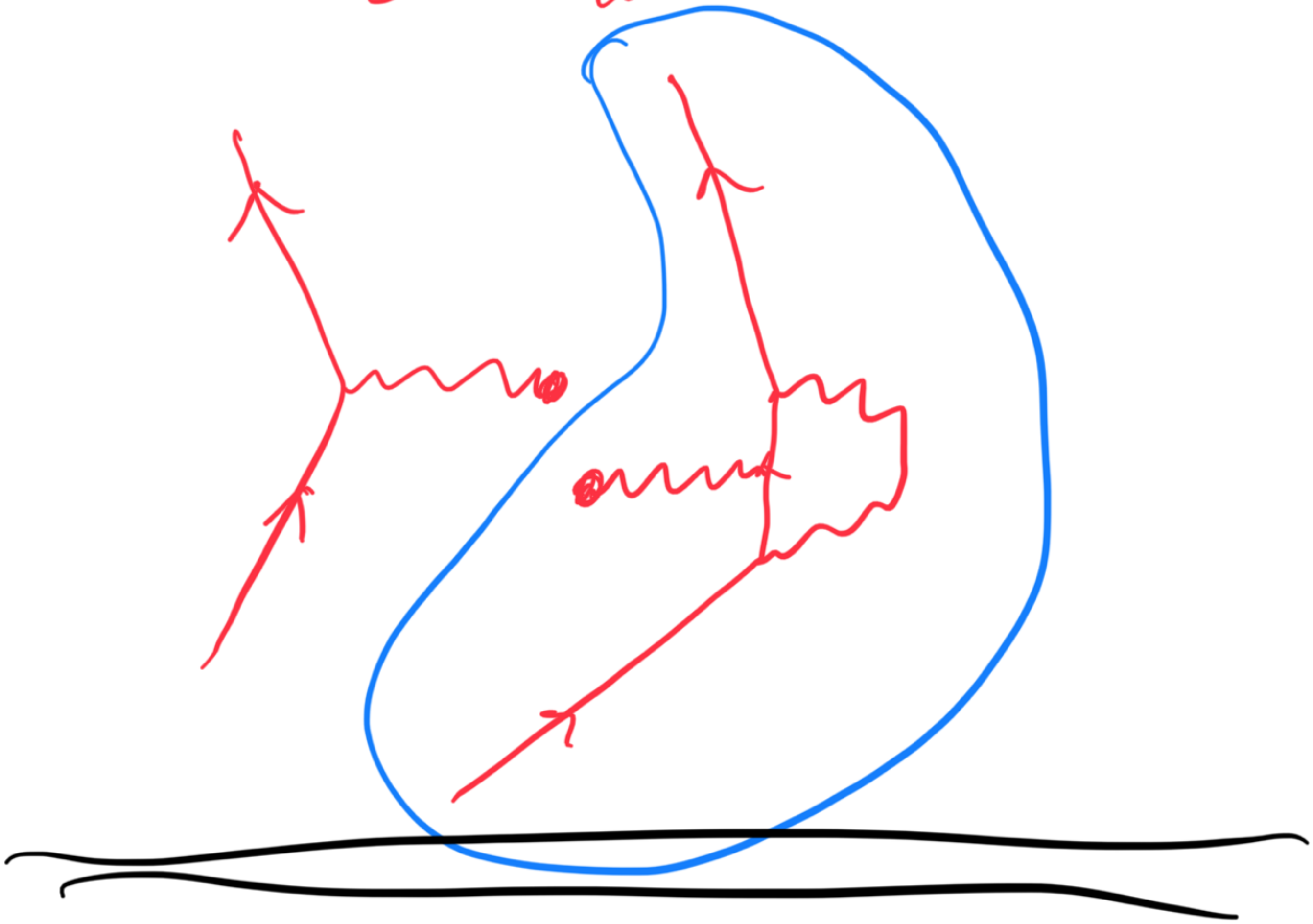
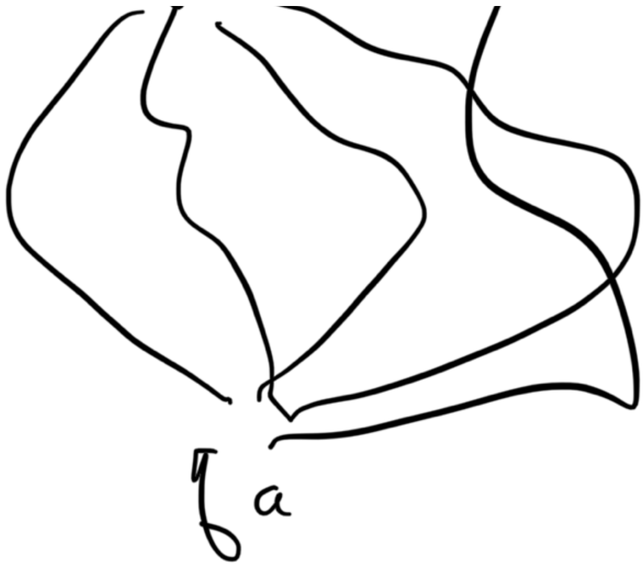
23 March 2023



