# 500 Seminar

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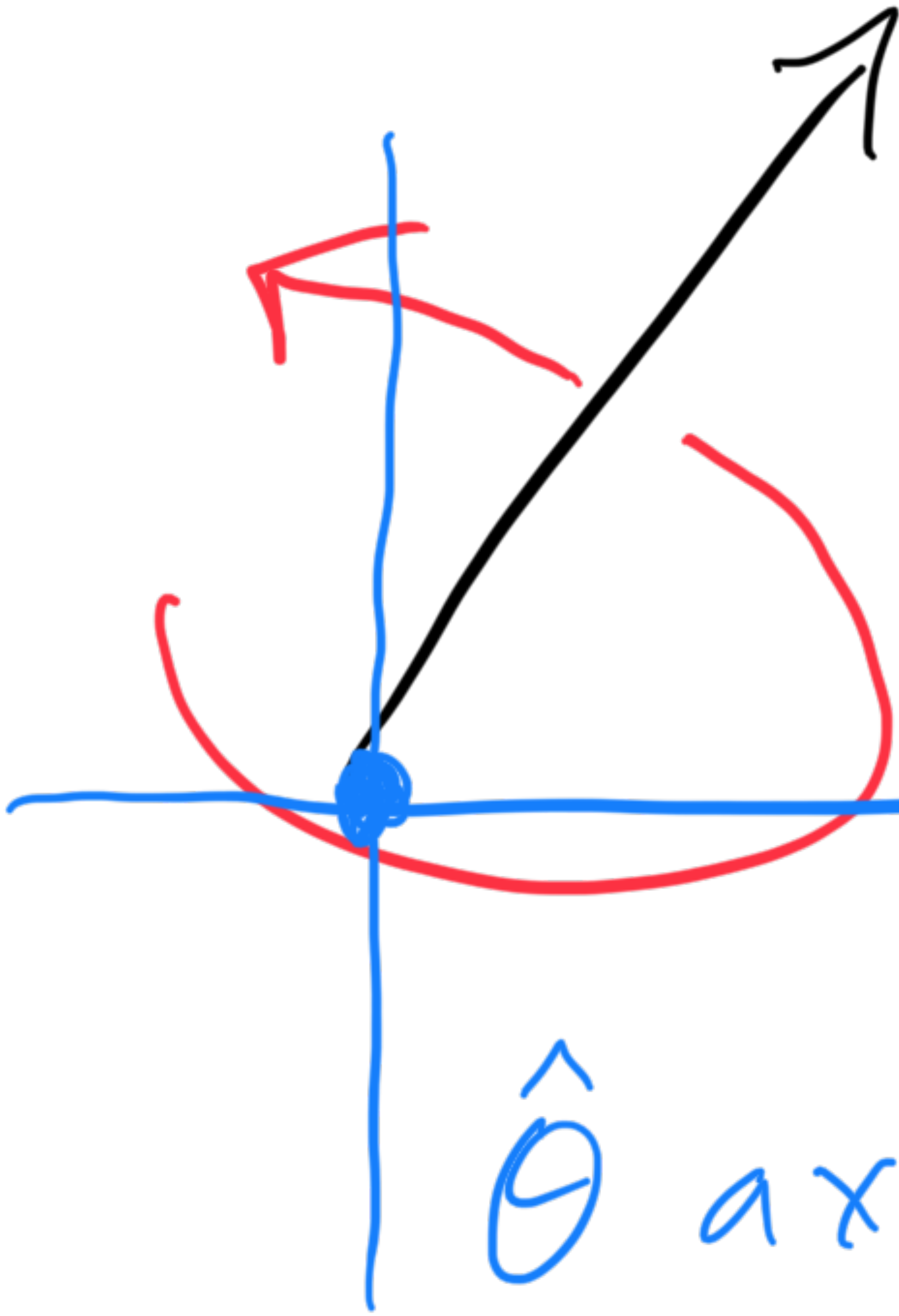
axis of rotation



