + g \psi^\dagger \psi \phi$

$$\int_{-\infty}^{\infty} V(t) dt = g \int \psi^\dagger(x) \psi(x) \phi(x) dx$$

in which the 3 fields have their free-field time dependence. T stands for time-ordered product.

Here ψ is a complex field, but ϕ is real:

$$\psi(x) = \psi^{(+)}(x) + \psi^{(-)}(x) = \frac{\int d^3 p}{(2\pi)^3 \sqrt{2p^0}} a_p e^{-ipx} + \frac{\int d^3 p}{(2\pi)^3 \sqrt{2p^0}} b_p^\dagger e^{ipx}$$

$$\phi(x) = \phi^{(+)}(x) + \phi^{(-)}(x) = \phi^{(+)}(x) + [\phi^{(+)}(x)]^\dagger, \quad \phi(x) = \int \frac{d^3 p}{(2\pi)^3 \sqrt{2p^0}} c_p e^{-ipx}$$

all in PS notation. All fields have spin zero.

The initial state is

$$|p_1 p_2\rangle = \sqrt{2p_1^0 2p_2^0} a_{p_1}^+ b_{p_2}^+ |0\rangle \text{ and the final one is}$$

$$|p'_1 p'_2\rangle = \sqrt{2p'_1 2p'_2} a_{p'_1}^+ b_{p'_2}^+ |0\rangle$$

in which all four momenta are different.

So

$$S = \langle p'_1 p'_2 | -i \int g \psi^+ \psi \phi d^4x - \frac{1}{2} g^2 \int T(\psi^+ \psi \phi(x_1) \psi^+ \psi \phi(x_2)) d^4x_1 d^4x_2 | p_1 p_2 \rangle$$

$$= -\frac{1}{2} g^2 \langle p'_1 p'_2 | \int T(\psi^+(x_1) \psi(x_1) \phi(x_1) \psi^+(x_2) \psi(x_2) \phi(x_2)) | p_1 p_2 \rangle$$

This term consists of several processes: Either $\psi^{(+)}(x_1)$ or $\psi^{(+)}(x_2)$ must cancel $a_{p_1}^+$. Either $\psi^{(-)}(x_1)$ or $\psi^{(-)}(x_2)$ must create $a_{p'_2}^+$. Either $\psi^{(+)}(x_1)^+$ or $\psi^{(+)}(x_2)^+$ must create $a_{p'_1}^+$. Either $\psi^{(-)}(x_1)^+$ or $\psi^{(-)}(x_2)^+$ must cancel $a_{p_2}^+$.

$$S = -\frac{1}{2} g^2 \sqrt{2^4 p_1^0 p_2^0 p'_1^0 p'_2^0} \langle 0 | a_{p'_1}^+ b_{p'_2}^+ \int T \left([\psi^{(+)}(x_1) + \psi^{(+)}(x_1)] [\psi^{(+)}(x_2) + \psi^{(+)}(x_2)] \phi(x_1) \right. \\ \times \left. [\psi^{(+)}(x_2) + \psi^{(+)}(x_2)] [\psi^{(+)}(x_2) + \psi^{(+)}(x_2)] \phi(x_2) \right) | a_{p_1}^+ b_{p_2}^+ | 0 \rangle.$$

Only terms with one $\psi^{(+)}$, one $\psi^{(-)}$, one $\psi^{(+)}\psi^{(+)}$, and one $\psi^{(-)}\psi^{(-)}$ survive. So instead of $2^4 = 16$ terms, we only get 4.

