G=

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 3 \\ 7 & 7 \end{pmatrix}$$

$$\vec{E}, \vec{B}$$

$$F_{ik} = \underline{\partial_i A_k - \partial_k A_i}$$

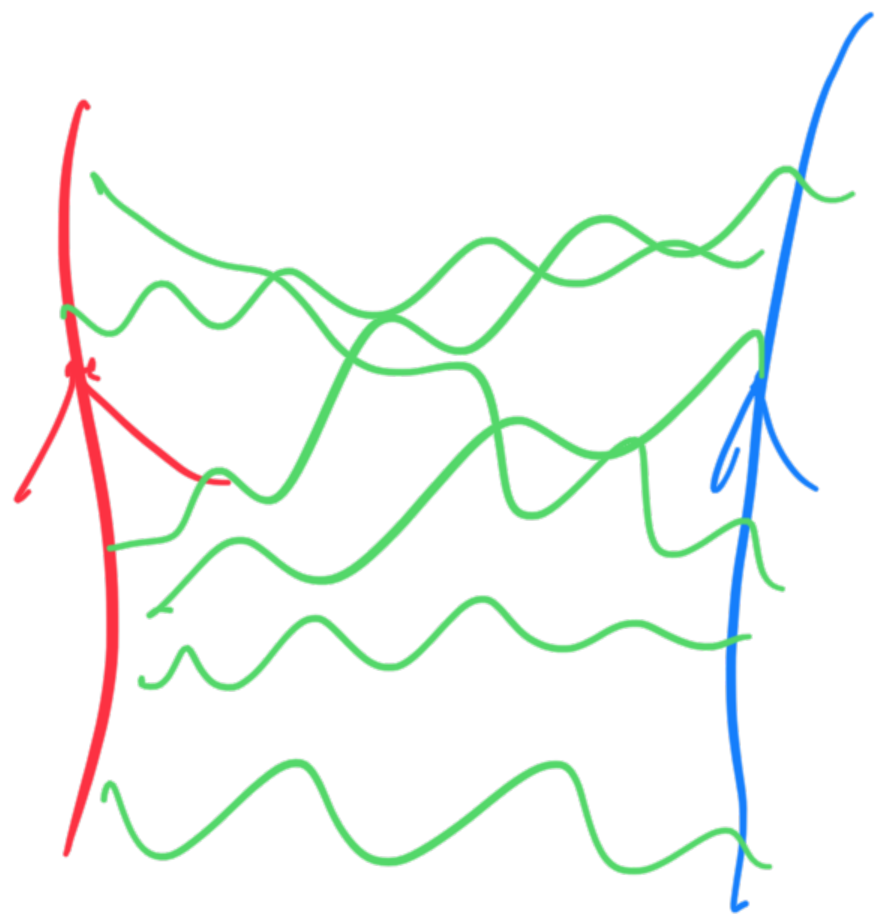
$$\begin{pmatrix} & E_1 & E_2 & E_3 \\ -E_1 & & B_1 & B_2 \\ -E_2 & -B_1 & & B_3 \\ -E_3 & -B_3 & -B_2 & \end{pmatrix} = F$$

quarks leptons

gauge fields

$A_i \rightarrow E, B$

$W^I, Z, G$



strong force

QCD

QED

$$\begin{pmatrix} u' \\ u' \\ u' \end{pmatrix} = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix} \begin{pmatrix} u \\ u \\ u \end{pmatrix}$$

3×3 unitary matrix

$$\text{prob} \sim |\psi(x)|^2 \, d^3x$$

$g$ = 3×3 unitary matrix

$$I = g^\dagger g = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$g^\dagger_{ij} = g^*_{ji}$$ hermitian adjoint