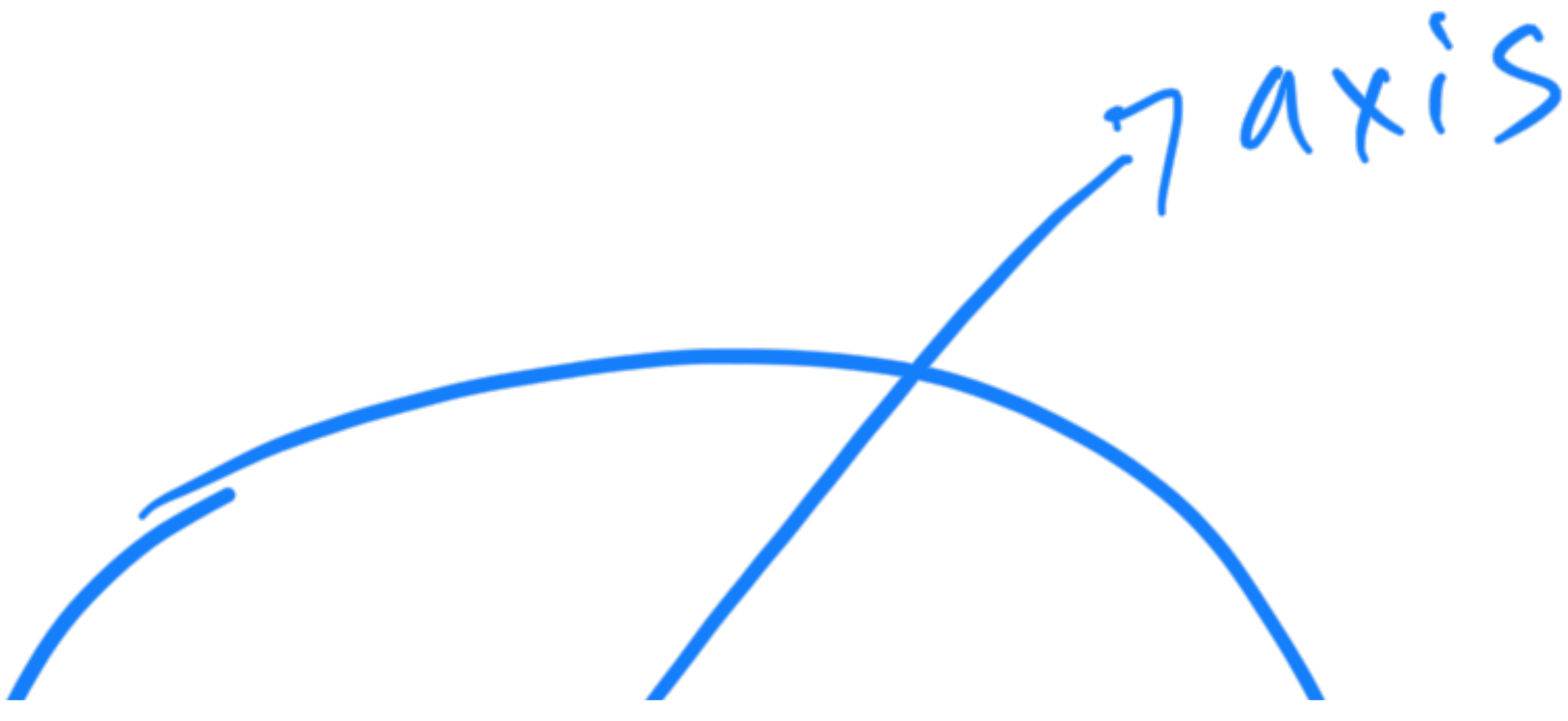
$\vec{r}$

$$d\vec{r} = d\vec{\theta} \times \vec{r}$$

$$\Delta\vec{r} = \vec{\theta} \times \vec{r}$$

$\hat{\theta}$ axis

$|\vec{\theta}|$ radians in rotation



$\hat{n}_i$ inertial, fixed unit vectors

$\hat{e}_i(t)$ unit vectors rotating with Earth

$$\vec{r}(t) = r_i \hat{n}_i = r_i(t)\, \hat{n}_i$$

$$\vec{r}_e(t) = r_{e_i}(t)\, \hat{e}_i(t)$$

$$\dot{r}(t) = \vec{v}(t) = \dot{r}_i(t)\, \hat{n}_i$$

$$\vec{r}(t) = r_e(t) = r_{e_i}(t)\, e_i(t)$$

$$\dot{\vec{r}}(t) = \dot{r}_{e_i}(t)\, \hat{e}_i(t) + r_{e_i}(t)\, \dot{\hat{e}}_i(t)$$

$$\dot{e}_i(t) = \dot{\theta} \times e_i$$

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$$\Delta\vec{r} = \vec{\theta} \times \vec{r} \qquad \dot{\theta} = \omega$$

$$\dot{r} = \omega \times r + \theta \times v$$

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$$\vec{r}_i(t) = e_i(t)\, r_i'$$

in rotating system

$$\vec{v} = \dot{r}_f = \dot{e}_j\, r_i' + e_j\, \dot{r}_j'$$

$$\dot{e}_j = \omega \times e_j \qquad \omega = \dot{\theta}$$

$$v = \dot{r}_f = (\omega \times e_i)\, r_i' + e_i\, \dot{r}_i'$$

$$v_f = \vec{\omega} \times \vec{r} + v'$$

$$= \omega \times r + \dot{r}'$$

$$\boxed{\frac{d}{dt_f} = \omega \times \; + \frac{d}{dt_r}}$$

real time derivative

apparant time derivative

$$a = \frac{d^2 r}{dt^2} = \left(\omega \times + \frac{d}{dt_r}\right)\left(\omega \times + \frac{d}{dt_r}\right) r$$

real

$= \omega \times (\omega \times \;) + \dot{\omega} \times$

$\dfrac{d}{dt_r}(\omega \times r) + \dfrac{d^2 r}{dt_r^2}$ $a$

Coriolis

$\vec{a}_f = \vec{a}' + \omega \times (\omega \times \vec{r}) + 2\omega \times v'$