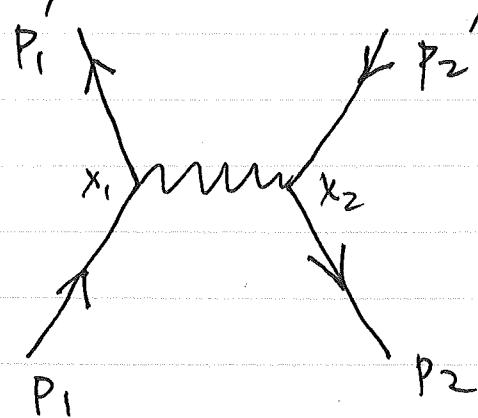
$$\begin{aligned}
 S = & -\frac{1}{2}g^2\sqrt{\pi p_1 p_2} \langle 0 | a_{p_1}^\dagger b_{p_2}^\dagger \left[\int T \left(\psi_{(x_1)}^{(+)} \psi_{(x_1)}^{(+)} \phi(x_1) d^4 x_1 \right. \right. \\
 & \times \left. \left. \psi_{(x_2)}^{(-)} \psi_{(x_2)}^{(-)} \phi(x_2) \right) dx_2 \right. \\
 & + \left. \int T \left(\psi_{(x_1)}^{(+)} \psi_{(x_1)}^{(-)} \phi(x_1) \psi_{(x_2)}^{(-)} \psi_{(x_2)}^{(+)} \phi(x_2) \right) dx_1 d^4 x_2 \right. \\
 & + \left. \int T \left(\psi_{(x_1)}^{(+)} \psi_{(x_1)}^{(+)} \phi(x_1) \psi_{(x_2)}^{(+)} \psi_{(x_2)}^{(-)} \phi(x_2) \right) d^4 x_1 d^4 x_2 \right. \\
 & \left. + \int T \left(\psi_{(x_1)}^{(-)} \psi_{(x_1)}^{(+)} \phi(x_1) \psi_{(x_2)}^{(+)} \psi_{(x_2)}^{(+)} \phi(x_2) \right) dx_1 d^4 x_2 \right] \\
 & \times a_{p_1}^\dagger b_{p_2}^\dagger | 0 \rangle
 \end{aligned}$$

Notice that the first and fourth terms differ only by the interchange of the dummy variables x_1 and x_2 . So they are the same. Also, the 2d and 3d terms are the same. So we get a factor of 2 :

$$\begin{aligned}
 S = & -g^2 \sqrt{p_1^\dagger p_2^\dagger} \langle 0 | a_{p_1}^\dagger b_{p_2}^\dagger \\
 & \times \left[\int T \left(\psi_{(x_1)}^{(+)} \psi_{(x_1)}^{(+)} \phi(x_1) \psi_{(x_2)}^{(-)} \psi_{(x_2)}^{(-)} \phi(x_2) \right) dx_1 d^4 x_2 \right. \\
 & \left. + \int T \left(\psi_{(x_1)}^{(+)} \psi_{(x_1)}^{(-)} \phi(x_1) \psi_{(x_2)}^{(-)} \psi_{(x_2)}^{(+)} \phi(x_2) \right) dx_1 d^4 x_2 \right] \\
 & \times a_{p_1}^\dagger b_{p_2}^\dagger | 0 \rangle.
 \end{aligned}$$

The 1st term cancels $a_{p_1}^\dagger$ at x_1 and creates $a_{p_1'}^\dagger$ at x_1 , and cancels $b_{p_2}^\dagger$ at x_2 and creates $b_{p_2'}^\dagger$ at x_2 .

Feynman drew this diagram

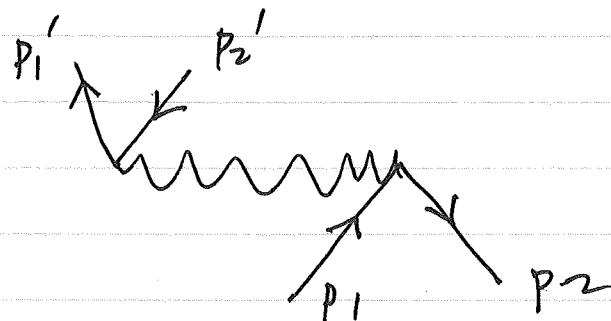


to represent that
it's first term.

Time runs up (\uparrow).

I took a and a^\dagger to be the particle annihilation and creation operators, and b and b^\dagger to be those for the anti-particle. That's why the arrows go up on the $p_1 \rightarrow p'_1$ line and down on the $p_2 \rightarrow p'_2$ line. The squiggle between x_1 and x_2 represent the boson field ϕ , about which more later.