$$q' = g_1 q$$

$$q'' = g_2 q' = g_2 g_1 q = g_{21} q$$

$$g_2 g_1 = g_{21} \in SU(3)$$

SU(3) all 3 x 3 unitary matrices with det 1

$$|g| = 1, \quad g^\dagger g = I$$

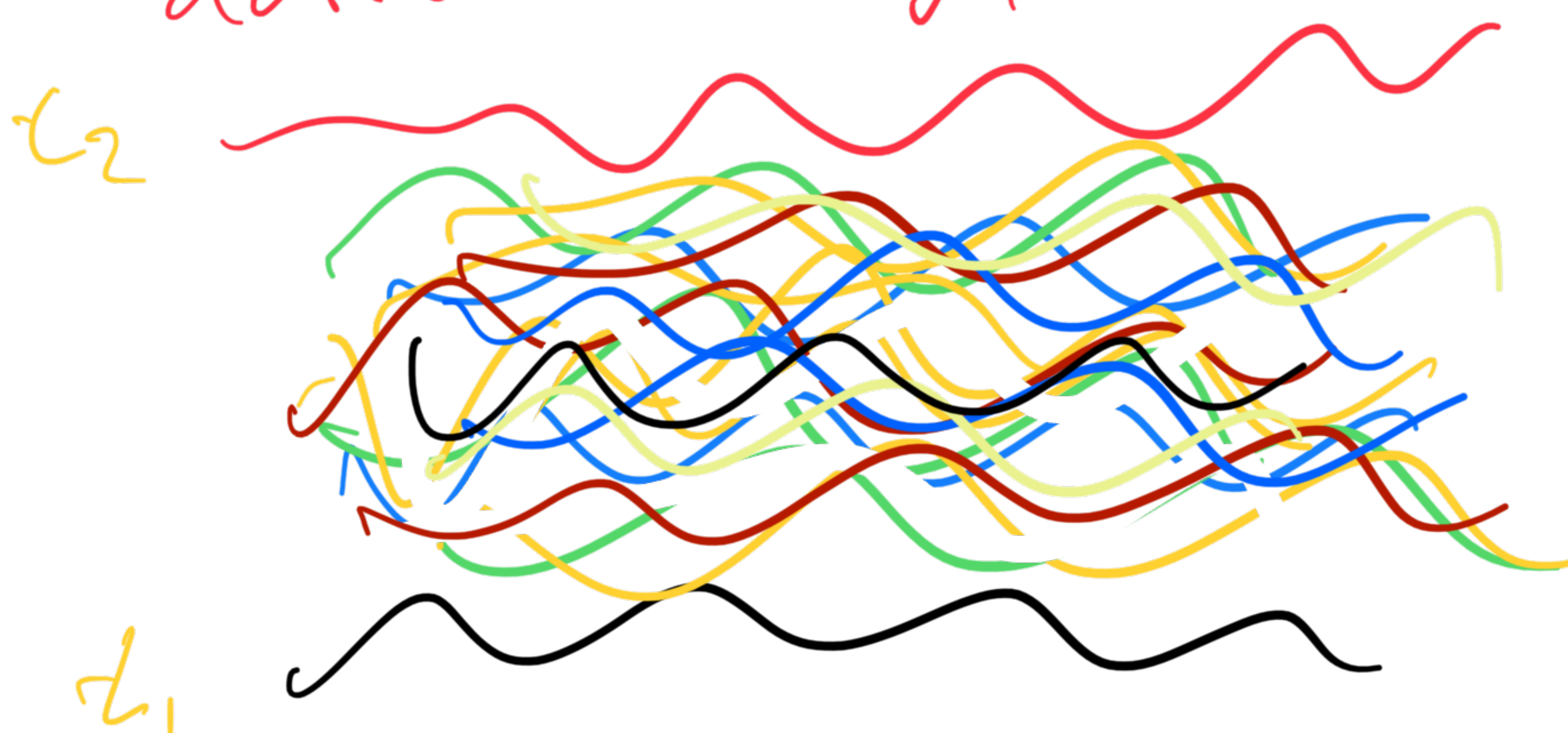
---

local symmetry

$$\text{Amplitude} = \boxed{\int e^{i\overset{\text{action}}{S}/\hbar} \, Dp}$$

$S[p]$ = classical action of the process $p$

action = Energy × Time




$$q'(x) = g(x)\, q(x)$$

$$x = (t, \vec{x})$$

$( \in SU(3))$

Can make action $S$ be unchanged under transformation $q(x) \to q'(x) = g(x) q(x)$.