$\dot{\omega} \times \vec{r}$

centrifugal acceleration

Euler's

$\ddot{a}' = \ddot{\vec{a}}_f - \dot{\omega} \times (\dot{\omega} \times r) - 2\dot{\omega} \times \dot{v}' - \dot{\omega} \times r$

$F_r' = m a' = m a_f + F_c + F_C + F_E$

$F_c = -m\, \omega \times (\omega \times r)$ $\qquad \omega = \dfrac{2\pi}{24 \cdot 3600}\, s^{-1}$

$F_C = -2m\, \Omega \times v'$

$$F_E = -m\omega \times r$$

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17 Feb.

$$\eta_{ik} = \begin{pmatrix} -1 & & & 0 \\ & 1 & & \\ & & 1 & \\ 0 & & & 1 \end{pmatrix} = \eta^{ik}$$

flat spacetime

$$g_{ik} = \eta_{ik}\, a(t) + h_{ik}$$

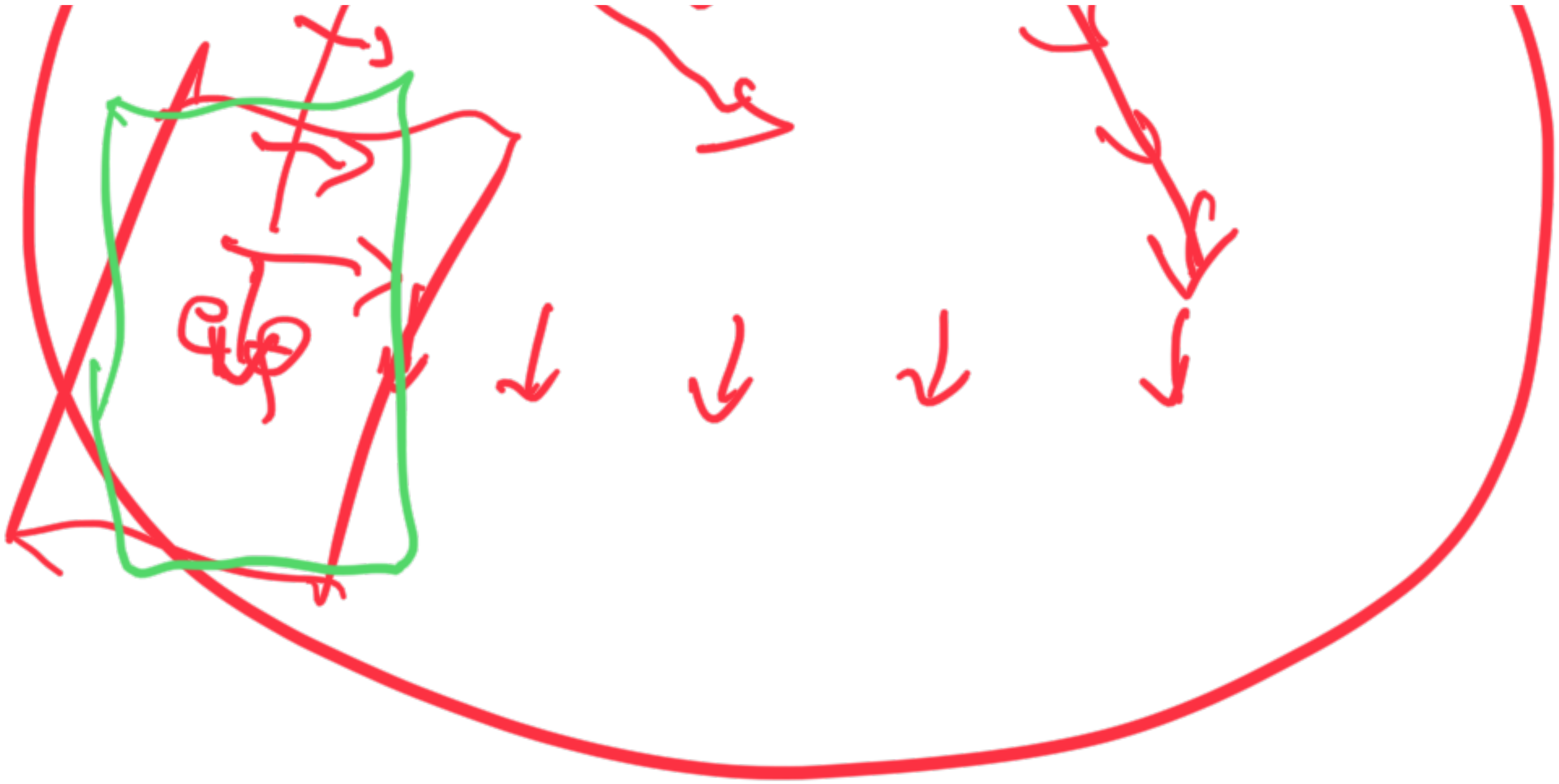
$0$

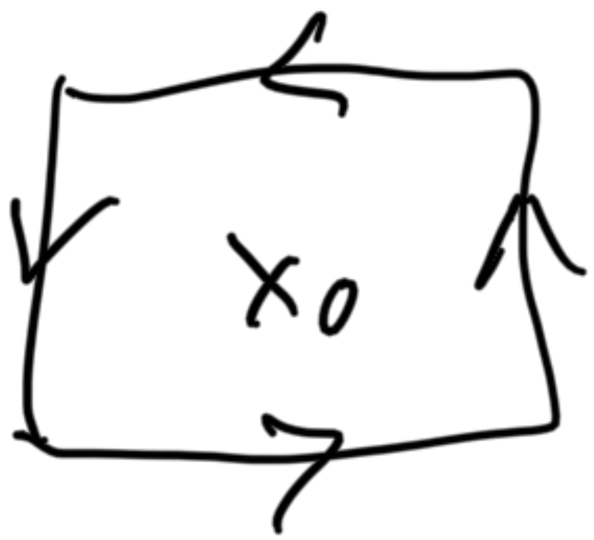
$$= a(t)\,\eta_{ik} + h_{ik}(\vec{x}, t)$$

scale factor

$$a_0 = a(\text{now}) = a(3.8\,10^9\,\text{yrs}) = 1$$

$$a(t) \simeq t^{\alpha} \qquad \alpha = \tfrac{2}{3}\ \text{mat}, \quad \tfrac{1}{2}\ \text{rad}$$






$$\eta = \begin{pmatrix} -1 & & \\ & 1 & \\ & & 1 \end{pmatrix}$$

$$R_{ie} = -\frac{1}{2}\Box h_{ie} + \cdots$$

can remove by a coordinate transformation

$$\Box = \nabla^2 - \partial_0^2$$

$$\Box = \partial^k \partial_k = \nabla^2 - \partial_0^2$$

$$x'^i = x^i + \epsilon^i(x)$$

$$\begin{pmatrix} 2 \\ 1 \\ 0 \\ -1 \\ -2 \end{pmatrix}$$

$$k^i \epsilon_i = 0$$

$$\vec{\nabla}\cdot\vec{A} = 0$$

$$\vec{\nabla}\cdot\vec{E} = 0 \qquad \vec{\nabla}\cdot\vec{B} = 0$$

$$k\cdot A$$

$$A_i = \sum_{k,\lambda} \epsilon_i(k,\lambda)\, a(k,\lambda)\, e^{ikx} + \epsilon^* a^\dagger(\quad) e^{-ikx}$$