The 2d term creates both $a_{p_1}^\dagger$ and $b_{p_2}^\dagger$ at x_1 and cancels both $a_{p_1}^\dagger$ and $b_{p_2}^\dagger$ at x_2 .
Feynman drew this as



Fd

The amplitude for $\gamma \nu \nu \bar{\nu}$ is

$$S_1 = -g^2 \sqrt{2p^0 s} \langle 0 | a_{p_1'} b_{p_2'}$$

$$\times \int T \int \frac{d^3 q_1' d^3 p_1' d^3 q_2' d^3 p_2'}{(2\pi)^4 \sqrt{2q_1'^0 2q_2'^0 2p_1'^0 2p_2'^0}} a_{q_1'}^+ e^{iq_1' x_1} a_{q_2'}^- e^{-iq_2' x_1} \phi(x_1)$$

$$\times b_{q_2'} e^{-iq_2' x_2} b_{q_2'}^+ e^{iq_2' x_2} \phi(x_2) a_{p_1'}^+ b_{p_2'}^+ \langle 0 | d\chi_1 d\chi_2$$

$$= -g^2 \sqrt{2E's} \langle 0 | b_{p_2'} \int T \int \frac{d^3 q_2' d^3 p_1' d^3 p_2'}{(2\pi)^4 \sqrt{2q_2'^0 s}} e^{ip_1' x_1} a_{q_1'}^- e^{-ip_1' x_1} \phi(x_1)$$

$$\times b_{q_2'} e^{-iq_2' x_2} b_{q_2'}^+ e^{iq_2' x_2} \phi(x_2) a_{p_1'}^+ b_{p_2'}^+ \langle 0 | d\chi_1 d\chi_2$$

Now since the final & initial momenta are all different, the $\delta(q_2 - q_2')$ that occurs when we move b_{q_2} part b_{q_2}' doesn't contribute.

So

$$S_1 = -g^2 \sqrt{\pi 2E's} \int \langle 0 | T(\phi(x_1) \phi(x_2)) \langle 0 | e^{i p_1' x_1 - i p_1 x_1} e^{-i p_2' x_2 + i p_2 x_2} d^4 x_1 d^4 x_2 .$$

On this page, I used $[a_p, a_{p'}^\dagger] = (2\pi)^3 S^{(3)}(\vec{p}, \vec{p}')$ repeatedly.

The mean value of the time-ordered product in the vacuum

$$\langle 0 | T(\phi(x_1) \phi(x_2)) | 0 \rangle = D_F(x_1 - x_2)$$

is the Feynman propagator

$$D_F(x - y) = \int \frac{d^4 p}{(2\pi)^4} \frac{i}{p^2 - m^2 + i\epsilon} e^{-ip(x-y)}$$

as a result (2.59) we'll derive presently. So

$$\begin{aligned} S_1 &= g^2 \int D_F(x_1 - x_2) e^{i(p_1' - p_1)x_1 - i(p_2' - p_2)x_2} d^4 x_1 d^4 x_2 \\ &= g^2 \int \frac{d^4 p d^4 k}{(2\pi)^4} \frac{i}{p^2 - m^2 + i\epsilon} e^{-ip(x_1 - x_2) + i(p_1' - p_1)x_1 + i(p_2' - p_2)x_2} \\ &= g^2 \int \frac{d^4 p}{p^2 - m^2 + i\epsilon} i d^4 x_2 \delta^{(4)}(p_1 + p) e^{i(p_1' - p_1)x_1 + i(p_2' - p_2)x_2} \\ &= g^2 \frac{i}{(p_1 - p_1')^2 - m^2 + i\epsilon} \int d^4 x_2 e^{i(p_1' - p_1 + p_2' - p_2)x_2} \\ &= g^2 (2\pi)^4 \delta^{(4)}(p_1' + p_2' - p_1 - p_2) \frac{i}{(p_1 - p_1')^2 - m^2} \end{aligned}$$

in which we may ignore the $i\epsilon$. We used the obvious fact that $(p_1' - p_1)^2 = (p_1 - p_1')^2$.