$$S = \int d^4x \; q^\dagger q$$

$$= \int d^4x \; q'^\dagger q'$$

$$= \int d^4x \; q^\dagger g^\dagger g q$$

$$= \int d^4x \; q^\dagger q$$

$$\partial_i q = \frac{\partial q(x)}{\partial x^i}$$

$\partial_i ( g(x) q(x) )$

$$= \frac{\partial}{\partial x^i} \left( g(x) q(x) \right)$$

$$= \frac{\partial g(x)}{\partial x^i} q(x) + g(x) \frac{\partial q(x)}{\partial x^i}$$

$$S = \int d^4x \, \frac{\partial q^\dagger}{\partial x^i} \frac{\partial q}{\partial x^i}$$

$$q(x) \longrightarrow q'(x) = g(x) q(x).$$

local symmetry

$$\psi'(x) = e^{i\theta(x)} \psi(x)$$

$$A'_i(x) = A_i(x) + \frac{\partial \lambda(x)}{\partial x^i}$$

$$F_{ik} = \partial_i A_k - \partial_k A_i \qquad \partial_i = \frac{\partial}{\partial x^i}$$

$$F'_{ik} = \partial_i \left( A_k + \frac{\partial \lambda}{\partial x^k} \right) - \partial_k \left( A_i + \frac{\partial \lambda}{\partial x^i} \right)$$

$$= F_{ik} + \frac{\partial^2 \lambda}{\partial x^i \partial x^k} - \frac{\partial^2 \lambda}{\partial x^k \partial x^i}$$

$$= F_{ik}$$

$$\left[(\partial_i + iA_i)\psi\right]' = e^{i\theta(x)}(\partial_i + iA_i)\psi$$

$$= (\partial_i + iA_i')\psi'$$

$$= (\partial_i + iA_i + i\partial_i\lambda)e^{i\theta}\psi$$

$$= e^{i\theta}\partial_i\psi + \psi\,\partial_i(e^{i\theta}) + e^{i\theta} iA_i\psi + i\partial_i\lambda\, e^{i\theta}\psi$$

$$= e^{i\theta}(\partial_i + iA_i)\psi$$

$$\boxed{i\partial_i\lambda = -\partial_i\theta}$$

$$\boxed{\psi\, i\partial_i\theta\, e^{i\theta} - i(\partial_i\lambda)e^{i\theta}\psi} = 0$$

$$\partial_i \lambda = -\partial_i \theta$$

$$\partial_i \theta = \partial_i \lambda$$

$$\partial_i q \longrightarrow (\partial_i + iG_i) q$$

$$[(\partial_i + iG_i) q]' = g[(\partial_i + iG_i) q]$$

$g$ ← SU(3)

$$= (\partial_i + iG_i') q'$$

$$= g(\partial_i + iG_i) q$$

$$= (\partial_i + iG_i') g q = g(\partial_i + iG_i) q$$

$$= (\partial_i g) q + \cancel{g \partial_i q} + iG_i' g q$$

$$\cancel{\;} + g iG_i q$$

$= g \cancel{\partial_i} q + g' G_i' q$

$(\partial_i g) q + i G_i' g q = i g G_i q$

$\partial_i g + i G_i' g = i g G_i$

$\partial_i g g^{-1} + i G_i' = i g G_i g^{-1}$

$$\boxed{G_i' = g G_i g^{-1} - (\partial_i g) g^{-1}}$$

$i = 0, 1, 2, 3$

$3 \times 3$ matrix $G(x)$

$$G_i'(x) = g(x) G_i(x) g^{-1}(x) - \left( \frac{\partial g(x)}{\partial x^i} \right) g^{-1}(x)$$