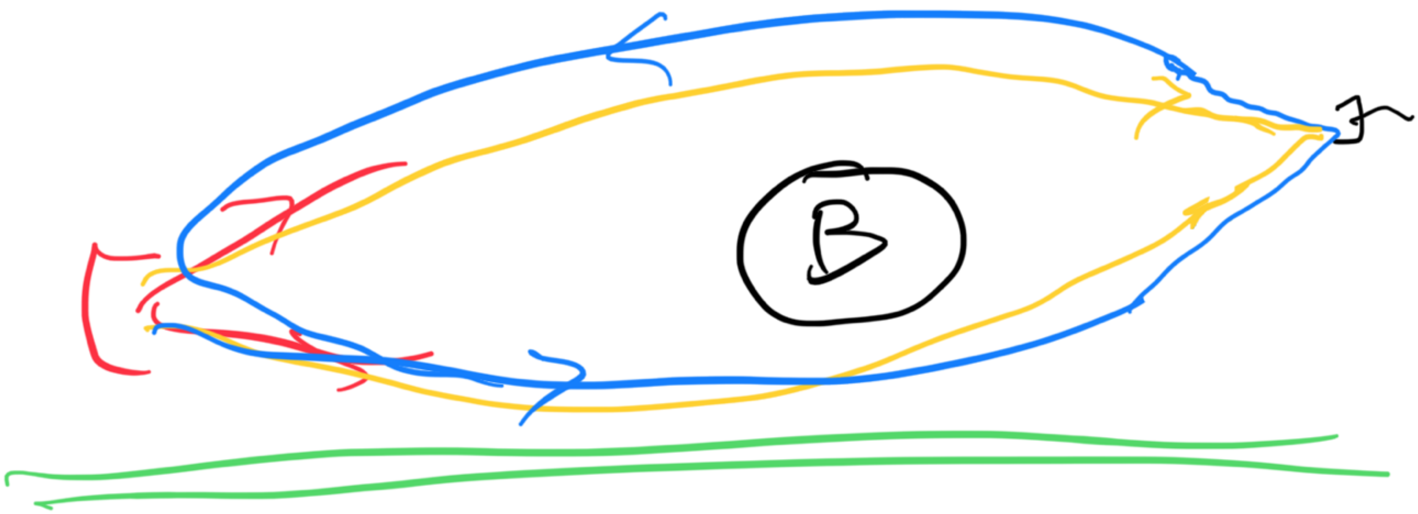
$$\beta = \frac{1}{kT}$$

$$S = \int (T - V)\, dt$$

$$e^{-i\epsilon k} \, e^{-i\epsilon v} \, I \, e^{-i\epsilon k} \, e^{-i\epsilon v}$$



