Alternatively, we might simply examine the continuous time trajectory in $\mathbf{y}$. For example, Figure 1.17 shows a result for an experiment involving chemical reactions (cf. Section 2.4.3). The vertical axis is the measured concentration $g(t)$ of one chemical constituant at time t and the horizontal axis is the same quantity evaluated at $t-$ (8.8 seconds). We see that the delay coordinates $\mathbf{y} = (g(t), g(t-8.8))$ traces out a closed curve indicating a limit cycle.

Problems

1. Consider the following systems and specify (i) whether chaos can or cannot be ruled out for these systems, and (ii) whether the system is conservative or dissipative. Justify your answer

(*a*) $\theta_{n+1} = [\theta_n + \Omega + 1.5\sin\theta_n]$ modulo 2π,

(*b*) $\theta_{n+1} = [\theta_n + \Omega + 0.5\sin\theta_n]$ modulo 2π,

(*c*) $x_{n+1} = [2x_n - x_{n-1} + k\sin x_n]$ modulo 2π,

(*d*) $x_{n+1} = x_n + k(x_n - y_n)^2$, $y_{n+1} = y_n + k(x_n - y_n)^2$,

(*e*) $\mathrm{d}x/\mathrm{d}t = v$, $\mathrm{d}v/\mathrm{d}t = -\alpha v + C\sin(\omega t - kx)$,

(*f*) $\mathrm{d}x/\mathrm{d}t = B\cos y + C\sin z$

$\mathrm{d}y/\mathrm{d}t = C\cos z + A\sin x$

$\mathrm{d}z/\mathrm{d}t = A\cos x + B\sin y$.

2. Consider the one-dimensional motion of a free particle which bounces elastically between a stationary wall located at $x = 0$ and a wall whose position oscillates with time and is given by $x = L + \Delta\sin(\omega t)$. Derive a map relating the times T_n of the nth bounce off the oscillating wall and the particle speed v_n between the nth bounce and the $(n+1)$th bounce off the oscillating wall to T_{n+1} and v_{n+1}. Assume that $L \gg \Delta$ so that $v_n(T_{n+1} - T_n) \approx 2L$. Is the map relating (T_n, v_n) to (T_{n+1}, v_{n+1}) conservative? Show that a new variable can be introduced in place of T_n, such that the new variable is bounded and results in a map which yields the same v_n as for the original map for all n.

3. Write a computer program to take iterates of the Hénon map. Considering the case $A = 1.4$, $B = 0.3$ and starting from an initial condition $(x_0, y_0) = (0, 0)$ iterate the map 20 times and then plot the next 1000 iterates to get a picture of the attractor.

4. Plot the first 25 iterates of the map given by Eq. (1.8) starting from $x_0 = 1/2$; (*a*) for $r = 3.8$ (chaotic attractor), (*b*) for $r = 2.5$ (period one attractor), and (*c*) for $r = 3.1$ (period two attractor).

5. For the map (1.8) with $r = 3.8$ plot the iterates of the two orbits orginating from the initial conditions $x_0 = 0.2$ and $x_0 = 0.2 + 10^{-5}$ versus iterate number. When does the separation between the two orbits first exceed 0.2?