$$g(x) = e^{i\theta_a t_a}$$

$$t_a^{\dagger} = t_a$$

$g(x) \in SU(3)$

$$e^{i\theta_a t_a} = g(x)$$

$$[t_a, t_b] = i f_{ab}^{c} t_c$$

$= t_a t_b - t_b t_a$

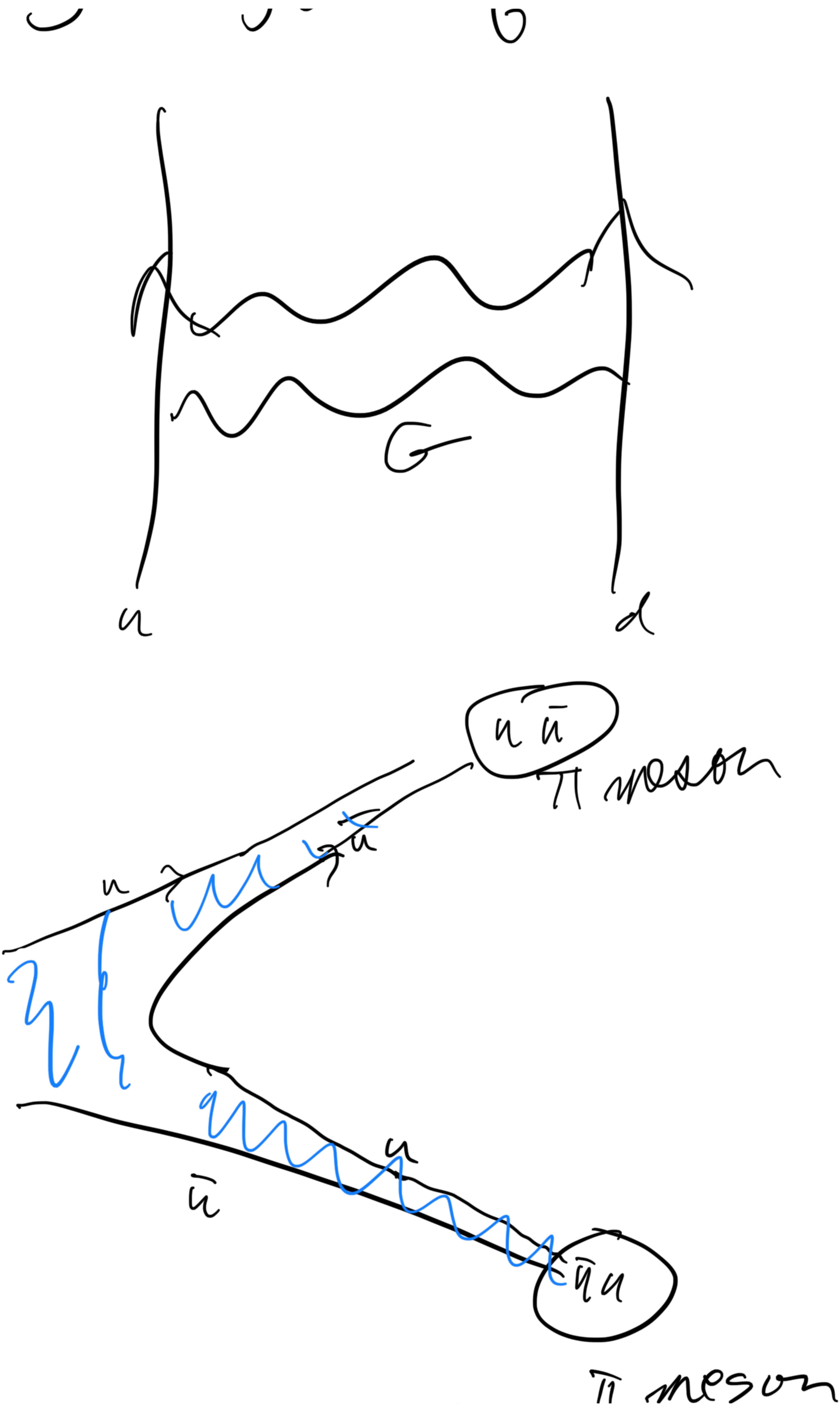
$3 \times 3$ hermitian matrices

generators

$$[A, B] = AB - BA$$

$f_{ab}^{c}$ structure constants

$$S = \int d^4x \, q^{\dagger} (\partial_i + G_i) q$$



G
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π meson

quarks confined
by strong force
ie by gluons.