$$\vec{k}\cdot\vec{\epsilon} = 0$$

$$H = \sum_{\vec{k}} \omega_k \left( a^\dagger(k)\, a(k) + \frac{1}{2} \right)$$

$$\langle 0 | H | 0 \rangle = \frac{1}{2} \sum_{k} \sqrt{\vec{k}^2 + m^2}$$

$\hbar = c = 1$

$$= \frac{1}{2} \int d^3k \, \sqrt{k^2 + m^2}$$

$\infty$

$$= 2\pi \int_0 k^2 dk \sqrt{k^2+m^2}$$

$$\simeq 2\pi \frac{\infty^4}{4} = \frac{\pi}{2}(\infty)^4 \quad *$$

$$\simeq \frac{\pi}{2} M_p^4 \sim (10^{19})^4$$

$$\langle H \rangle = (10^{-3} eV)^4$$

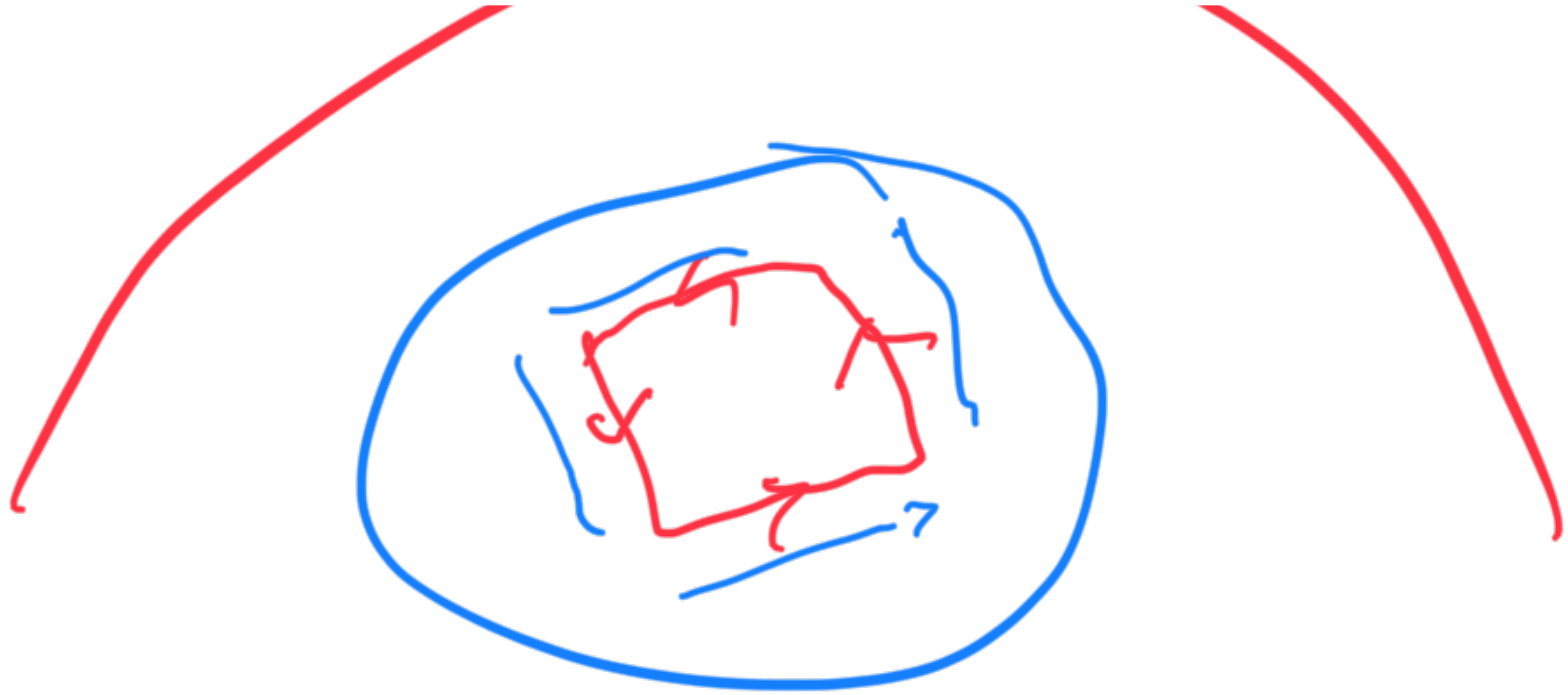
$$\left(\frac{10^{19}}{10^{-3}}\right) = 10^{22\cdot 4}$$

