# 1

# Masses

## 1.1 Masses

Masses occur in the action density as the coefficients of terms quadratic in the fields. Thus the action density of a neutral, spin-zero field $\phi$ is

$$L = -\tfrac{1}{2}\partial_a\phi\partial^a\phi - \tfrac{1}{2}\mu^2\phi^2, \tag{1.1}$$

and the mass is $\mu$. The equation of motion is

$$\left(\partial_a\partial^a - \mu^2\right)\phi(x) = 0. \tag{1.2}$$

The field obeying this equation is a linear combination of $a$ and $a^\dagger$

$$\phi(x) = \int \left[a(k)e^{ikx} + a^\dagger(k)e^{-ikx}\right] \frac{d^3k}{\sqrt{(2\pi)^3 2k^0}} \tag{1.3}$$

which obey the commutation relation

$$[a(k), a^\dagger(p)] = a(k)\, a^\dagger(p) - a^\dagger(p)\, a(k) = \delta(\boldsymbol{k} - \boldsymbol{p}). \tag{1.4}$$

The charged spin-zero field is a complex linear combination of two equal-mass real fields

$$\phi = \frac{1}{\sqrt{2}}\left(\phi^{(1)} + i\phi^{(2)}\right). \tag{1.5}$$

Its action density is

$$L = -\partial_a\phi^*\partial^a\phi - \mu^2|\phi|^2, \tag{1.6}$$

and its equation of motion is

$$\left(\partial_a\partial^a - \mu^2\right)\phi(x) = 0. \tag{1.7}$$

The charged field is

$$\phi(x) = \int \left[a(k)e^{ikx} + b^\dagger(k)e^{-ikx}\right] \frac{d^3k}{\sqrt{(2\pi)^3 2k^0}}. \tag{1.8}$$