$$S = -mc^2 \int \sqrt{1 - v^2/c^2}\, dt$$



$$\beta = \frac{1}{kT}$$

$$L = T - V$$

$$= \frac{1}{2m} - V(q)$$

$$S = \int L\,dt$$

$$\langle \quad \rangle = \int e^{iS[q]}\, Dq$$

$$\langle \{q\}_b | e^{-itH/\hbar} | \{q\}_a \rangle$$

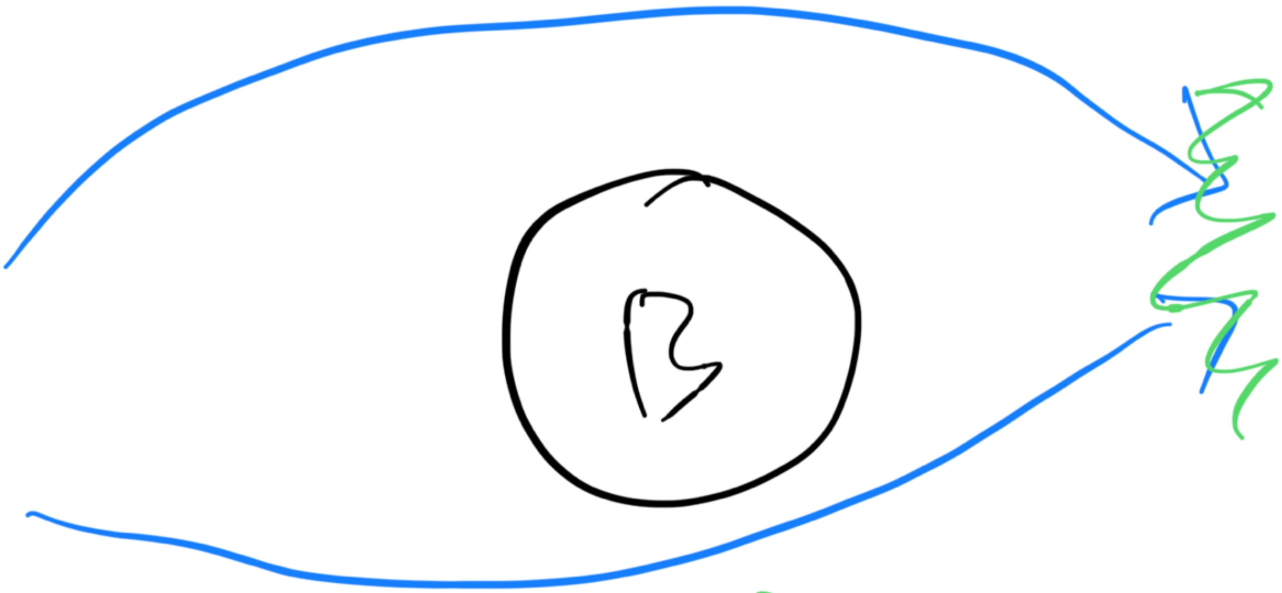
$$= \int e^{\frac{i}{\hbar}\int dt\, \frac{1}{2} m \dot{q}_j^2 - V(\{q\})}\, Dq$$

$$V(q) = V(q_0) + V'(q_0)(q - q_0)$$

$$+ \frac{1}{2} V''(q_0)(q - q_0)^2$$

$$\boxed{+ \frac{1}{3!} V'''(q_0)(q - q_0)^3 + \cdots}$$

small



$$\int A \cdot dq = \int (\nabla \times A) \cdot d\sigma$$

$$= \int B \cdot ds = \Phi$$

$$Tr(AB) = Tr(BA)$$

$$Tr(e^{-\beta H}) = Tr(uu^{\dagger} e^{-\beta H})$$

$$Tr(u^{\dagger} e^{-\beta H} u)$$

$$\sum_n \langle n| e^{-\beta H} |n\rangle = \sum_\alpha \langle \alpha | e^{-\beta H} |\alpha\rangle$$