The amplitude for  is

$$\begin{aligned}
 S_2 &= -j^2 \sqrt{2E's} \langle 0 | a_{p_1} b_{p_2}^\dagger \\
 &\times \left[T \left(\psi_{(x_1)}^{(+)\dagger} \psi_{(x_1)}^{(-)} \phi(x_1) \psi_{(x_2)}^{(-)\dagger} \psi_{(x_2)}^{(+)} \phi(x_2) \right) dx_1 dx_2 \right] a_{p_1}^\dagger b_{p_2} | 0 \rangle \\
 &= -j^2 \sqrt{2E's} \langle 0 | a_{p_1} b_{p_2}^\dagger \int T \left(\frac{\int d^3q_1 d^3q_2 d^3q'_1 d^3q'_2}{(2\pi)^{12}} a_{q'_1}^\dagger e^{iq'_1 x_1} + b_{q'_2}^\dagger e^{iq'_2 x_1} \right. \\
 &\quad \left. - i q'_2 x_2 \right. \\
 &\quad \left. \times \phi(x_1) b_{q'_2} e^{-iq'_1 x_2} \right) a_{q_1}^\dagger e^{ip_1' x_1} b_{p_2}^\dagger | 0 \rangle.
 \end{aligned}$$

As before, $\langle a_{q_1}^\dagger a_{p_1}^\dagger | 0 \rangle = (2\pi)^3 \delta^3(q_1 + p_1) | 0 \rangle$, etc., so

$$\begin{aligned}
 S_2 &= -j^2 \int \langle 0 | T(\phi(x_1) \phi(x_2)) | 0 \rangle e^{ip_1' x_1 + ip_2' x_2} \\
 &\times e^{-ip_2 x_2 - ip_1 x_2} \frac{d^4 x_1 d^4 x_2}{(2\pi)^4 p^2 - m^2/c^2}
 \end{aligned}$$

in which all the $\sqrt{2E's}$ were cancelled by the $1/\sqrt{2E's}$.

We now use P&S's (2.59)

$$\langle 0 | T(\phi(x_1) \phi(x_2)) | 0 \rangle = D_F(x_1 - x_2) = \frac{1}{(2\pi)^4} \frac{i}{p^2 - m^2/c^2} e^{-ip(x_1 - x_2)}$$

where m is the mass of the particle of $\phi(x)$.

S_0

$$S_2 = -g^2 \int d^4x_1 d^4x_2 d^4p \frac{i}{(2\pi)^4 (p^2 - m^2 + i\epsilon)} e^{ix_1(p'_1 + p'_2 - p)} e^{ix_2(p - p_1 - p_2)} x e^{ix_1(p'_1 + p'_2 - p)}$$

$$= -g^2 \int d^4x_1 d^4p \frac{\delta^{(4)}(p - p_1 - p_2) i e}{(p^2 - m^2 + i\epsilon)}$$

$$= -g^2 \int d^4x_1 \frac{i}{(p_1 + p_2)^2 - m^2 + iG} e^{ix_1(p'_1 + p'_2 - p_1 - p_2)}$$

$$= -i g^2 (2\pi)^4 \frac{\delta^{(4)}(p'_1 + p'_2 - p_1 - p_2)}{(p_1 + p_2)^2 - m^2}$$

So the full amplitude is

$$S = S_0 + S_2 = -i g^2 (2\pi)^4 \delta^{(4)}(p'_1 + p'_2 - p_1 - p_2)$$

$$\times \left[\frac{1}{(p_1 - p'_1)^2 - m^2} + \frac{1}{(p_1 + p_2)^2 - m^2} \right].$$

Scattering of a "charged" particle by a neutral boson.

$$\begin{aligned}
 \mathcal{H}(x) &= g \psi^+(x) \psi(x) \phi(x) \\
 S &= \langle p_1' p_2' | \int (e^{-i \int \mathcal{H}(x) d^4x}) | p_1 p_2 \rangle \\
 &= \sqrt{2 p_1^0 s} \langle 0 | a_{p_1'} c_{p_2'} | \int (e^{-i \int \mathcal{H}(d^4x)}) a_{p_1'}^\dagger c_{p_2'}^\dagger | 0 \rangle \\
 &= \sqrt{2 E' s} \langle 0 | a_{p_1'} c_{p_2'} | \frac{(-i)}{2} \int \int (\psi_{(k_1)}^\dagger \psi_{(k_1)} \phi_{(k_1)} \psi_{(k_2)}^\dagger \psi_{(k_2)} \phi_{(k_2)}) d_{k_1} d_{k_2} a_{p_1'}^\dagger c_{p_2'}^\dagger | 0 \rangle
 \end{aligned}$$

