There is some arbitrariness in the η's.

$$P' = P\, e^{i\alpha B + i\beta L + i\gamma Q}$$

is as conserved as P.

By choosing $\alpha \beta \gamma$, we may fix $\eta_p = \eta_n = \eta_e = 1$.

But η for π^0 is unique — carries no conserved quantum numbers.

P^2 is (like) an internal symmetry

$$P^2 \psi^{\pm}_{\alpha} = \eta^2_{n_1} \eta^2_{n_2} \eta^2_{n_3} \cdots \psi^{\pm}_{\alpha}$$

If we can set $\eta^2_\alpha = e^{i \vec{\alpha} \cdot \vec{Q}}$

with $[Q, H] = 0$, then we can say

$$P^2 = e^{i\alpha \cdot Q}$$

and $$P' = P\, e^{-i\alpha \cdot Q/2}$$

satisfies $$P'^2 = P^2 P^{-2} = I.$$

Call P' parity and have

$$P'^2 = 1,$$

and so $\eta_\alpha = \pm 1$.

But such an $\alpha \cdot Q$ may not be available. If ν is Majorana, $B_\nu = L_\nu = 0$, then $\eta_\nu = \pm i$ is possible.

Note (3.3.42) says

$$S_{\alpha\beta} = \eta_\alpha^* \eta_\beta S_{P\alpha\, P\beta} .$$

So if $\eta_\alpha = \pm \eta_\beta$, then

$$S_{\alpha\beta} = \pm S_{P\alpha\, P\beta}$$

or S is even / odd in the $\vec{p}$'s.

1951 $\quad (\pi^- d) \longrightarrow n\, n$

$\ell = 0$, $j = 1$ $\qquad \ell = 1$, $s = 1$

So $\quad S_{\alpha\beta} = - S_{P\alpha P\beta}$

So $\quad \eta_d \eta_{\pi^-} = - \eta_n^2 = -1$

d is $\ell = 0$ $\quad \eta_p = \eta_n$ so $\eta_d = 1$

$$\eta_{\pi^-} = -1 .$$

pseudoscalar ($\pi^{\pm,0}$ all have $\eta_\pi = -1$)

(go to 89)

If $\quad X \to \pi\,\pi\,\pi$,

then in rest frame $\quad \vec{p}_1 + \vec{p}_2 + \vec{p}_3 = \vec{0}$

So S being rotationally invariant just depends on $\quad p_1 \cdot p_2 \quad$ etc $\quad p_2 \cdot p_3$.

the only available scalars made from p_i that are rotationally invariant. So

$$S_{\pi\pi\pi X} = (\eta_\pi^*)^3 \, \eta_X \, S_{P(\pi\pi\pi X)}$$

$$= - \eta_X \, S_{\pi\pi\pi X}$$

So

$$\eta_X = -1.$$

And if $\quad Y \to \pi\,\pi$, then $\quad \eta_Y = +1$.

τ, θ puzzle late 1940's

same masses and lifetimes $\tau, \theta \to \pi\pi,\ \pi\pi\pi$

Lee-Yang 1956 : $\nexists$ P in weak int's.

Rate $\alpha \to \beta \;\propto\; |S_{\beta\alpha}|^2$

factors are just E's.

invariant under P.

if α, β have fixed #'s of particles, the η's are irrelevant in the rates

So $P \Rightarrow$ $\quad S_{\beta\alpha} = \eta_\beta^* \eta_\alpha S_{P\beta\,P\alpha}$

$$\text{Rate}(\alpha \to \beta) = \text{Rate}(P\alpha \to P\beta)$$

rates invariant under reversal of all 3-momenta $\vec{p}_i$

Lee, Yang, Wu

$$Co^{60} \to Ni^{60} + e^- + \bar{\nu}$$

found e^-'s came out more in direction opposite $\vec{\sigma}$ of Co^{60}.

Also seen in

$\pi^+ \to \mu^+ + \nu$

(polarized $\qquad \mu^+ \to e^+ + \bar{\nu}_\mu + \nu_e$

But in vector gauge theories parity is conserved. (12.5) as long as $e_L = e_R$ in QED e.g.

from

Time Reversal

$$T\,\Psi^{\pm}_{p_1\sigma_1 n_1,\, p_2\sigma_2 n_2} = \zeta_{n_1}(-1)^{j_1-\sigma_1}\,\zeta_{n_2}(-1)^{j_2-\sigma_2}\cdots$$

$$\Psi^{\mp}_{\mathcal{P}p_1,-\sigma_1,n_1;\ \mathcal{P}p_2,-\sigma_2,n_2;\ \cdots}$$

in $n>0$ case.

$$T\,\Psi^{\pm}_{\alpha} = \Psi^{\mp}_{\mathcal{T}\alpha} \qquad (3.3.44)$$

includes phases!

go to 91

T is antiunitary $= (\Psi^{+}_{\alpha}, \Psi^{-}_{\beta})^{*}$ (2.2.4)

$$S_{\beta\alpha} = (\Psi^{-}_{\beta}, \Psi^{+}_{\alpha}) = (T\Psi^{+}_{\alpha}, T\Psi^{-}_{\beta})$$

$$= (\Psi^{-}_{\mathcal{T}\alpha}, \Psi^{+}_{\mathcal{T}\beta}) = S_{\mathcal{T}\alpha,\mathcal{T}\beta}$$

$$S_{p_1'\sigma_1'n_1'\cdots\ p_1\sigma_1 n_1\cdots}$$

$$= \zeta_{n_1'}(-1)^{j_1'-\sigma_1'}\cdots\ \zeta^{*}_{n_1}(-1)^{j_1-\sigma_1}\cdots$$

$$S_{\mathcal{P}p_1,-\sigma_1,n_1\cdots\ \mathcal{P}p_1',-\sigma_1',n_1'\cdots}$$

If $T_0 \Phi_\alpha = \Phi_{\mathcal{T}\alpha}$

and $T_0^{-1} H_0 T_0 = H_0$

If also $T_0^{-1} V T_0 = V$

Then $T_0^{-1} H T_0 = H$ and

we take $T = T_0$ and get by (3.1.13)

$$
\begin{aligned}
T \Psi_\alpha^\pm &= T \Omega(\mp\infty) \Phi_\alpha \\
&= T e^{\mp i H T_B} e^{\pm i H_0 T_B} \Phi_\alpha \\
&= e^{\pm i H T_B} e^{\mp i H_0 T_B} T \Phi_\alpha \\
&= \Omega(\pm\infty) \Phi_{\mathcal{T}\alpha} = \Psi^\mp_{\mathcal{T}\alpha}
\end{aligned}
$$

which is (3.3.44) and which implies the covariance of the S-matrix under time reversal, i.e., the behavior of the S-matrix in a theory that has time-reversal symmetry.

One may also use (3.1.16) and (3.3.48-50). In either case

$$S_{\beta,\alpha} = S_{\mathcal{T}\alpha, \mathcal{T}\beta}$$

i.e. rate $R(\alpha \to \beta) = R(\mathcal{T}\beta \to \mathcal{T}\alpha)$.