$$[K_0, U] = -e^{iH_0T}\, W\, e^{-iH_0T}\, U + e^{iH_0T}\, e^{-iH(T-T_0)}\, e^{-iH_0T_0}$$
$$\times\, e^{iH_0T_0}\, W\, e^{-iH_0T_0}$$

$$[\vec{K}_0, U(T,T_0)] = -W(T)U + U\,W(T_0) \qquad (3.3.22)$$

where

$$W(t) \equiv e^{iH_0t}\, W\, e^{-iH_0t} \qquad 23$$

If $(\Phi_\alpha, W\Phi_\beta)$ are smooth enough in E_α, E_β,

then (see 76,5)

$$\int d\alpha \int d\beta\, g^*(\alpha)\, g(\beta)\, (\Phi_\alpha, W(t)\Phi_\beta) \to 0 \text{ as } t \to \pm\infty$$

and so

$$[\vec{K}_0, S] = 0, \qquad 24$$

Much like conditions on $V(t)$.

9/8 Now recall $\Omega(t) = e^{iHt}\, e^{-iH_0t} = U(0,t)$. And

$$[K_0, U(0,\mp\infty)] = -W(0)U(0,\mp\infty) + U(0,\mp\infty)W(\mp\infty) \to 0$$

$$\Omega(\tau) = e^{iH\tau}\, e^{-iH_0\tau} = U(0,\tau) \qquad \text{so (see 76.5b)}$$

$$(\vec{K}_0 + \vec{W})\,\Omega(\mp\infty) = \Omega(\mp\infty)\,\vec{K}_0$$

$$\vec{K}\,\Omega(\mp\infty) = \Omega(\mp\infty)\,\vec{K}_0 \qquad (3.3.25)$$