The boson c_{p_2} can be stopped at x_1 or x_2 .
 Pick x_2 and get a factor of 2.

$$\begin{aligned}
 S &= -\sqrt{2 E' s} g \langle 0 | a_{p_1'} c_{p_2'} | \int T(\psi_{(k_1)}^\dagger \psi_{(k_1)} \phi_{(k_1)} \psi_{(k_2)}^\dagger \psi_{(k_2)} \phi_{(k_2)}) d_{k_1} d_{k_2} a_{p_1'}^\dagger c_{p_2'}^\dagger | 0 \rangle \\
 &= -\sqrt{2 E' s} g \langle 0 | a_{p_1'} c_{p_2'} | \int T(\psi_{(k_1)}^\dagger \psi_{(k_1)} \phi_{(k_1)} \psi_{(k_2)}^\dagger \psi_{(k_2)}) \int \frac{d_{k_2}^3}{(2\pi)^3 \sqrt{2q_2^0}} c_{q_{k_2}} e^{-i p_2' x_2} a_{p_1'}^\dagger c_{p_2'}^\dagger | 0 \rangle \\
 &= -\frac{\sqrt{2 E' s} g^2}{\sqrt{2 p_2^0}} \langle 0 | a_{p_1'} c_{p_2'} | \int T(\psi_{(k_1)}^\dagger \psi_{(k_1)} \phi_{(k_1)} \psi_{(k_2)}^\dagger \psi_{(k_2)}) e^{-i p_2' x_1} a_{p_1'}^\dagger | 0 \rangle \int \frac{d_{k_2}^3}{(2\pi)^3 \sqrt{2q_2^0}} c_{q_{k_2}} e^{i q_2 x_1} a_{p_1'}^\dagger c_{p_2'}^\dagger | 0 \rangle \\
 &= -\frac{\sqrt{2 E' s} g^2}{\sqrt{2 p_2^0 a_{p_2'}^0}} \langle 0 | a_{p_1'} | \int T(\psi_{(k_1)}^\dagger \psi_{(k_1)} \psi_{(k_2)}^\dagger \psi_{(k_2)}) e^{i p_2' x_1 - i p_2' x_2} a_{p_1'}^\dagger | 0 \rangle \int \frac{d_{k_2}^4}{d_{x_1}^4 d_{x_2}^4} d_{x_1}^4 d_{x_2}^4
 \end{aligned}$$

The "charged" particle $a_{p_1}^+$ can be stopped at x_1 or x_2 .
So

$$S = S_1 + S_2$$

stop $a_{p_1}^+$
at x_2

$$S_1 = \frac{-\sqrt{2E's}g^2}{\sqrt{2p_2^0 2p_2'^0}} \langle 0 | a_{p_1}' \int T \left(\psi^{+(-)}(x_1) \psi(x_1) \psi^+(x_2) \psi^{(+)}(x_2) \right) e^{a_{p_1}^+ 10} d^4x_1 d^4x_2 | i p_2' x_1 - i p_2 x_2 \rangle$$

stop $a_{p_1}^+$
at x_1

$$S_2 = \frac{-\sqrt{2E's}g^2}{\sqrt{2p_2^0 2p_2'^0}} \langle 0 | a_{p_1}' \int T \left(\psi^+(x_2) \psi^{(+)}(x_2) \psi^{+(-)}(x_1) \psi(x_1) \right) e^{a_{p_1}^+ 10} d^4x_1 d^4x_2 | i p_2' x_1 - i p_2 x_2 \rangle$$