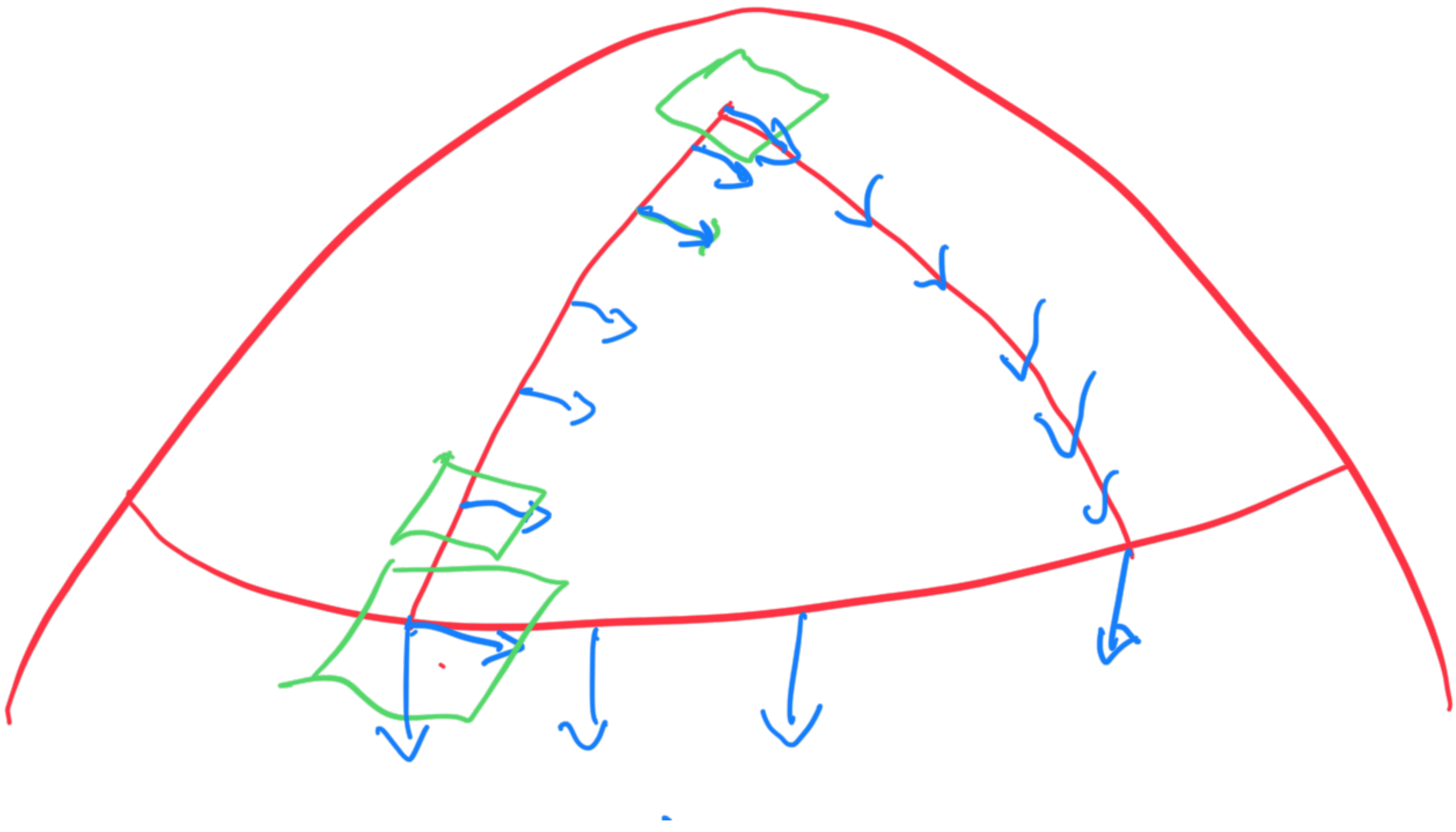
$$= 10^{88}.$$

28 Feb 2022

$$S = \frac{c^3}{16\pi G}\int (R + \Lambda)\sqrt{g}\, d^4x$$

cosmological constant





$$R = R^i{}_i$$

$$S \propto \int R \sqrt{g}\, d^4x$$

$$\propto \int (R+\Lambda) \sqrt{g}\, d^4x$$

$$g = |\det(g_{ik})|$$

$$\Lambda \overset{?}{=} \text{dark energy}$$

$$w = \sum_i \frac{p_i}{c^2 \rho_i}$$

Peebles

Steinhardt

Carroll

quintessence

$\Lambda \Leftrightarrow w = -1$

$$[a(p), a^\dagger(k)] = \delta^3(\vec{p} - \vec{k})$$

$$= \int \frac{d^3x}{(2\pi)^3} e^{i(\vec{p} - \vec{k}) \cdot \vec{x}}$$

$$\int \frac{d^3x}{} = V$$

$$[a(p), a^\dagger(p)] = \delta \quad (2\pi)^3 \quad \overline{(2\pi)^3}$$

$$\int^{P} d^3p \sqrt{p^2 + m^2}$$

$$= 4\pi \int_0^{P} dp \; p^2 \sqrt{p^2 + m^2}$$

$$\hat{=} \; 4\pi \int_0^{P} dp \, p^3 = 4\pi \frac{P^4}{4}$$

quantically divergent energy

$$\rho \approx 7 \times 10^{-30} \frac{g}{cm^3} = 7 \times 10^{-27} \frac{kg}{m^3}$$