$$|\alpha\rangle = U |n\rangle$$

$$Z(\beta) = \sum_n e^{-\beta E_n}$$

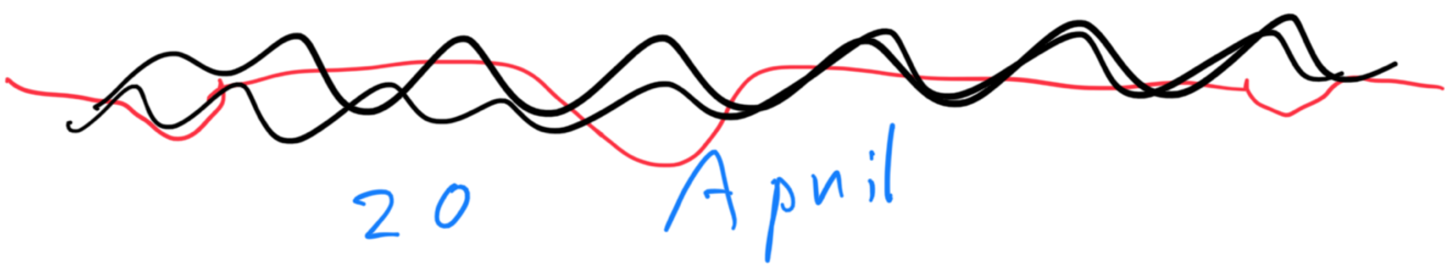
$$\int \boxed{q_n} \, dq_n$$

$$\beta = \frac{1}{kT} \qquad T \text{ temp.}$$

$$T \to 0 \iff \beta \to \infty$$

$$\int_0 du \left[\frac{m \dot{q}_j^2}{2} + V(q_j) \right]$$

$$[\hbar \beta] = \frac{ET}{E} = \text{Time}$$



$$\int f(x)\, \delta'(x)\, dx$$

$$= \int -f'(x)\, \delta(x)\, dx$$

$$= -f'(0)$$

$$\int f(x)\,\delta''(x-y)\,dx$$

$$= \int f''(x)\,\delta(x-y)\,dx$$

$$= f''(y).$$

$$\delta Z[j][\delta_y][\delta_{\cdot}]$$

$$\frac{d}{dx}\frac{y'}{\sqrt{1+y'^2}} = \frac{y''}{\sqrt{1+y'^2}} - \frac{1}{2}\frac{y''\,2y''y'}{(1+y'^2)^{3/2}} = 0$$

$$y''=0 \qquad = \frac{y''}{\sqrt{1+y'^2}}\left(1 - \frac{y'y''}{1+y'^2}\right)$$

$$\binom{a}{b} = \frac{a!}{b!\,(a-b)!}$$

$$\langle q' | f \rangle = f(q')$$

$$\langle q' | \psi \rangle = \psi(q')$$

$$\langle q' | pq | f \rangle = \langle q' | \frac{\hbar}{i} + q' p | f \rangle$$

$$= \langle q' | \frac{\hbar}{i} + q p | f \rangle$$

$$[q, p] = i\hbar \qquad qp - pq = i\hbar$$

$$[\phi(\vec{x}, t), \phi(\vec{y}, t)] = 0$$

$$\phi(x,t) | \phi' \rangle = \phi'(x) | \phi' \rangle$$

hermitian   real

$$\langle \phi' | \phi'(x) = \langle \phi' | \phi^{\dagger}(x,t) = \langle \phi' | \phi(x,t)$$

$$\hbar = 1 = c$$

$$-\sqrt{-\nabla^2 + m^2}\,\phi'(x)\,\langle \phi' | 0 \rangle \qquad \langle \phi' | 0 \rangle$$

$$= -\int \delta(x - x')\sqrt{-\nabla'^2 + m^2}\,\phi(x')\,d^3x'$$

$$= -\sqrt{-\nabla^2 + m^2}\,\phi'(x)\,\langle \phi' | 0 \rangle$$

$$\sqrt{-\nabla^2 + m^2}\int e^{+i\vec{p}\cdot\vec{x}}\,\tilde{\phi}(p)\,\frac{d^3p}{(2\pi)^{3/2}}$$