Here $\psi^{+(-)}(x) = \int \frac{d^3 p}{(2\pi)^3 \sqrt{2p^0}} a_{p_1}^+ e^{ipx} = \left[\psi^{(+)}(x) \right]^+, \text{ so}$

$$S_1 = \frac{-\sqrt{2E's}g^2}{\sqrt{2p_2^0 2p_2'^0}} \int \int \frac{d^3 p_1'}{(2\pi)^3 \sqrt{2p_1'^0}} a_{q_1'}^+ e^{i p_1' x_1 + i p_2' x_1} T(\psi(x_1) \psi(x_2)) \int \int \frac{d^3 p_1}{(2\pi)^3 \sqrt{2p_1^0}} a_{q_1}^+ e^{i p_1 x_1 + i p_2 x_1} e^{a_{p_1}^+ 10} d^4x_1 d^4x_2$$

$$= -g^2 \int d^4x_1 d^4x_2 \langle \psi(x_1) \psi^+(x_2) | e^{i x_1(p_1' + p_2') - i x_2(p_1 + p_2)} | 10 \rangle$$

Now

$$\langle 0 | T(\psi(x_1) \psi^+(x_2)) | 10 \rangle = \langle 0 | \Theta(x_1^0 - x_2^0) \psi(x_1) \psi^+(x_2) + \Theta(x_2^0 - x_1^0) \psi^+(x_2) \psi(x_1) | 10 \rangle$$

$$= \langle 0 | \Theta(x_1^0 - x_2^0) \psi^{(+)}(x_1) \psi^{+(-)}(x_2) + \Theta(x_2^0 - x_1^0) \psi^{+(-)}(x_2) \psi^{(+)}(x_1) | 10 \rangle$$

$$= \langle 0 | \Theta(x_1^0 - x_2^0) [\psi^{(+)}(x_1), \psi^{+(-)}(x_2)] + \Theta(x_2^0 - x_1^0) [\psi^{+(-)}(x_2), \psi^{(+)}(x_1)] | 10 \rangle$$

Now as for free field $\phi(x)$

$$[\psi^{(+)}(x_1), \psi^{(-)}(x_2)] = \int \frac{d^3 p_1 d^3 p_2}{(2\pi)^6 \sqrt{2p_1^0 2p_2^0}} [a_{p_1} e^{-ip_1 x_1}, a_{p_2}^+ e^{ip_2 x_2}]$$

$$= \int \frac{d^3 p_1}{(2\pi)^3 2p_1^0} e^{-ip_1(x_1 - x_2)} = \Delta_{+}(x_1 - x_2) \text{ by (0.24)}$$

And

$$[\psi^{(+)}(x_2), \psi^{(-)}(x_1)] = \int \frac{d^3 p_1 d^3 p_2}{(2\pi)^6 \sqrt{2p_1^0 2p_2^0}} [b_{p_1} e^{-ip_1 x_2}, b_{p_2}^+ e^{ip_2 x_1}]$$

$$= \int \frac{d^3 p_1}{(2\pi)^3 2p_1^0} e^{-ip_1(x_2 - x_1)} = \Delta_{+}(x_2 - x_1) \text{ ((0.24) again)}$$

Thus by (0.20) & (0.5)

$$\langle 0 | \bar{\psi}(\psi(x_1) \psi^+(x_2)) | 0 \rangle = \theta(x_1^0 x_2^0) \Delta_{+}(x_1 - x_2) + \theta(x_2^0 x_1^0) \Delta_{+}(x_2 - x_1)$$

$$= -i \Delta_F(x_1 - x_2) = D_F(x_1 - x_2) = \int \frac{e^{-ik(x_1 - x_2)}}{k^2 - m^2 + i\epsilon} \frac{d^4 k}{(2\pi)^4}$$

$$S_0 = -g^2 \int \frac{-ik(x_1 - x_2) + i x_1(p_1' + p_2') - i x_2(p_1 + p_2)}{k^2 - m^2 + i\epsilon} d^4 k d^4 x_1 d^4 x_2$$

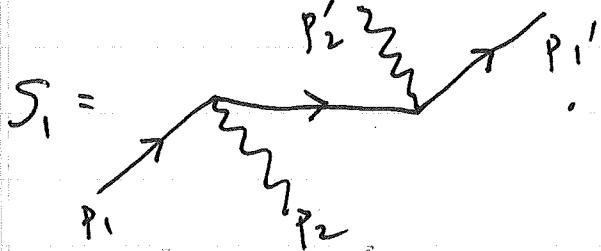
$$S_1 = -g^2 \int \frac{\delta^{(4)}(p_1 + p_2 - k)}{k^2 - m^2 + i\epsilon} d^4 k d^4 x_1 e^{-ik x_1 + i x_1(p_1' + p_2')}$$