$$( \quad {}^{-3} \quad )^4$$

$\simeq (2\times 10^{'}eV)$ $\hbar = c = 1$

$10^{-3} eV \sim m_\nu$

$m^2_{\nu 1} - m^2_{\nu 2} =$

$m^2_{\nu 2} - m^2_{\nu 3} =$

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Witten

exact susy $\Rightarrow E_0 = 0.$

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string theory

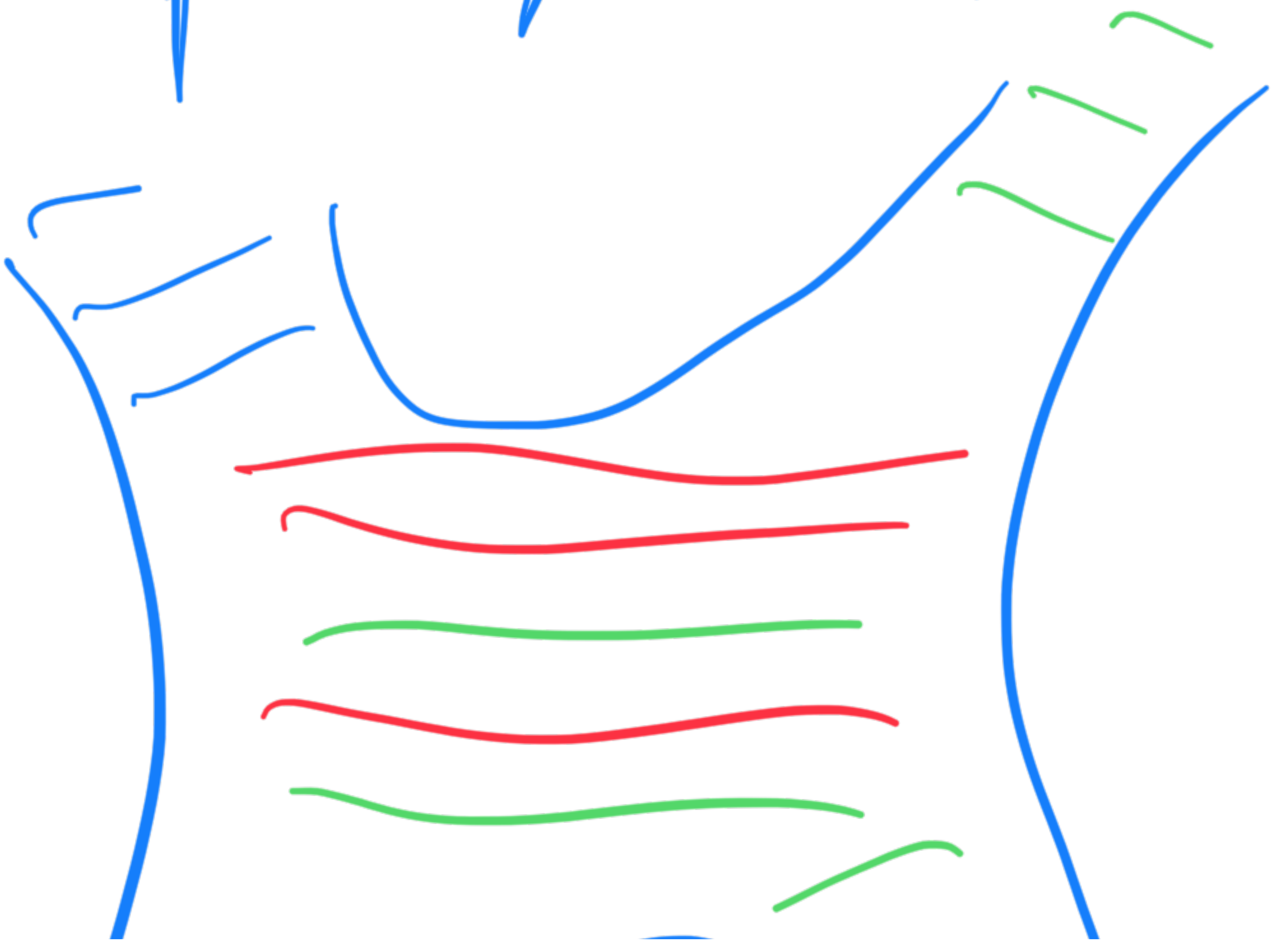
AVOIDS

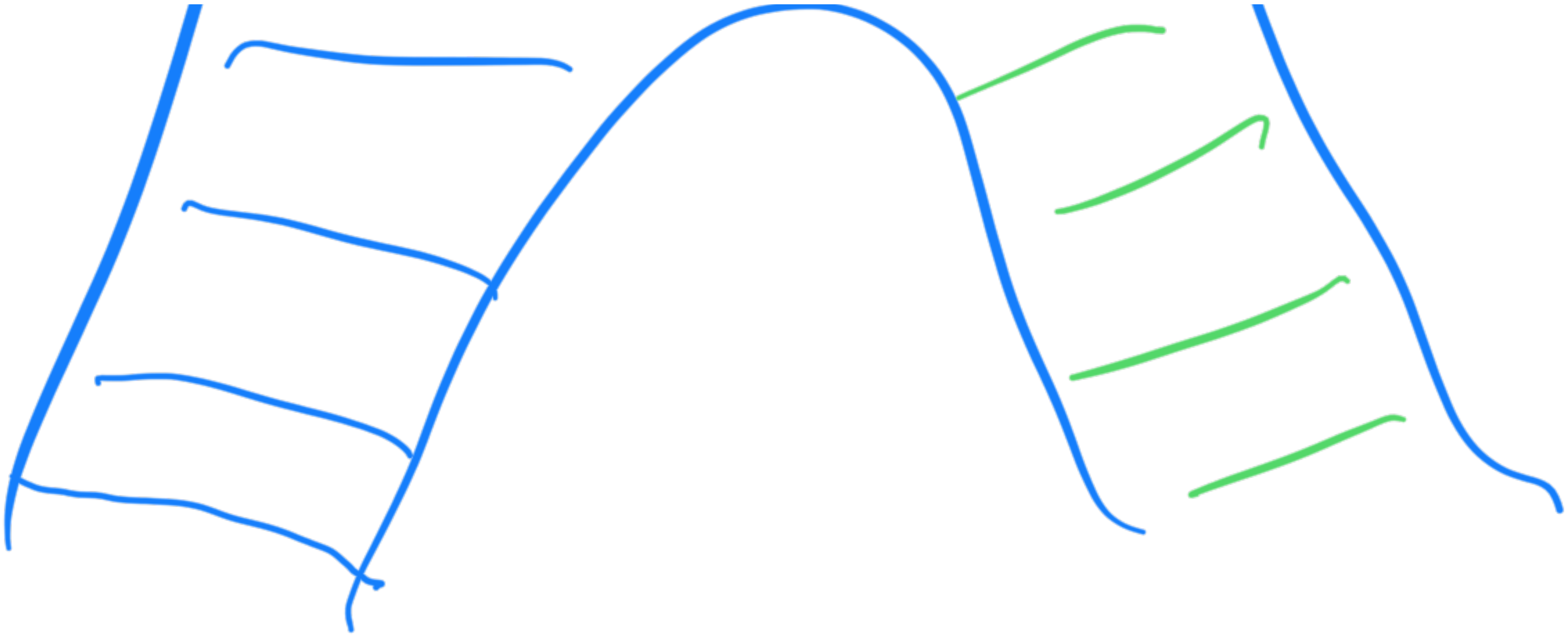
$$[\phi(x,t), \phi(y,t)] = i\,\delta^3(\vec{x-y})$$


