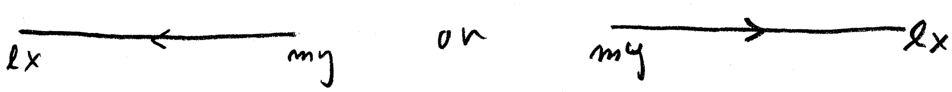
1 Feynman Diagrams: Arrows and Momenta

The Feynman propagator, SW's Eq.(6.2.1),

$$\begin{aligned}
-i\Delta_{\ell m}(x,y) &= \langle 0|T\left\{\psi_\ell(x)\psi_m^\dagger(y)\right\}|0\rangle \\
&= \theta(x-y)(2\pi)^{-3}\int d^3p\sum_s u_\ell(\mathbf{p},s)u_m^*(\mathbf{p},s)e^{ip\cdot(x-y)} \\
&\pm\ \theta(y-x)(2\pi)^{-3}\int d^3p\sum_s v_\ell(\mathbf{p},s)v_m^*(\mathbf{p},s)e^{ip\cdot(y-x)} \\
&= \frac{-i}{(2\pi)^4}\int d^4q\frac{P_{\ell m}(q)e^{iq\cdot(x-y)}}{q^2+m^2-i\epsilon} \qquad (1)
\end{aligned}$$

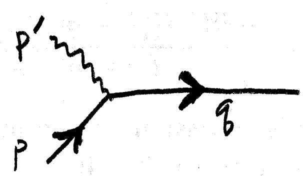
represents a particle carrying momentum p from y,m to x,ℓ or an anti-particle carrying momentum p from x,ℓ to y,m. The space-time diagram is



(2)

and we see that the arrow points from y,m to x,ℓ. So for a particle, the arrow points in the direction of the momentum transport, but for an anti-particle, the arrow points in the direction opposite to that of the momentum transport. The off-shell momentum q is from y,m to x,ℓ which is the direction of the arrow.

To see how to find the sign of q, which matters only for fermions, let's consider two vertices. The first vertex y,m has an incoming particle, a fermion, of momentum p and an outgoing boson of momentum p'



(3)

Leaving out irrelevant factors, this vertex gives

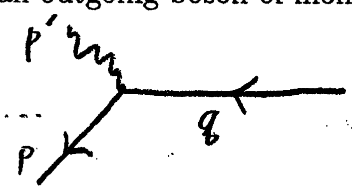
$$\begin{aligned}
\int d^4y\,\frac{-i}{(2\pi)^4}\int d^4q\frac{P_{\ell m}(q)e^{iq\cdot(x-y)}}{q^2+m^2-i\epsilon}\,e^{i(p-p')\cdot y} &= -i\int d^4q\frac{P_{\ell m}(q)e^{iq\cdot x}}{q^2+m^2-i\epsilon}\,\delta^4(p-p'-q) \\
&= \frac{-iP_{\ell m}(p-p')e^{i(p-p')\cdot x}}{(p-p')^2+m^2-i\epsilon}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{-i[(-i\gamma_a(p^a - p'^a) + m)\beta]_{lm} e^{i(p-p')\cdot x}}{(p-p')^2 + m^2 - i\epsilon} \\
&= \frac{-i[(-i(\not{p} - \not{p}') + m)\beta]_{lm} e^{i(p-p')\cdot x}}{(p-p')^2 + m^2 - i\epsilon} \qquad (4)
\end{aligned}
$$

for a Dirac or Majorana field. So the rule is to assign to the incoming and outgoing particles their physical momenta and to have the internal line taking momentum q from the vertex, as shown by its arrow. The sum of all momenta *entering* the vertex should be zero, so

$$
p - p' - q = 0 \quad \Rightarrow \quad q = p - p'. \qquad (5)
$$

The second vertex x, ℓ has an incoming anti-particle, a fermion, of momentum p and an outgoing boson of momentum p'



(6)

Leaving out irrelevant factors, this vertex gives

$$
\begin{aligned}
\int d^4x \frac{-i}{(2\pi)^4} \int d^4q \frac{P_{\ell m}(q) e^{iq\cdot(x-y)}}{q^2 + m^2 - i\epsilon} e^{i(p-p')\cdot x} &= -i \int d^4q \frac{P_{\ell m}(q) e^{-iq\cdot y}}{q^2 + m^2 - i\epsilon} \delta^4(p - p' + q) \\
&= \frac{-iP_{\ell m}(p' - p) e^{-i(p'-p)\cdot y}}{(p'-p)^2 + m^2 - i\epsilon} \\
&= \frac{-i[(-i\gamma_a(p'^a - p^a) + m)\beta]_{lm} e^{i(p-p')\cdot y}}{(p'-p)^2 + m^2 - i\epsilon} \\
&= \frac{-i[(-i(\not{p}' - \not{p}) + m)\beta]_{lm} e^{i(p-p')\cdot y}}{(p'-p)^2 + m^2 - i\epsilon} \qquad (7)
\end{aligned}
$$

for a Dirac or Majorana field. So the rule is to assign to the incoming and outgoing particles their physical momenta and to have the internal line taking momentum q from the vertex, as shown by its arrow. The sum of all momenta *entering* the vertex should be zero, so

$$
p - p' - q = 0 \quad \Rightarrow \quad q = p - p'. \qquad (8)
$$

So the rule is to assign to the incoming and outgoing particles their physical momenta and to have the internal line taking momentum q into the vertex, as shown by its arrow. The sum of all momenta *entering* the vertex should be zero, so

$$p - p' + q = 0 \quad \Rightarrow \quad q = p' - p. \tag{9}$$

The general rule then is to set the sum of the momenta entering minus the sum of the momenta leaving each vertex equal to zero, counting initial-state particles and anti-particles as entering the vertex and all final-state particles and anti-particles as leaving the vertex and counting internal lines with arrows pointing toward the vertex as entering the vertex and those with arrows pointing away from the vertex as leaving the vertex. SW states this rule in greater generality in the text around Eq.(6.3.1).