$$= \int \sqrt{p^2 + m^2}\,e^{ip\cdot x}\,\tilde{\phi}(p)\,\frac{d^3p}{(2\pi)^{3/2}}$$

$$\int d^3x \int \frac{d^3p\,d^3p'}{(2\pi)^3}\,\tilde{\phi}(p')\,e^{ip'\cdot x}\,\tilde{\phi}(p)\,e^{ipx} \times \sqrt{p^2 + m^2}$$

$$\int d^3p\,d^3p'\,\sqrt{p^2 + m^2}\,\delta(\vec{p} + \vec{p}')$$

$$= \int d^3p\, d^3p' \cdots$$

$$\tilde{\phi}(p')\,\tilde{\phi}(p)$$

$$= \int d^3p\, \tilde{\phi}(-p)\tilde{\phi}(p)\sqrt{p^2+m^2}$$

$$= \int d^3p\, \tilde{\phi}^*(p)\tilde{\phi}(p)\sqrt{p^2+m^2}$$

19.54

$$\langle \phi' | 0 \rangle = N\, e^{-\frac{1}{2}\int |\tilde{\phi}'(\vec{p})|^2 \sqrt{\vec{p}'^2+m^2}\, d^3p}$$

$$\phi'(x)$$

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27 April

$$f(x)\delta'(x-y) \to -\delta(x-y)\frac{\partial f(x)}{\partial x} = -\frac{\partial f(y)}{\partial y}$$

$$f(x)\delta''(x-y) \to \delta(x-y)\frac{\partial^2 f(x)}{\partial x^2} = +\frac{\partial^2 f(y)}{\partial y^2}$$

$$0 = H|0\rangle = \frac{1}{2}\int [\quad][\sqrt{\quad}\,\phi + i\pi]\, d^3x\, |0\rangle$$

$$\langle\phi|\left[\sqrt{-\nabla^2 + m^2}\,\phi + i\pi\right]|0\rangle = 0$$

$$\langle\phi|\pi|0\rangle = \frac{1}{i}\frac{\delta\langle\phi|0\rangle}{\delta\phi}$$

$$* \qquad \langle q|p|\psi\rangle = \frac{1}{i}\frac{d}{dq}\langle q|\psi\rangle$$

$$\phi(\vec{x},0)|\phi'\rangle = \phi'(\vec{x})|\phi'\rangle$$

any of ∞ly many commuting fields

$$\ldots\ \phi(\vec{x}_k, 0)\ \ldots\ \phi(\vec{x}_n, 0)\ \ldots$$

$$\partial^a e^{ip\cdot x} = \frac{\partial}{\partial x_a} e^{ip^b x_b}$$

$= i\phi'^{a}$

$\phi(\vec{x}, 0)$

field $\quad \phi(\vec{x},0)|\phi'(\vec{x},t)\rangle$

$= \phi'(\vec{x},t)|\phi'(\vec{x},t)\rangle$

$(x+a)^2 = x^2 + 2xa + a^2$

$(x+a)^2 - a^2 = x^2 + 2xa$

$(x+a)^2 b^2 = x^2 b^2 + 2xab^2 + a^2 b^2$

$(x+a)^2 b^2 - a^2 b^2 = x^2 b^2 + 2xab^2$

$$\left(\frac{1}{i}\right)^2 \frac{\delta}{\delta j(x)} \frac{\delta}{\delta j(x')} Z_0[j] \Big|_{j=0}$$

$$= \langle 0 | T\left(\phi(x)\phi(x')\right) | 0 \rangle$$

4 May 2023

local $\Longleftrightarrow$ depends upon spacetime $x = (ct, \vec{x})$ $\Longleftrightarrow$ gauge

$$\psi' = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_n \end{pmatrix}' = \underbrace{U}_{n \times n}(x)\,\psi(x)$$