21 March

$$a_k a_p^\dagger + a_p^\dagger a_k = \delta^3(p-k)$$

$$p^0 = \sqrt{m^2 + \vec{p}^{\,2}}$$





$$H = \omega\left(a^\dagger a + \frac{1}{2}\right)$$

$$a = \frac{q + ip}{\sqrt{2}}$$

$$H = \frac{\omega}{2}(\hat{q} - i\hat{p})(\hat{q} \ldots$$

$$= \frac{\omega}{2}(q^2 + p^2) + \frac{i\omega}{2}[q, p]$$

q
q ^
==

$$= \omega \quad a$$

$$= \omega\left(a^\dagger a + \frac{1}{2}\right) - \frac{\omega}{2}$$

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$$= \omega a^{+} a$$

$$\langle q | H | 0 \rangle = 0$$

$$\langle q | (q + i p) | 0 \rangle = 0$$

$$q \langle q | 0 \rangle + i \langle q | p | 0 \rangle = 0$$

$$q \langle q | 0 \rangle + \frac{d}{dq} \langle q | 0 \rangle = 0$$

$$\frac{d}{dq}\langle q|0\rangle = -q\,\langle q|0\rangle$$

$$\langle q|0\rangle = e^{-q^2/2}$$

$$H = (W - ip)(W + ip)$$

$$\frac{d}{dq}\langle q|0\rangle = -W_q\,\langle q|0\rangle$$

$$\langle q|0\rangle = e^{-\int W(q')\,dq'}$$

$$|\langle q|0\rangle| \to 0 \text{ as } q \to \infty$$

$$\int_0^q q'^3\,dq' = \frac{q^4}{4}$$

$$-\frac{q^4}{4}$$

$$\langle q|0\rangle = e$$

$\langle q | 0 \rangle \sim e$

$$\int_0^q q'^4 \, dq' = \frac{q^5}{5}$$

$$\langle q | 0 \rangle = e^{-q^5/5}$$

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$$H = \int dx\,(W - i\pi)(W + i\pi(x))$$

$$= \int d^3x\,(W - i\pi)\left(\sqrt{p^2+m^2} + i\phi + i\pi\right)$$

$$\pi(x) \to \frac{1}{i}\frac{\delta}{\delta\phi(x)}$$

$$\langle\phi|(W + i\pi)|\Omega\rangle = 0$$

$$\delta \quad \langle\phi|\Omega\rangle = 0$$

$$= W(\phi)\langle\phi|\Omega\rangle + \frac{\delta}{\delta\phi}\langle\ \ \ \rangle$$

$$\frac{\delta}{\delta\phi}\langle\phi|\Omega\rangle = -W\langle\phi|\Omega\rangle$$

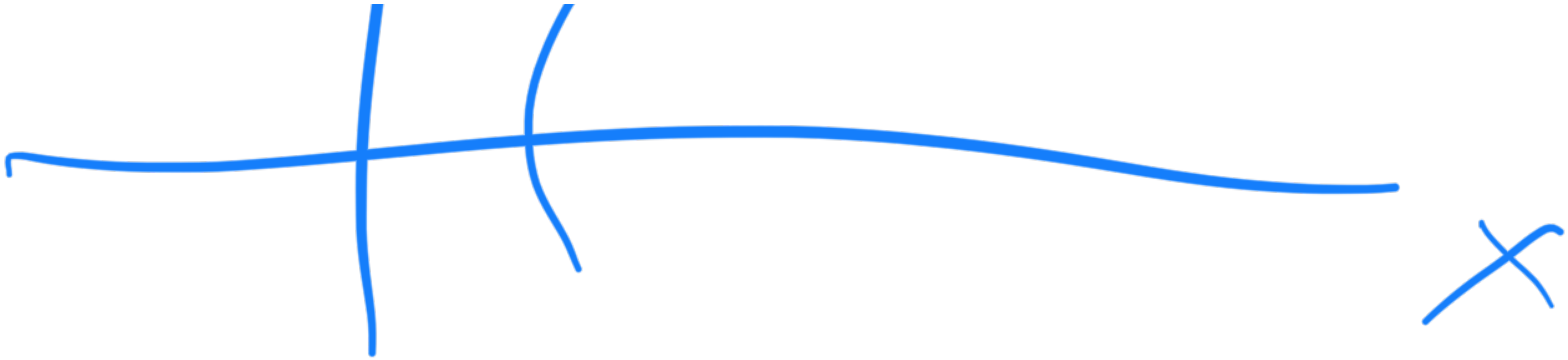
$$\int dx\, n(f+\epsilon h)^{n-1} h$$

$$\to \int n f^{n-1} h\, dx$$

$$G = \int dx\, f^{n}(x)$$

$$\frac{\delta G}{\delta f(y)} = \delta G[f][\delta_y]$$

$$\delta_y(x) = \delta(x-y)$$




$$\sqrt{dx^2+dy^2} = dx\sqrt{1 + \frac{dy^2}{dx^2}}$$

$$= dx\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$