So

$$S_1 = -ig^2 \int \frac{i\lambda_i(p'_1 + p'_2 - p_1 - p_2)}{(p_1 + p_2)^2 - m^2} d^4x_1$$

$$= -ig^2 \frac{(2\pi)^4 \delta(p'_1 + p'_2 - p_1 - p_2)}{(p_1 + p_2)^2 - m^2}$$

Feynman's diagram for this quantity is



$$S_2 = -g^2 \frac{\sqrt{2g_s}}{\sqrt{2p_1^0 2p_2^0}} \langle 0 | a_{p_1'}^\dagger \int \frac{d^3 q_1}{(2\pi)^3 \sqrt{2q_1^0}} a_{q_1} e^{-ip_1 x_1 + i p'_1 x_1} \int \frac{d^3 q'_1}{(2\pi)^3 \sqrt{2q'_1^0}} a_{q'_1}^\dagger e^{-ip_1' x_1 + i p'_1' x_1} \bar{J}(\psi_{(x_1)} \psi_{(x_2)}) a_{p_1'}^{10} \rangle d^4x_1 d^4x_2$$

$$= -g^2 \int \langle 0 | \bar{J}(\psi_{(x_1)}^\dagger \psi_{(x_2)}) | 10 \rangle e^{-ip_1 x_1 + ip'_1 x_1 + ip'_2 x_1 - ip_2 x_2} d^4x_1 d^4x_2$$

$$= -g^2 \int \langle 0 | \bar{J}(\psi_{(x_1)}^\dagger \psi_{(x_2)}) | 10 \rangle e^{-i(p_1 - p'_1)x_1 - i(p_2 - p'_2)x_2} d^4x_1 d^4x_2$$

$$\text{Now } \langle 0 | \bar{J}(\psi_{(x_1)}^\dagger \psi_{(x_2)}) | 10 \rangle = \langle 0 | \bar{J}(\psi_{(x_2)} \psi_{(x_1)}^\dagger) | 10 \rangle \\ = D_F(x_2 - x_1).$$

S_0

$$S_2 = -g^2 \int D_F(x_2 - x_1) e^{-i(p_1 - p_2')x_1 - i(p_2 - p_1')x_2} d^4x_1 d^4x_2$$

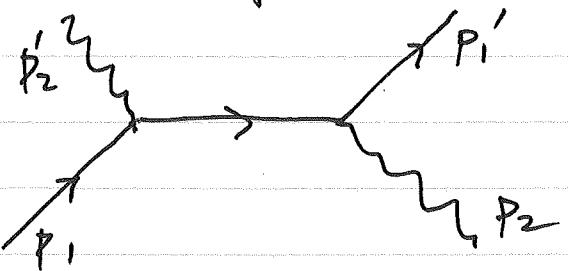
$$= -g^2 \int \frac{i}{k^2 - m^2 + i\epsilon} e^{-ik(x_2 - x_1)} e^{-i(p_1 - p_2')x_1 - i(p_2 - p_1')x_2} d^4x_1 d^4x_2$$

$$= -ig^2 \int \frac{\delta^4(k + p_2 - p_1')}{k^2 - m^2 + i\epsilon} e^{ik \cdot (p_1' + p_2')} d^4k d^4x_1$$

$$= -ig^2 \int \frac{e^{i(p_1' - p_2 - p_1 + p_2')x_1}}{(p_1' - p_2)^2 - m^2} d^4x_1$$

$$= -ig^2 \frac{(2\pi)^4 \delta^{(4)}(p_1' + p_2' - p_1 - p_2)}{(p_1' - p_2)^2 - m^2}$$

for which Feynman's diagram is



$$\text{Since } p_1' = p_1 + p_2 - p_2', \quad (p_1' - p_2)^2 = (p_1 - p_2')^2.$$

So the sum of the two amplitudes is

$$S = S_1 + S_2 = -g^2(2\pi)^4 \delta^{(4)}(p'_1 + p'_2 - p_1 - p_2)$$