$$\psi' = U\psi$$

$$\left(\psi^\dagger_{(x)} \psi_{(x)}\right)' = \psi^\dagger U^\dagger U \psi = \psi^\dagger \psi$$

$$U^\dagger U = I$$

$$\left[(D_i\psi)^\dagger D^i\psi\right]' = (D_i\psi)^\dagger D^i\psi$$

$$(\psi^\dagger D_i\gamma^i\psi)'$$
$$= \psi^\dagger u^\dagger (D_i\gamma^i\psi)'$$
$$= \psi^\dagger u^\dagger u\, D_i\gamma^i\psi$$
$$= \psi^\dagger D_i\gamma^i\psi$$

$$\{\gamma^a,\gamma^b\} = \gamma^a\gamma^b + \gamma^b\gamma^a = 2\eta^{ab}$$
$$= 2\begin{pmatrix} -1 & & & 0 \\ & 1 & & \\ & & 1 & \\ 0 & & & 1 \end{pmatrix}$$

$$u^\dagger \rightleftarrows u^{-1}$$

$$u^\dagger = u^{-1}$$

$iq\theta(x)$

electrodynamics $U(x) = e$

$$[t_a, t_b] = t_a t_b - t_b t_a$$
$$= i f^{c}_{ab} t_c$$

structure constants

$$A'_i(x) = A_i(x) + \partial_i \lambda(x)$$

QED

$$D'_i = U D_i U^{-1} = U D_i U^{\dagger}$$

$$F_{ik} = (\partial_i + A_i)(\partial_k + A_k) - (\partial_k + A_k)(\partial_i + A_i)$$

$$= \partial_i A_k - \partial_k A_i + A_i A_k - A_k A_i$$

$$= \partial_i A_k - \partial_k A_i + [A_i, A_k]$$

$n \times n$

$$\vec{p} = \frac{\hbar}{i} \vec{\nabla}$$

herm. antiherm.

$$\psi = \begin{pmatrix} l \\ r \end{pmatrix} \qquad e^{z \cdot \sigma} \qquad e^{\bar{z} \cdot \sigma}$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$L_e = \begin{pmatrix} l_\nu \\ l_e \end{pmatrix} \begin{matrix} \text{neutrino} \\ \text{electron} \end{matrix}$$

$SU_l(2)$

$$D_i L_e = (\partial_i + A_i) L_e$$

$$= \left(\partial_i - ig\frac{\vec{\sigma}}{2}\cdot\vec{A}_i\right) L_e$$

$U_Y(1)$

$$D_i L_e = (\partial_i - ig' Y B_i) L_e$$

$$D_i R_e = (\partial_i - ig' Y B_i) R_e$$

$$R_e = \begin{pmatrix} r_\nu \\ r_e \end{pmatrix}$$

$$\mathcal{H} = \ldots \quad \lambda^2 (\phi^\dagger \phi - \mu^2)^2$$

$$\langle 0 | \phi^\dagger \phi | 0 \rangle = \mu^2$$

$i\varphi$

$$\langle 0|\phi(x)|0\rangle = \mu e^{'}$$

$$\langle 0|\phi(x)|0\rangle = \mu$$

$$S = -\frac{1}{2}\mu^2\phi^2$$

$$\text{mass} = \mu$$

$$H = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix} =$$

$$\langle 0||H||0\rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$(D_i\psi)^\dagger(D^i\psi)$$

$$(D_iH)^\dagger(D^iH)$$

$$\vec{u} = \begin{pmatrix} \vec{l}_u \\ \vec{r}_u \end{pmatrix} = \begin{pmatrix} \psi_u \\ \psi_u \\ \psi_u \end{pmatrix} = \Psi_u$$

$SU_c(3)$

$$D_i \Psi_u = \partial_i \Psi_u + A_i \Psi_u$$

$$A_i = -i g_s \lambda^\alpha A^\alpha_i$$

$u_r \bar{u}_b$

$u_n \bar{u}_n$

$u_b \bar{u}_r$

$\bar{u}_n u_n$

$$A_i(x) \longrightarrow U(x)$$